

# Non-Linear Adaptive Control on Reusable Launch Vehicle Due to Actuator Stuck

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**Abstract:** *In this paper a nonlinear Lyapunov based controller is designed to compensate for the actuator stuck in Reusable Launch Vehicle during its descent phase. During re-entry phase, atmospheric contact forces become comparable with the gravitational forces. In order to cope up with the non linearities and uncertainties of the system, the control strategy based on adaptive state feedback and backstepping control is employed. Simulation results show that the proposed controller works satisfactorily.*

**Keywords:** Adaptive Backstepping controller (ABC), Angle of Attack, Backstepping controller, Reusable Launch Vehicle (RLV), Side slip angle, Yaw rate

## 1. Introduction

The flight of a Reusable Launch Vehicle during its descent phase is subjected to a huge variation in Mach numbers and adverse flight envelopes and the system must be stabilized in the midst of these uncertainties. Reusable Launch Vehicle is a launch system used for deploying artificial satellites into space for multiple times. The re-entry phase of RLV is very critical due to enormous amount of heat generation during re-entry phase and for high structural load. During re-entry phase, atmospheric contact forces become comparable with the gravitational forces. To address these critical issues of reentry system and to guide the vehicle along the re-entry path given by guidance algorithm, the vehicle has to be equipped with a high performance and reliable flight control system. In the literature, not many non-linear control schemes have been proposed for the control of a Reusable Launch Vehicle. Though powerful linear design tools are available, nonlinear design tools such as feedback linearization, gain scheduling, and backstepping can come up with accurate results retaining useful nonlinearities.

Benaskeur A and Desbiens [1] proposed a nonlinear Lyapunov based controller where inner loop uses a backstepping approach to stabilize the inverted pendulum. Adaptive backstepping controller is designed by treating every constant parameters in the system as unknowns in [2]. Flight control design is discussed in [3]. Adaptive position control for an electrohydraulic actuator based on adaptive backstepping control scheme is proposed in [4]. Adaptive backstepping design for strict feedback systems are proposed by Kritic et al, [5]. The autopilot design for reentry vehicle is discussed in [7]. Adaptive backstepping control of RLV is proposed in [8]. Backstepping control design has been proposed for electrohydraulic servo system and spacecraft attitude control in [9] [10].

In this paper Adaptive Backstepping controller is designed to compensate for the fault due to actuator stuck in Reusable Launch Vehicle for lateral motion.

The Paper has been organized as follows. Section 2 deals with the modelling of Reusable Launch Vehicle (RLV). Section 3 deals with the Theory of controller design. Section 4 gives a detailed description of the controller design for lateral motion. In Section 5 the Simulation results are shown with some discussions on it. Section 6 is the Conclusion part.

## 2. Modelling of Reusable Launch Vehicle

The system under consideration is a Reusable Launch Vehicle (RLV) during its re-entry phase [6]. During this time, the aerodynamic forces become comparable to the gravitational forces. The equation of motion can be derived from Newton's Second law of motion

$$\begin{aligned}\sum F &= \frac{d}{dt} (m V_T) \\ \sum M &= \frac{dH}{dt}\end{aligned}\quad (1)$$

Where  $m$  is the mass of the aircraft,  $V_T$  is the terminal velocity of the aircraft.  $M$  is the external moment acting on the body and  $H$  is the angular momentum.

A conventional aircraft is usually equipped with three control surfaces viz: a rudder, an elevator, and an aileron. But, the X-38 has only two sets of control surfaces: rudders and elevons. The X-38 vehicle has a pair of rudders on top of each of the two vertical fins, a pair of elevon on lower rear of the vehicle. Each surface can be controlled independently to obtain the required control action.

The elevon angle for pitch control is given by averaging the elevon deflections.

$$\delta_e = \frac{(\delta_{eL} + \delta_{eR})}{2}\quad (2)$$

The aileron angle for roll control is given by taking the average of the difference of the elevon deflections.

$$\delta_a = \frac{(\delta_{eL} - \delta_{eR})}{2}\quad (3)$$

Similarly, the total rudder angle for yaw control is given by taking the average of the rudder deflections.

$$\delta_r = \frac{(\delta_{rL} + \delta_{rR})}{2} \quad (4) \quad \zeta_k' = f_k(x, \zeta_1, \dots, \zeta_k) + g_2(x, \zeta_1, \dots, \zeta_k)u \quad (11)$$

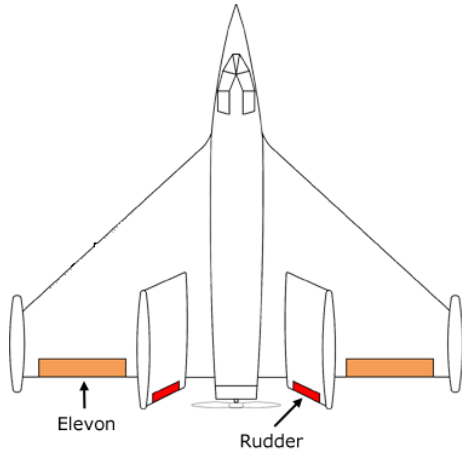


Figure 1: Control Surfaces

The non-linear set of equations of which describes the motion of the vehicle is as follows [6]:

$$\dot{p} = \frac{1}{I_{xx}}(L_{\beta} \sin \beta + L_p P + L_{\delta a} \sin \delta a + L_{\delta r} \sin \delta r + L_r r) \quad (5)$$

$$\dot{q} = \frac{1}{I_{yy}}(M_{\alpha} \sin \alpha + M_q q + M_{\delta a} \sin \delta a + M_{\delta r} \sin \delta r) \quad (6)$$

$$\dot{r} = \frac{1}{I_{zz}}(N_{\beta} \beta + N_r r + N_{\delta a} \delta a + N_{\delta r} \delta r + N_p p) \quad (7)$$

$$\dot{\alpha} = \frac{Z_{\alpha}}{V_T} \sin \alpha - g \frac{\sin \gamma}{V_T} \theta + \left( \frac{Z_{\alpha}}{V_T} + 1 \right) q \quad (8)$$

$$\dot{\beta} = \frac{Y_{\beta} \beta}{V_T} + \frac{Y_{\beta p}}{V_T} + \left( \frac{Y_r}{V_T} - 1 \right) r + \frac{g}{V_T} \phi \quad (9)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (10)$$

In the above set of equations p, q and r are the roll rate, pitch rate and yaw rate respectively. Here,  $\alpha$  is the angle of attack,  $\beta$  is the side slip angle, and  $\gamma$  is the flight path angle. The equations are represented in terms of aerodynamic forces and moments, where L is called the rolling moment, M the pitching moment and N is the yawing moment.  $I_{xx}, I_{yy}, I_{zz}$  are the moment of inertia in the x, y and z directions respectively.  $\phi$  is the roll angle, g is the acceleration due to gravity and  $V_T$  is the vehicle velocity. The coefficients Y and Z in the equations represents side force and downward force respectively. The dependence of the aerodynamic forces on the angle-of-attack and the side slip angle is crucial to stability and control.

### 3. Theory of Controller Design

#### 3.1 Basic Backstepping Technique

Backstepping designs by breaking down complex nonlinear systems into smaller subsystems, then designing control Lyapunov functions and virtual controls for these subsystems and finally integrating these individual controllers into the actual controller, by stepping back through the subsystems [5].

Consider a system of the form

$$\dot{x} = f(x) + g(x)\zeta_1$$

$$\zeta_1' = f_1(x, \zeta_1) + g_1(x, \zeta_1)\zeta_2$$

$$\zeta_2' = f_2(x, \zeta_1, \zeta_2) + g_2(x, \zeta_1, \zeta_2)\zeta_3$$

⋮

#### 3.2 Adaptive Backstepping Controller Design

Adaptive Backstepping Controllers are dynamic and more complex than the static controllers. What is achieved with this complexity is that, an Adaptive Backstepping Controller guarantees not only that the plant 'x,' remains bounded, but also regulation and tracking of a reference signal. In its basic form, the Adaptive Backstepping Control design employs overparametrization and this means that the dynamic part of the controller is not of minimal order. Consider

$$\begin{aligned} x_1' &= x_2 + \theta \phi(x_1) \\ x_2' &= u \end{aligned} \quad (12)$$

Where  $\theta$  is a known constant parameter and  $x_2$  as the first control input. Denote  $\theta_0$  as the estimated value for the parameter  $\theta$  and the estimation error  $\theta_e$  is given by

$$\theta_e = \theta - \theta_0 \quad (13)$$

Next the candidate Lyapunov function is selected as

$$v_2(x, \theta_e) = \frac{1}{2} x_1^2 + \frac{1}{2\gamma} \theta_e^2 \quad (14)$$

Where  $\gamma$  is the adaptation gain. With the control law.

$$x_2 = -k_1 - \theta \phi(x_1) = \alpha_1(x_1, \theta) \quad (15)$$

And the adaptation law

$$\dot{\theta}_0 = \gamma \phi(x_1) x_1 \quad (16)$$

The derivative of the candidate Lyapunov function becomes negative definite and is given by

$$\dot{v}_1 = -k_1 x_1^2 < 0 \quad (17)$$

In the eqn. (15)  $\alpha_1$  is called a stabilizing function for  $x_2$

The deviation of  $x_2$  from the stabilizing function is given by

$$Z = x_2 - \alpha_1(x_1, \theta) \quad (18)$$

Augmenting the Lyapunov function by adding the error Variable

$$v_2(x, Z, \theta_e) = v_1(x_1, \theta_e) + \frac{1}{2} Z^2 \quad (19)$$

By the proper selection of 'u,' the overall Lyapunov function '  $V_2$ ,' becomes negative definite which implies that as  $x_1$  tends to zero, then z also tends to zero asymptotically.

#### 4. Adaptive Backstepping Controller Design for Lateral Dynamics

The side slip angle  $\beta$  and the yaw rate r are used to completely define the lateral dynamics of the system. The equations of motion for the lateral dynamics are as follows

$$\begin{aligned} \dot{\beta} &= \frac{Y_{\beta}}{V_T} \sin \beta - x_2 \\ \dot{r} &= \frac{1}{I_z} [N_{\beta} \sin \beta + N_{\delta r} \sin \delta r] \end{aligned} \quad (20)$$

The state variables are selected as  $x_1 = \beta, x_2 = r$ . the control variable is selected as  $u = \delta_r$ . The system equation can now be expressed as

$$\begin{aligned} \dot{x}_1 &= \frac{Y_{\beta}}{V_T} \sin x_1 - x_2 \\ \dot{x}_2 &= \frac{1}{I_z} [N_{\beta} \sin x_1 + N_{\delta r} \sin u] \end{aligned} \quad (21)$$

In the case of actuator stuck

$$U = 2\delta r \quad (22)$$

The system dynamics now becomes

$$\begin{aligned} \dot{x}_1 &= \frac{Y\beta}{V_t} \sin x_1 - x_2 \\ \dot{x}_2 &= \frac{1}{I_z} [N_\beta \sin x_1 + N_{\delta r} \sin 2u] \end{aligned} \quad (23)$$

The control law is to be designed such that the system stabilizes for whatever be the initial conditions. For applying the Adaptive Backstepping Control design procedure, the system can now be expressed as

$$\begin{aligned} \dot{x}_1 &= \Phi_1 \sin x_1 - x_2 \\ \dot{x}_2 &= \Phi_2 \sin x_1 + \Phi_3 \sin 2u \end{aligned} \quad (24)$$

Where  $\Phi_1, \Phi_2, \Phi_3$  are the unknown parameters in the system.

The first error variable is defined as

$$e = x_1 - \theta_{sp} \quad (25)$$

Where  $\theta_{sp}$  is the desired set point using the Lyapunov function

$$V_1 = \frac{1}{2} e^2 \quad (26)$$

And using the derivative of the Lyapunov function, the virtual control law can be formulated as

$$x_{2des} = \Phi_1 \sin x_1 + k_1 e - \theta_{sp}' \quad (27)$$

Where  $k_1 > 0$  and is a design parameter which guarantees  $\dot{V}_1 < 0$ . The second error variable  $\zeta$  is defined as

$$\zeta = x_2 - x_{2des} \quad (28)$$

By augmenting the Lyapunov function  $V_1$  with the error variable  $\zeta$  and the unknown parameters in the system, we get

$$V_2 = \frac{1}{2} e^2 + \frac{1}{2} \zeta^2 + \frac{1}{2\gamma_1} \Phi_{1e}^2 + \frac{1}{2\gamma_2} \Phi_{2e}^2 + \frac{1}{2\gamma_3} \Phi_{3e}^2 \quad (29)$$

Where  $\Phi_{1e}, \Phi_{2e}, \Phi_{3e}$  are the parameter estimation errors of  $\Phi_1, \Phi_2, \Phi_3$  where  $\Phi_{*e} = \Phi - \Phi_{*0}$  and \* stands for 1, 2, 3. The variables  $\Phi_{10}, \Phi_{20}, \Phi_{30}$  are the parameter estimates with  $\gamma_1, \gamma_2, \gamma_3$  are the adaptation gain constants. With the control law

$$u_{des} = \frac{1}{2\Phi_{30}} \sin^{-1}(-k_2 \zeta + \theta_{sp}'' + k_1 \dot{e} + \Phi_{10} \cos x_1 \sin x_1) \quad (30)$$

And the parameter update laws given by

$$\begin{aligned} \dot{\Phi}_{10} &= -\gamma_1 \zeta \cos x_1 \\ \dot{\Phi}_{20} &= -\gamma_2 \zeta \sin x_1 \\ \dot{\Phi}_{30} &= \gamma_3 \zeta \sin 2u \end{aligned} \quad (31)$$

The derivative of the augmented Lyapunov function becomes negative definite

$$V_2' = -k_1 e^2 - k_2 \zeta^2 \leq 0 \quad (32)$$

Where  $k_1 > 0; k_2 > 0$ . Therefore by Laselles theorem, the system is globally asymptotically stable at the equilibrium point of the system.

## 5. Simulation Results and Discussion

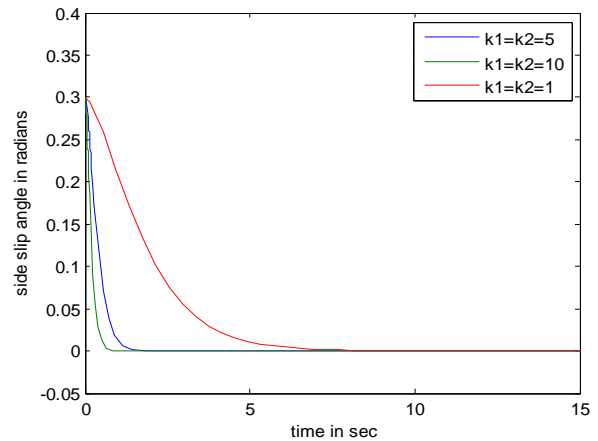


Figure 2: Side slip angle vs time

Fig. 2 shows the variation of side slip angle with time in seconds for an initial value of 17.18 degrees and different values of  $k_1$  and  $k_2$ . Fig. 3 shows the variation of yaw rate with respect to  $k_1$  and  $k_2$ .

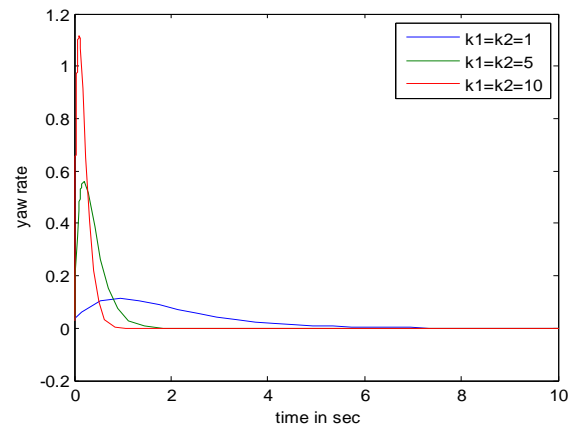


Figure 3: Yaw rate vs time

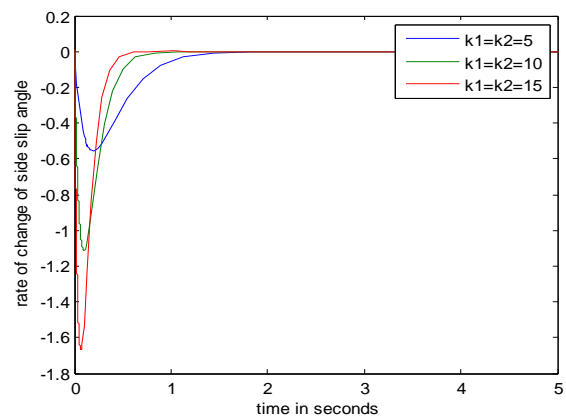
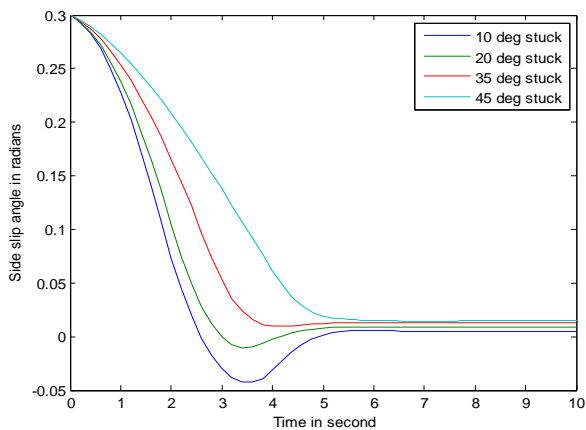


Figure 4: Rate of change of Side slip angle



**Figure 5:** Actuator stuck error tolerance for lateral motion

Fig. 4 shows the variation of rate of change of side slip angle for different values of  $k_1$  and  $k_2$ . Fig. 5 shows the fault tolerance of RLV for lateral motion at different values of actuator stuck. It is inferred that the system tolerates the fault within 5% upto an actuator stuck upto 45 degree.

## 6. Conclusion

In this paper a nonlinear adaptive control has been implemented on Reusable Launch Vehicle. Apart from the Backstepping design procedure in which only non-linearities had been taken care of, in the Adaptive Backstepping design uncertainties associated with the constant parameters of the system is also dealt with. Simulation results shows that the proposed controller compensates for fault due to actuator stuck within acceptable tolerance level. The relatively large estimation time and over parameterization are the two disadvantage of this control scheme.

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