Model Predictive Controller Based on Laguerre Function for Binary Distillation Column

Milu Solaman[1], Riya Mary Francis[2]

[1]Department of Electrical and Electronics Engineering, TKM College of Engineering, Kollam

[2]Professor, Department of Electrical and Electronics Engineering, TKM College of Engineering, Kollam

Abstract: ***A performance improved Model Predictive Control for binary distillation column is proposed in this work. A MPC is designed to provide a timely control action ensuring optimum performance from the controller. Predictive control is now one of the most widely used advanced control methods in industry, especially in the control of processes that are constrained, multivariable and uncertain. In MPC, at each control interval an MPC algorithm attempts to optimize future plant behaviour by computing a sequence of future manipulated variable adjustments. The improvement in performance here we consider is the design of the MPC using Laguerre function, a discrete orthonormal basis function. It will help us in reformulating the predictive control problem and simplifying the solutions, in addition to tuning the predictive control system.***

Keywords: Binary distillation column, Model predictive controller (MPC), Laguerre function.

1. Introduction

Distillation column is considered to be the important process equipment widely used in petroleum industries. It is used for the separation of mixture of components by the application of heat. So that the mixture get separated based on each of its boiling point. Distillation column can be divided into batch and continuous based on how the feed is given into the system and can be binary and multi-component distillation column based on the number of products obtained from it.

The distillation column here considered is a binary distillation column which produces two products as distillate and bottom product specifically for methanol and water separation. The figure 1 shows a binary distillation column. The distillate and bottom product composition is maintained at a setpoint value by manipulating reflux flow rate from condenser and boilup flow rate of reboiler respectively. The disturbances that we have considered here is the feed flow rate and the feed composition. Usually a column has N trays including reboiler as first tray and condenser as last (N+1) tray and feed given at N_F.

The mathematical model of binary distillation column is extracted from Skogestad , [1] which involves the dynamics based on mass balance and liquid dynamics on each tray. The steady state value for binary distillation colum is taken from [3].A set of assumption s are used to reduce the complexity of the modelling [2]. The model predictive controller in [5],[4]and the Laguerre function axplanation in [4],[6],[7] was helpful for the work.

The Paper has been organised as follows. Section II deals with the mathematical modelling of binary distillation column. Section III deals with the theory and design of

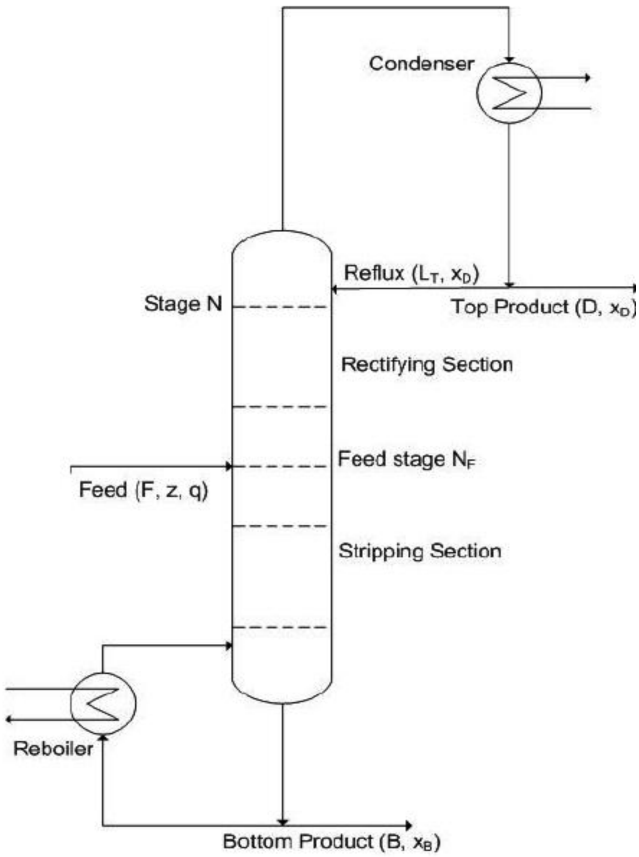

Figure 1: Binary distillation column

Model predictive controller. Section IV deals with the theory and design of Laguerre function based MPC controller. In Section V the Simulation results are shown with some discussions on it. Section VI is the Conclusion part.

2. Mathematical Modeling of Binary Distillation Column

The modelling of distillation column can be fundamental, empirical and hybrid modeling. The fundamental modeling gives the complete idea about the process dynamics, empirical one uses only input output data and no information about the inner dynamics. But the hybrid modeling combines the advantages of both fundamental and empirical modeling. The fundamental modelling is commonly used which can be simulated and understand the column dynamics.

Methanol-water distillation column considered has 20 trays including condenser tray and reboiler tray. The feed is given to the 10 th tray with flow rate F and feed composition Z_F. The dynamics of each tray is represented by dynamic equations. The model is formed based on some assumption [2]. They are

1) Binary mixture, the feed contains two components.
2) Pressure inside the column is fixed by controlling the cooling water.
3) Constant relative volatility, α=1.5
4) Constant molar flow
5) No vapor hold up, vapor hold up on each tray is negligible
6) Linear liquid dynamics
7) Equilibrium on all stage

Total Condenser (i=N+1):

$$M_D\dot{x}_i = V_{i-1}Y_{i-1} - L_iX_i - DX_i \tag{1}$$

Each tray (i=N_F +2 to N):

$$M_i\dot{x}_i = L_{i+1}X_{i+1} + V_{i-1}Y_{i-1} - L_iX_i - V_iY_i \tag{2}$$

Above feed location tray (i=N_F +1):

$$M_i\dot{x}_i = L_{i+1}X_{i+1} + V_{i-1}Y_{i-1} - L_iX_i - V_iY_i + F_VY_F \tag{3}$$

Below feed location tray (i=N_F):

$$M_i\dot{x}_i = L_{i+1}X_{i+1} + V_{i-1}Y_{i-1} - L_iX_i - V_iY_i + F_Lx_F \tag{4}$$

Each tray (i=2 to N_F-1):

$$M_i\dot{x}_i = L_{i+1}X_{i+1} + V_{i-1}Y_{i-1} - L_iX_i - V_iY_i \tag{5}$$

Reboiler (i=1):

$$M_Bx_i = L_{i+1}X_{i+1} + V_iY_i - BX_i \tag{6}$$

where M_B, M_D, M_i are liquid holdup on reboiler,condenser and on other trays.L_i, V_i are the liquid flow rate and vapour flow rate on each tray.x_i and y_i are composition of liquid and vapour components on each tray. The vapour-liquid equilibrium on each tray (i=1, N) can be expressed as

$$y_i = \alpha x_i/[(1+(\alpha-1)x_i] \tag{7}$$

Even though the model is simplified the vapour liquid relation on each tray makes the system non-linear due to vapour liquid equilibrium relationship (7). The vapour flow rate on each tray can be

Above feed (i >N_F):

$$L_i = L \text{ and } V_i = V + F_v \tag{8}$$

Where F_v is the vapour feed flow rate

Below feed ($i \le N_F$) :

$$L_i = L + F_L \text{ and } V_i = V \tag{9}$$

F_L is the feed liquid flow rate.

$$F_L = q_F \;\; and \;\; F_V = F - F_L \tag{10}$$

$$B = L_2 - V_1 = L + F_L \text{-V} \tag{11}$$

$$D = V_N - L = V + F_V - L \tag{12}$$

Compositions x_F and y_F in the liquid and vapor phase of the feed are obtained by solving the flash equations:

$$FZ = F_Lx_F + F_Vy_F \tag{13}$$

$$y_F = \alpha x_F/(1+(\alpha-1)x_F) \tag{14}$$

The simplified model for liquid flow dynamics with negligible vapor holdup are decoupled from the dynamic composition, which the formula for the liquid holdup on tray (*i =N_F+1 to N*) is:

$$L_i = L_0 + \frac{M_i - M_{0i}}{\tau} + (V_{i-1} - V_{0T})\gamma \tag{15}$$

and for tray (*i =2 to N_F*) is:

$$L_i = L_{0B} + \frac{M_i - M_{0i}}{\tau} + (V_{i-1} - V_{0T})\gamma \tag{16}$$

where L_0 is the initial reflux flow, $M_{0i}\,i$ is the initial reboiler holdup (kmol) on tray *i*, τ is the time constant for liquid dynamics. In this model it is chosen to be 0.063 min, and γ represents the effect of vapor flow on liquid flow which it ignore by setting γ=0. L_{0B} is the initial liquid flow below feed, given by the formula:

$$L_0 = L_{0B} + q_0 + F_0 \tag{17}$$

The initial feed rate F_0 is taken 1 kmol/min and q_0 = 1 is the initial liquid feed fraction. The state space model of the system is formed Taylor series linearization method [8], which is used in the predictive control design.

3. Theory of Model Predictive Controller

Model predictive controller uses open loop optimization for online feedback control.so it relies on both open loop and closed loop control. It tries to minimize the future predicted error and control moves by predicting the future plant response with the help of process model [5].

The set value is called target and the error is called the residuals, which is the difference between the actual and the predicted output. The residual act as the feedback signal to the prediction block. A process model is used to predict the current values of output. The control signal i.e. the change in the input variable can be calculated from the current measurement and the predicted output.

The actual output of the system is y and the predicted output is $y^\wedge$.At each sampling instant, k the MPC controller calculate M control inputs for P prediction horizon.

$$\{u(k+i-1)\,,\, i=\{1,2,3........P\}\} \tag{18}$$

So P predicted outputs are produced by the controller so that the output reaches the set value.

$$\{y^\wedge(k+i),\, i=\{1,2,3........P\}\} \tag{19}$$

The prediction horizon, P is the number for future control intervals the controller must evaluate by prediction when optimizing its manipulated variable at control interval k. The control horizon is the number of manipulated variable moves to be optimized at control interval k. Among the control moves the first one is implemented when a new measurement is available.

Model predictive control system designed here based on a mathematical model of the plant. The model to be used in the control system design is taken to be a state-space model. By using a state-space model, the current information required for predicting ahead is represented by the state variable at the current time. The state space model of the discrete system is defined by

$$\begin{aligned} x_m(k+1) &= A_mx_m(k) + B_mu(k) \\ y_m &= C_mx_m(k) \end{aligned} \tag{20}$$

where u is the manipulated variable and y the process output. Then by taking the difference of the state equation and producing the output based on that its formed the augmented model, which will be used in the design of the predictive control. The augmented model for MIMO system with *m* inputs and *q* outputs has the form

$$\begin{bmatrix}\Delta x_m(k+1)\\ y(k+1)\end{bmatrix}=\begin{bmatrix}A_m & o_m^T\\ C_mA_m & I_{qxq}\end{bmatrix}\begin{bmatrix}\Delta x_m(k)\\ y(k)\end{bmatrix}+\begin{bmatrix}B_m\\ C_mB_m\end{bmatrix}\Delta u(k)$$

$$y(k)=\begin{bmatrix}o_m & I_{qxq}\end{bmatrix}\begin{bmatrix}\Delta x_m\\ y(k)\end{bmatrix} \quad (21)$$

where $o_m=[0\ 0\ 0\ldots\ldots 0]$ which can be related to the equation (20), has size equal to the size of state matrix. The future control moves and the future state variables are calculated from the augmented matrix and based on this the output is predicted. Given by

$$Y=Fx(k_i)+\emptyset\Delta u \quad (22)$$

where the matrices F and $\emptyset$ can be formulated from the matrices of augmented state space model.

$$F=[C\quad CA^2\ \ldots\ldots\ldots\ldots\ldots\quad CA^{N_p}]^T \quad (23)$$

$$\emptyset=\begin{bmatrix}CB & 0 & 0 & \cdots & 0\\ CAB & CB & 0 & \cdots & 0\\ CA^2B & CAB & CB & \cdots & 0\\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \cdots & CA^{N_p-N_c}B\end{bmatrix} \quad (24)$$

The incremental optimal control with one optimization window is

$$\Delta u=(\emptyset^T\emptyset+\bar{R})^{-1}(\emptyset^T\overline{R_s}r(k_i)-\emptyset^TFx(k_i)) \quad (25)$$

where $\overline{R_s}$ is a matrix with ones has a dimension of mNpxq $\bar{R}$ is a block matrix with dimension equal to the dimension of $\emptyset^T\emptyset$ and $r(k_i)=r_1(k_1)\quad r_2(k_2)\ldots\quad r_q(k_i)$ is the *q* set point signals. The receding horizon control principle by taking the first m elements in Δu is

$$\Delta u(k_i)=\overbrace{[I_m\quad O_m\cdots\quad O_m]}^{N_c}(\emptyset^T\emptyset+\bar{R})^{-1}(\emptyset^T\overline{R_s}r(k_i)-\emptyset^TFx(k_i))$$

i.e $\Delta u(k_i)=K_yr(k_i)-K_{mpc}x(k_i)$ (26)

Identity and zero matrix has dimension equal to *m* x *m*

4. Model Predictive Control Based on Laguerre Function

The main idea in the design of discrete MPC is optimizing the difference of control signal $\Delta u(k_i)$ i.e. the future control trajectory. Usually we consider the $\Delta u(k)$ for k=0,1,2…..Nc-1.But there were some cases in which the rejected $\Delta u(k_i)$is not zero. So here we introduce a discrete orthonormal basis function i.e. discrete time Laguerre function into the design of predictive control system.

The discrete-time Laguerre network can be generated from the discretization of continuous-time Laguerre network. In the design of predictive control we use Laguerre function in time domain. A compact realization of function can be done based on state space realization of the networks. The set of discrete time Laguerre function expressed in vector form as

$$L(k)=[l_1(k)\quad l_2(k)\ldots.\quad l_N(k)]^T \quad (27)$$

The network satisfy the following difference equation,

$$L(k+1)=A_lL(k) \quad (28)$$

where A_l is an *N*x*N* matrix and the function of the parameters *a* and $\beta=(1-a^2)$,a is the pole of discrete Laguerre network also called the scaling factor required to select by the user. The value of *a* is $0<a<1$,for the stability of the network. The initial condition is given by

$$L(0)^T=\sqrt{\beta}[1\quad -a\quad a^2\ -a^3\quad \ldots\quad (-1)^{N-1}a^{N-1}] \quad (29)$$

$$\text{For N=5, } A_l=\begin{bmatrix}a & 0 & 0 & 0 & 0\\ \beta & a & 0 & 0 & 0\\ -a\beta & \beta & 0 & 0 & 0\\ a^2\beta & -a\beta & \beta & a & 0\\ -a^3\beta & a^2\beta & -a\beta & \beta & a\end{bmatrix} \quad (30)$$

In the design of predictive control system Laguerre functions are used to capture dynamic response of the process. The future samples at instant k can be

$$\Delta u(k_i+k)=\sum_{j=1}^{N}c_j(k_i)l_j(k) \quad (31)$$

k_i is the initial time of moving horizon window and k be the future sampling instant,N is the number of terms in expansion.In the vector form

$$\Delta u(k_i+k)=L(k)^T\acute{\eta} \quad (32)$$

where $\acute{\eta}=[c_1\quad c_2\ldots..\quad c_N]^T$

By minimizing it with the cost function

$$\acute{\eta}=-\Omega^{-1}\Psi x(k_i) \quad (33)$$

$$\Omega=\sum_{m=1}^{N_p}\emptyset(m)Q\emptyset(m)^T+R_L \quad (34)$$

$$\Psi=\sum_{m=1}^{N_p}\emptyset(m)QA^m \quad (35)$$

$$\emptyset(m)^T=\sum_{i=0}^{m-1}A^{m-i-1}BL(i)^T \quad (36)$$

$Q>0$ and $R_L>0$are the weight matrices, *Q* has a dimension equal to number of state variables and R_L has dimension equal to dimension of $\acute{\eta}$.

Then the receding horizon law can be

$$\Delta u(k)=-K_{mpc}x(k) \quad (37)$$

For SISO system,

$$K_{mpc}=L(0)^T\Omega^{-1}\Psi \quad (38)$$

But in the case of MIMO system

$$\acute{\eta}^T=[\acute{\eta}_1{}^T\quad \acute{\eta}_2{}^T\ \ldots.\quad \acute{\eta}_m{}^T] \quad (39)$$

$$K_{mpc}=\begin{bmatrix}L_1(0)^T & 0_2^T & \cdots & 0_m^T\\ 0_1^T & L_2(k)^T & \cdots & 0_m^T\\ \vdots & \vdots & \ddots & \vdots\\ 0_0^T & 0_2^T & \cdots & L_m(k)^T\end{bmatrix}\Omega^{-1}\Psi \quad (40)$$

The state feedback gain for the predictive controller is produced so as to maintain the process output in the reference point.

5. Simulation Results and Discussions

The binary distillation column modeled here is simulated in MATLAB and taken the step response. The steady state response shows a stable behaviour in which the distillate product, y_D reach steady state value of 98.9% and bottom product reach 4% of purity in composition. The steady state behaviour is shown below in fig.6.

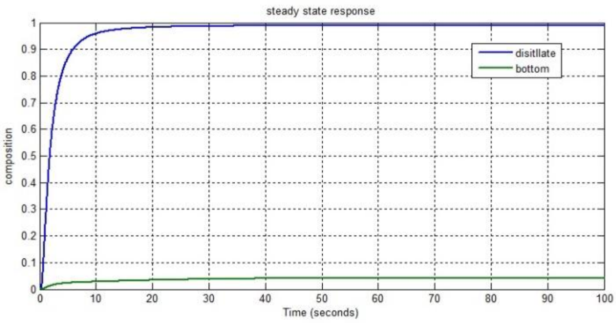

Figure 2: Steady state response

The system attains steady state response only when no disturbance affects the system. The disturbance factors the we consider here is the feed composition and feed flow rate disturbance. If there is any change occurs in the nominal value of feed flow rate and feed composition the product composition will not attain the steady state value. An increase or decrease in the feed composition or flow rate cause changes in the composition of final products. Now it is considered a 10% change in the disturbances. An increase in feed flow rate F, make large increase in bottom product and small change in top product and decrease in feed flow rate F, cause large drop in both top and bottom product. The disturbance in feed composition were felt very quickly in the bottom of the column. The effect of feed composition disturbance is more than feed flow rate disturbance.Fig.3 shows an example. So to maintain the composition setpoint at the desired value an MPC controller is used.

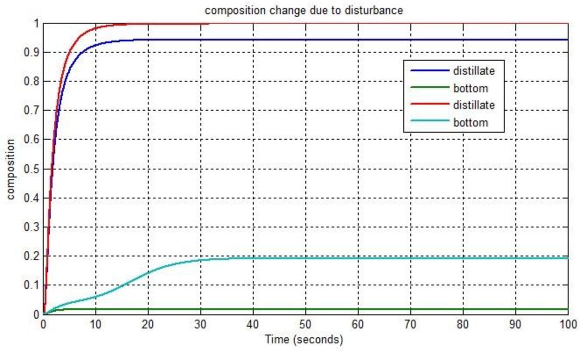

Figure 3: Change in composition with disturbance

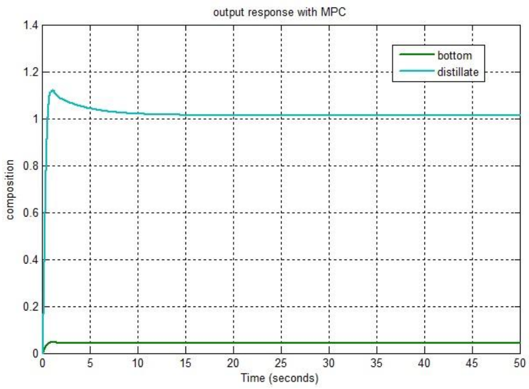

Figure 4.Output response with MPC

The predictive controller designed here is a discrete state feedback MPC. The output response shown in fig,4 has a steady sate error of some minute value. In this case we consider that all the states are available for the measurement. The fig 9 gives the response with an observer, which shows a better response. The steady state error is reduced

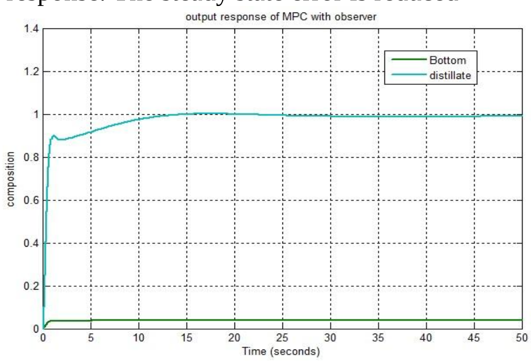

Figure 5: Output response of MPC with observer

The below given fig.10, 11 is the response with the MPC with the Laguerre function. In the case of MPC we take only future trajectory of up to N_c moves. In some case the rejected trajectory may not be zero. Such situation cause the deviation of output from the set point. so we use Laguerre function to make all the trajectory to zero. Also the performance of the MPC is increased shown by the below response. The advantage of using this method is that we can use separate scaling factor and control horizon for each input, which is useful in the case of a MIMO system.

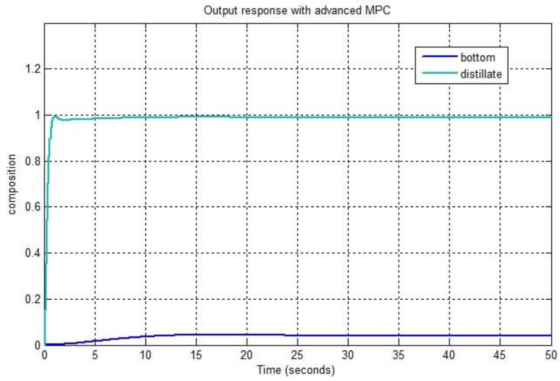

Figure 6: Output response of MPC with Laguerre function

6. Conclusion

In this paper, a MPC controller based on Laguerre function is formed for a binary distillation column. MPC is an advanced control strategy which can be very effectively used for a multivariable control. In case of high process dynamics and closed loop performance the satisfactory approximation of $\Delta u(k)$ need a large number of samples. By adding Laguerre function into MPC, a long control horizon can be realised without using large number of parameter samples, also we have the freedom to choose the parameter that suits the input of the system. We can say that the performance of the MPC with the Laguerre function is better than the conventional MPC.

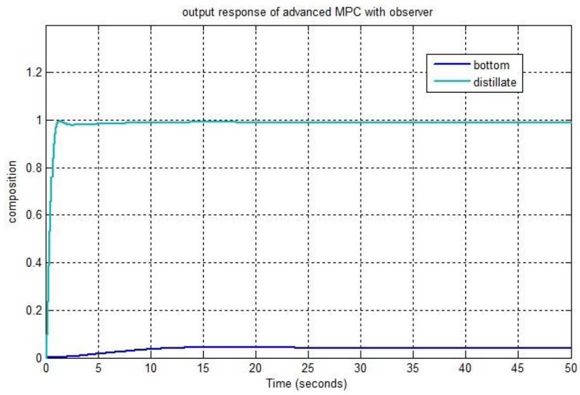

Figure 7: Output response of Laguerre MPC with observer

References

[1] S.Skogestard ," Dynamics and control of distillation column-Acritical survey", IFAC symbisium DYCORD+'92,Maryland April 27-29 ,1992.

[2] H. S. Truong, I. Ismail and R. Razali," Fundamental modelling and simulation of a binary continuous

ISSN (Online): 2347-3878, Impact Factor (2014): 3.05

distillation column", IEEE International conference on intelligent and advanced systems, 2010.
[3] R.H Perry, D.W Green and J.O Maloney," Perry's Chemical Engineer's Handbook 7 ed McGraw Hill,", 1997.
[4] Liuping Wang" Model predictive control design and implementation using MATLAB", Springer
[5] Zhiyun Zou and Dehong Yu, Zhen Hu , Ning Guo, Luping Yu and Wenqiang Feng, "State space modelling and predictive control of a binary batch distillation column", Proceedings of 6 th world congress on intelligent control and automation ,June 2006.
[6] Hai-Tao Zang,Chen Zonghai,Yogji Wang,"Adaptive predictive control algorithm based on laguerre function", International journal od adaptive control and signal proessing,March 2006
[7] M.A Masnadi-Shirazi,M.Ghazemi,"Adaptive Laguerre network realization", Signal processing Elsevier,2000
[8] Sigurd Skogestard and Manfred Morai,''Understanding the dynamic behaviour of distillation column'',California Institute of Chemical Engineering 206-41,Pasadena,Califronia 91125

Author Profile

Milu Solaman was born in Kerala,India in the year 1990 has received his B.Tech degree from College of Engineering Kidangoor, Kottayam, kerala in 2012.She is pursuing his M.tech degree in Industrial Instrumentation and Control from TKM College of Engineering, Kollam. Her research interest include control theory and process control.