Weakly b-δ Open and Closed Functions

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Abstract: ***In this paper, we introduce and study two new classes of functions called weakly b-δ -open functions and weakly b-δ-closed functions by using the notions of b-δ-open sets and b-δ-closed sets. The connections between these functions and other related functions are investigated***

Keywords: b-open set, δ -open set, b- δ-open set, weakly b- δ -open function, weakly-b- δ -closed function

1. Introduction

The notions of δ -open sets, δ -closed set where introduced by Velicko [11] for the purpose of studying the important class of H-closed spaces. 1996, Andrijevic´ [3] introduced a new class of generalized open sets called b-open sets in a topological space. This class is a subset of the class of β-open sets [1]. Also the class of b-open sets is a superset of the class of semi-open sets [5] and the class of preopen sets [6]. The purpose of this paper is to introduce and investigate the notions of weakly b-δ-open functions and weakly b-δ-closed functions. We investigate some of the fundamental properties of this class of functions. we recall some basic Definitions and known results.

2. Preliminary

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space (X, τ). We denote closure and interior of A by cl(A) and int(A), respectively.

Definition 1.1. A subset A of a space X is said to be b-open [3] if A ⊆ cl(int(A)) ∪ int(cl(A)).The complement of a b-open set is said to be b-closed. The intersection of all b-closed sets containing A ⊆ X is called the b-closure of A and shall be denoted by bcl(A). The union of all b-open sets of X contained in A is called the b-interior of A and is denoted by bint(A). A subset A is said to be b-regular if it is b-open and b-closed. The family of all b-open (resp. b-closed, b-regular) subsets of a space X is denoted by BO(X) (resp. BC (X), BR(X)) and the collection of all b-open subsets of X containing a fixed point x is denoted by BO(X, x). The sets BC (X, x) and BR(X, x) are defined analogously.

Definition 1.2. A point x ∈ X is called a δ -cluster [11] point of A if int(c l(U)) ∩ A ≠ φ for every open set U of X containing x.

The set of all δ -cluster points of A is called the δ - closure of A and is denoted by δ -cl (A)). A subset A is said to be δ -closed if δ -cl(A) = A. The complement of a δ -closed set is said to be δ-open. The δ -interior of A is defined by the union of all δ -open sets contained in A and is denoted by δ-int (A)).

Definition 1.3. A point x ∈ X is called a b- δ -cluster [8] point of A if int(bc l(U)) ∩ A ≠ φ for every b-open set U of X containing x. The set of all b-δ -cluster points of A is called the b-δ - closure of A and is denoted by b-δ -cl (A)). A subset A is said to be b-δ -closed if b-δ-cl(A) = A. The complement of a b-δ -closed set is said to be b-δ-open. The b-δ -interior of A is defined by the union of all b-δ -open sets contained in A and is denoted by b-δ-int (A)). The family of all b- δ -open (resp. b- δ -closed) sets of a space X is denoted by BδO(X, τ) (resp. BδC(X, τ)).

Definition 1.4. A subset A of a space X is said to be α-open [7] (resp. semi-open [5], preopen[6], β-open[1] or semi-preopen [2]) if A ⊆ int(cl(int(A))) (resp. A ⊆ c l(int(A)), A ⊆ int(cl(A)), A ⊆ cl (int (cl(A))).

Lemma 1.5. [3] For a subset A of a space X, the following properties hold:

(1) b i nt(A) = sint(A) ∪ pint(A);
(2) bcl(A) = scl(A) ∩ pcl(A);
(3) bcl(X − A) = X - bint(A);
(4) x ∈ bcl(A) if and only if A ∩ U = φ for every U ∈ BO(X, x);
(5) A ∈ BC (X) if and only if A = bcl(A);
(6) pint(bcl(A)) = bcl(pint(A)).

Lemma 1.6. [2] For a subset A of a space X, the following properties are hold:

(1) αint(A) = A ∩ int(cl(int(A)));
(2) sint(A) = A ∩cl(int(A));
(3) pint(A) = A ∩int(cl(A)).

Lemma 1.7. [8] Let A and Aα (α ∈ Λ) be any subsets of a space X . Then the following properties hold:

1) if Aα ∈ BδO(X) for each α ∈ Λ, then ∪α∈Λ Aα ∈ BδO(X);
2) b-δ-cl(A) is b-δ-closed;
3) A is b- δ -open in X if and only if for each x ∈ A there exists V ∈ BR(X, x) such that x ∈ V ⊆ A.

Lemma 1.8. [4] f : (X, τ) → (Y, σ) is said to be strongly continuous if for every subset A of X, f (cl(A)) ⊆ f (A).

3. Weaklty b-δ Open Functions

Definition 3.1. A function f : (X, τ) → (Y, σ) is said to be b-δ -open if for each open set U of (X, τ), f (U) is b-δ -open.

Definition 3.2. A function f: (X, τ) → (Y, σ) is said to be weakly b- δ -open if f(U) ⊆ b-δ-int(f(cl(U))) for each open set U of (X, τ).

Theorem 3.3. Every b- δ -open function is also weakly b- δ –open function.

Proof: Follows from Definitions of b- δ -open function and weakly b- δ –open function.

The Converse of the above theorem need not be true as shown in the following example.

Example 3.4. Let X = {a, b, c} and τ = {φ, {a}, {b}, {a, b}, {a, c}, X}. Let f : (X, τ) → (Y, σ) be a function defined by f (a) = c, f (b) = b and f (c) = a. Then BδO(X) = {φ, {b}, {a, c}, X}. Then f is a weakly b- δ -open function but f is not b- δ –open function, since for U = {a} and U = {a, c} f (U) is not b-δ -open in (X, τ).

Theorem 3.5. Every b- δ –open function is also b-θ-open function.

Proof: Every b- δ –open set is b- θ –open set. Hence the proof follows from Definitions of b- δ –open function and b-θ-open function.

Theorem 3.6. For a function f: (X, τ) → (Y, σ), the following conditions are equivalent:

1) f is weakly b-δ-open,
2) f (δ-int (A)) ⊆b-δ-int (f (A)) for every subset of A of (X, τ),
3) δ-int (f^{-1} (B))⊆ f^{-1} (b-δ-int (B)) for every subset B of (Y, σ),
4) f^{-1} (b-δ-cl(B) ⊆δ-cl(f^{-1} (B)) for every subset of B of (Y, σ).

Proof. (1)⇒(2): Let A be any subset of (X, τ) and x ∈ δ-int (A). Then, there exists an open set U such that x ∈ U ⊆ c l(U) ⊆ A. Then, f (x) ∈ f (U) ⊆ f (c l(U)) ⊆ f (A). Since f is weakly b-δ -open, f (U) ⊆ b-δ-int (f (c l(U))) ⊆ b-δ-int(f (A)).This implies that f (x) ∈ b-δ-int (f (A)). This shows that x ∈ f^{-1} (b-δ-int (f (A))).Thus, δ-int (A) ⊆ f^{-1} (b-δ-int(f (A))), and so, f (δ-int (A)) ⊆b-δ-int(f (A)).

(2)⇒(3): Let B be any subset of (Y, σ).Then by (2), f (δ-int(f^{-1} (B)))⊆b-δ-int(f(f^{-1} (B))⊆b-δ-int(B). Therefore δ-int(f^{-1} (B) ⊆ f^{-1} (b-δ-int(B)).

(3)⇒(4): Let B be any subset of (Y, σ).

Using (3), we have X–δ-cl(f^{-1} (B))= δ-int (X- f^{-1} (B)) = δ-int(f^{-1} (Y− B))⊆ f^{-1} (b-δ-int (Y−B))= f^{-1} (Y−b-δ-cl (B)) = X− f^{-1} (b-δ-cl(B)).

Therefore, we obtain f^{-1} (b-δ-cl(B))⊆δ-cl(f^{-1} (B)).

(4)⇒(1): Let V be any open set of (X, τ) and B=Y− f (c l(V)). By (4), f^{-1} (b-δ-cl (Y−f (c l(V))))⊆δ-cl(f^{-1} (Y−f(c l(V)))).Therefore, we obtain f^{-1} (Y−b-δ-int (f (cl(V))))⊆δ-cl (X− f^{-1} (f (c l(V))))⊆δ-cl(X−cl(V)).Hence V⊆δ-int(cl(V)) ⊆ f^{-1} (b-δ-int (f (c l(V)))) and f (V)⊆b-δ-int(f (c l(V))). This shows that f is weakly b-δ -open.

Theorem 3.7. For a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

1) f is weakly b-δ -open;
2) For each x ∈ X and each open subset U of (X, τ) containing x, there exists a b- δ-open set V containing f (x) such that V ⊆ f (cl(U)).

Proof. (1)⇒(2): Let x ∈ X and U be an open set in (X, τ) with x ∈ U . Since f is weakly b-δ-open, f (x) ∈ f (U) ⊆ b-δ-int(f(cl(U))). Let V = b-δ-int (f (cl(U))). Then V is b-δ-open and f (x) ∈ V ⊆ f (c l(U)).

(2)⇒(1): Let U be an open set in (X, τ) and let y ∈ f (U). It follows from (2) that V ⊆ f (cl (U)) for some b-δ-open set V in (Y, σ) containing y. Hence, we have y ∈ V⊆b-δ-int(f(cl(U))).This shows that f(U)⊆b-δ-int(f(cl(U))).Thus f is weakly b-δ-open.

Theorem 3.8. For a bijective function f: (X, τ) → (Y, σ), the following conditions are equivalent:

1) f is weakly b-δ-open,
2) b-δ-cl(f(int(F))) ⊆f(F) for each closed set F in (X, τ),
3) b-δ-cl(f(U))) ⊆f(cl(U)) for each open set U in (X, τ).

Proof. (1)⇒(2): Let F be a closed set in (X, τ).Then since f is weakly b-δ-open, f (X−F)=⊆b-δ-int(f (cl(X− F)))= b-δ-int(f(cl(X− F))) and so Y− f (F)⊆Y −b-δ-cl(f (int(F))). Hence b-δ-cl (f (int(F)))⊆ f (F) .

(2)⇒(3): Let U be an open set in (X, τ). Since c l(U) is a closed set and U⊆ int(cl(U)), by (2) we have b-δ-cl(f(U))⊆ b-δ-cl(f(int(cl(U))))⊆f(cl(U)).

(3)⇒(1): Let V be an open set of(X, τ). Then we have Y - b-δ-int(f(cl(V)))=b-δ-cl(Y–f(cl(V)))=b-δ-cl(f(X–cl(V))) ⊆f(cl(X–cl(V)))=f(X–int(cl(V)))⊆f(X–V)=Y–f(V).

Therefore, we have f(V)⊆b-δ-int(f(cl(V))) and hence f is weakly b-δ-open

Theorem 3.9. For a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

1) f is weakly b-δ open;
2) f(U) ⊆ b-δ-int(f(cl(U))) for each preopen set U of (X, τ),
3) f(U) ⊆ b-δ-int(f(cl(U))) for each α -open set U of (X, τ),
4) f(int(cl(U))) ⊆ b-δ-int(f(cl(U))) for each open set U of (X, τ),
5) f(int(F)) ⊆ b-δ-int(f(F)) for each closed set F of (X, τ).

Proof: Follows from Definitions open, pre-open and α -open sets.

Theorem 3.10. Let X be a regular space. A function f : (X, τ) → (Y, σ) is weakly b-δ -open if and only if f is b-δ -open.

Proof. The sufficiency is clear.

For the necessity, let W be a nonempty open subset of X . For each x in W, let Ux be an open set such that x ∈ Ux ⊆ cl(Ux) ⊆ W . Hence we obtain that W = ∪{Ux : x ∈ W } = ∪{cl(Ux) : x ∈ W } and f (W) = ∪{f (Ux) : x ∈ W } ⊆ ∪{b-δ-int (f (cl(Ux))) : x ∈ W } ⊆b-δ-int (f (∪{c l(Ux) : x ∈ W })) = b-δ-int (f (W)). Thus f is b-δ -open.

Theorem 3.11. If f: (X, τ) → (Y, σ) is weakly b-δ -open and strongly continuous, then f is b-δ -open.

Proof. Let U be an open subset of (X, τ). Since f is weakly b-δ -open, f (U) ⊆ b-δ-int (f (cl(U))). However, because f is strongly continuous, f (U) ⊆ b-δ-int (f (U)). Therefore f (U) is b-δ -open.

Definition 3.12. A function f : (X, τ) → (Y, σ) is said to be contra b-δ-closed if f(U) is a b-δ-open set of Y, for each closed set U in (X, τ).

Theorem 3.13. If f: (X, τ) → (Y, σ) is a contra b-δ -closed function, then f is weakly b-δ -open.

Proof. Let U be an open subset of (X, τ). Then, we have f (U) ⊆ f (c l(U)) = b-δ-int(f(cl(U))).

The converse of the above theorem need not be true as shown in the following example.

Example 3.14 Let X = {a, b, c} and τ = {φ, {a}, {b}, {a, b}, {a, c}, X}. Let f : (X, τ) → (Y, σ) be a function defined by f (a) = c, f (b) = b and f (c) = a. Then f is a weakly b- δ -open function but f is not b- δ –open function, since for U = {b, c} and U = {a, b, } f (U) is not b- δ -open in (X, τ).

Definition 3.15.[10] A space X is said to be hyperconnected if every non-empty open subset of X is dense in X .

Theorem 3.16. If X is a hyperconnected space, then a function f: (X, τ) → (Y, σ) is weakly b-δ -open if and only if f (X) is b-δ -open in (Y, σ).

Proof. Let U be an open subset of (X, τ).Then since f(x) is b-δ –open, f(x)=b-δ -int(f(x)). Now f(U) ⊆ f(X) = b-δ –int(f(X))= b-δ –(f(cl(U)). Hence f is weakly b-δ –open.

Conversely, let f is weakly b-δ–open.Then f(x)⊆b-δ–int(f(cl(x))= b-δ–int(f(x)).Thus f(x) ⊆ b-δ–int(f(x)). But b-δ–int(f(x)) ⊆ f(x).Hence f(x)= b-δ–int(f(x) which implies f(x) is b-δ–open in (Y, σ).

Definition 3..17.. A function f: (X, τ) → (Y, σ) is called complementary weakly b-δ -open if for each open set U of X, f (Fr(U)) is b-δ -closed in (Y, σ), where Fr(U) denotes the frontier of U .

Theorem 3.18. Let B δ O(X, τ) be closed under finite intersection. If f : (X, τ) → (Y, σ) is bijective weakly b-δ-open and complementary weakly b-δ -open, then f is b-δ-open.
Proof. Let U be an open subset in (X, τ) and y ∈ f (U). Since f is weakly b-δ -open, by Theorem 2.5, for some x ∈ U there exists a b-δ -open set V containing f (x) = y such that V ⊆ f (cl(U)). Now Fr(U)=c l(U)− U and thus x ∈/ Fr(U). Hence y ∈/ f (Fr(U)) and therefore y ∈ V \ f (Fr(U)). Put Vy =V− f (Fr(U)). Then Vy is a b-δ-open set since f is complementary weakly b-δ-open. Furthermore, y ∈ Vy and Vy =V −f (Fr(U))⊆ f (cl(U))−f (Fr(U))=f (cl(U)−Fr(U))=f(U).Therefore f (U)=∪{Vy :Vy ∈ BδO(Y, σ), y ∈ f (U)}.Hence by Lemma 1.7, f is b-δ-open.

Theorem 3. 19. If a function f : (X, τ) → (Y, σ) is weakly b-δ -open and precontinuous, then f is β-open.

Proof. Let U be an open subset of X . Then by weak b-δ -openness of f, f(U) ⊆ b-δ-int(f(cl(U))). Since f is precontinuous, f(cl(U)) ⊆ cl(f(U)). Hence we obtain that

f (U) ⊆ b-δ-int(f (cl(U))) ⊆ b-δ-int (c l(f (U)))
= bint(cl(f (U)))
= sint(cl(f (U))) ∪ pint(cl(f (U))), by lemma 1.5
⊆ c l(int(cl(f (U)))) ∪ int(cl(f (U))) ⊆ c l(int(cl(f (U))))

which shows that f (U) is a β-open set in Y . Thus f is a β-open function.

Definition 3.20. A topological space X is said to be b-δ -connected if it cannot be written as the union of two nonempty disjoint b-δ -open sets.

Theorem 3.21. If f : (X, τ) → (Y, σ) is a weakly b-δ -open bijective function of a space X onto a b-δ -connected space Y, then X is connected.

Proof. Suppose that X is not connected. Then there exist non-empty open sets U and V such that U ∩ V = φ and U ∪ V =X. Hence we have f (U)∩ f (V) =φ and f (U) ∪ f (V)=Y . Since f is weakly b-δ-open, we have f (U) ⊆ b-δ-int(f(cl(U))) and f(V) ⊆ b-δ-int(f(cl(V))). Moreover U, V are open and also closed. we have f (U)=b-δ-int (f (U)) and f (V)= b-δ-int (f (V)). Hence, f (U) and f (V) are b-δ -open in Y . Thus, Y has been decomposed into two non-empty disjoint b-δ -open sets. This is contrary to the hypothesis that Y is a b-δ -connected space. Thus, X is connected.

Theorem 3.22. Let X be a regular space. Then for a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

(1) f is weakly b-δ-open;
(2) For each δ -open set A in X, f (A) is b- δ -open in Y;
(3) For any set B of Y and any δ -closed set A in X containing f^{-1}(B), there exists a b-δ -closed set F in Y containing B such that f^{-1}(F) ⊆ A.

Proof. (1)⇒(2): Let A be a δ-open set in X . Since X is regular, by Theorem 3.10, f is b-δ -open and A is open. Therefore f (A) is b-δ-open in Y.

(2)⇒(3): Let B be any set in Y and A a δ -closed set in X such that f^{-1} (B) ⊆ A. Since X−A is δ -open in X, by (2), f (X−A) is b-δ -open in Y . Let F=Y − f (X − A). Then F is b-δ -closed and B⊆F.

Now f^{-1} (F)= f^{-1} (Y−f (X−A))=X− f^{-1} (f (X−A)) ⊆ A.

(3)⇒(1): Let B be any set in Y . Let A = δ-cl (f^{-1} (B)).Since X is regular, A is a δ -closed set in X and f^{-1} (B)⊆ A. Then there exists a b-δ - closed set F in Y containing B such that f^{-1}(F)⊆A. Since F is b-δ closed, f^{-1} (b-δ-cl(B)⊆ f^{-1} (F)⊆A=δcl(f^{-1} (B)). Therefore by Theorem 3.5, f is weakly b-δ-open.

4. Weakly b-δ Closed Functions

Definition 4.1. A functions f : (X, τ) → (Y, σ) is said to be b-δ -closed if for each closed set F of (X, τ), f (F) is b-δ -closed.

Definition 4.2. A functions f : (X, τ) → (Y, σ) is said to be weakly b-δ -closed if b-δ-cl (f (int(F))) ⊆ f (F) for each closed set F of (X, τ) .

Theorem 4.3: Every b-δ –closed function is also weakly b-δ –closed function.

Proof: Follows from Definitions.

The converse of above theorem need not be true as shown in the following example.

Example 4.4. Let X ={a, b, c} and τ = {φ, {a}, {b}, {a, b}, {a, c}, X}. Then BδC(X) = {φ, {b}, {a, c}, X}. Let f : (X, τ) → (Y, σ) be a function defined by f (a) = a, f (b) = b and f (c) = c. Then f is a weakly b- δ -closed function which is not b- δ -closed, since for U = {b, c} and U = {c}, f(U) is not f (U) is not b- δ -closed in (X, τ).

Theorem 4.5. Every b-δ –closed function is also b- θ -closed function.

Proof. Every b-δ –closed is also b- θ -closed set. Hence the Proof follows from the Definitions of b-δ –closed and b- θ –closed sets.

Theorem 4.6. For a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

(1) f is weakly b-δ -closed;
(2) b-δ-cl(f (U))⊆f(c l(U)) for each open set U in (X, τ).

Proof. (1)⇒(2): Let U be an open set in X . Since c l(U) is a closed set and U ⊆ int(c l(U)), we have b-δ-cl (f (U)) ⊆ b-δ-cl (f (i nt(C l(U)))) ⊆ f (c l(U)).

(2)⇒(1): Let F be a closed set of X . Then, we have b-δ-cl(f(i nt(F)))⊆f (cl(i nt(F)))⊆f (c l(F))=f (F) and hence f is weakly b-δ -closed.

Corollary 4.7. A bijective function f : (X, τ) → (Y, σ), is weakly b-δ-open if and only if f is weakly b-δ-closed.

Proof. This is an immediate consequence of Theorems 3.6 and 4.6.

Theorem 4.8. For a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

(1) f is weakly b-δ –closed,
(2) b-δ-cl(f (int(F)))⊆ f (F) for each preclosed set F in
(3) b-δ-cl(f (int(F)))⊆f (F) for each α-closed set F in (X, τ),
(4) b-δ-cl(f (int(cl(U)))) ⊆ f (c l(U)) for each subset U in (X, τ),
(5) b-δ-cl (f (U))⊆f (c l(U)) for each preopen set U in (X, τ).
Proof: Follows from Definitions

Theorem 4.9. For a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

1) f is weakly b-δ –closed,
2) b-δ-cl (f (U))⊆ f (cl(U)) for each regular open set U in (X, τ),
3) For each subset F in Y and each open set U in X with f^{-1} (F)⊆U, there exists a b-δ -open set A in Y with F ⊆ A and f^{-1} (A) ⊆ c l(U),
4) For each point y in Y and each open set U in X with f^{-1} (y) ⊆ U, there exists a b-δ -open set A in Y containing y and f^{-1} (A) ⊆ cl(U).

Proof. (1)⇒(2): Let U be a regular open subset of (X, τ). Then U is open and so U = int(U). Since cl(U) is closed and f is weakly b-δ –closed, b-δ –cl(f(U))= b-δ –cl(f(int(U))) ⊆ b-δ –cl(f(int(cl(U)))) ⊆f(cl(U)). Hence b-δ –cl(f(U))=f(cl(U))

(2)⇒(3): Let F be a subset of Y and U an open set in X with f^{-1} (F)⊆ U . Then f^{-1} (F)∩ c l(X−c l(U))= φ and consequently, F∩f (c l(X−c l(U)))=φ. Since X− cl(U) is regular open, F∩b-δ-cl(f (X− c l(U))) = φ. Let A=Y−b-δ-cl (f (X− C l(U))). Then A is a b-δ -open set with F⊆ A and we have f^{-1}(A)⊆X− f^{-1}(b-δ-cl(f (Y− c l(U))))⊆X− f^{-1}(Y− c l(U)) ⊆ c l(U).

(3)⇒(4): This is obvious.
(4)⇒(1): Let F be closed in X and let y ∈ Y −f (F). Since f^{-1}(y)⊆ X−F, by (4) there exists a b-δ-open set A in Y with y ∈ A and f^{-1}(A) ⊆ c l(X−F) =X-int(F). Therefore A ∩f (int(F)) = φ, so that y ∈/ b-δ-cl (f (int(F))).Thus b-δ-cl (f (int(F))) ⊆ f (F). Hence f is weakly b-δ-closed.

Theorem 4.10. If f : (X, τ) → (Y, σ) is a bijective weakly b-δ -closed function, then for every subset F in Y and every open set U in X with f^{-1}(F) ⊆ U, there exists a b-δ -closed set B in Y such that F ⊆ B and f^{-1}(B) ⊆ cl(U).

Proof. Let F be a subset of Y and U be an open subset of X with f^{-1}(F)⊆U . Put B=b-δ-cl(f (i nt(c l(U)))). Then B is a b-δ -closed set in (Y, σ) such that F ⊆ B, since F ⊆f (U)⊆ f (i nt(c l(U))) ⊆ b-δ-cl (f (int(c l(U))))=B. Since f is weakly b-δ –closed, by Theorem 4.6, we have f^{-1}(B) ⊆ cl(U).

Theorem 4.11. If f : (X, τ) → (Y, σ) is a weakly b-δ –closed surjection and all pairs of disjoint fibers are strongly separated then (Y, σ) is b-T2.

Proof. Let y and z be two points in Y.Let U and V be open set in (X, τ) such that f-1(y) ∈ U and f-1(z) ∈ V with cl(U) ∩ cl(V)= φ Since f is weakly b-δ –closed, by Theorem 4.9, there are b-δ –open sets F and B in (Y, σ)such that y ∈ F and z ∈ B, f-1(F) ⊆ cl(U) and f-1(B) ⊆ cl(V).

Therefore F∩ B= φ, because cl(U) ∩cl(V) = φ and f is surjective. Since every b-δ –open is b-open. Then Y, σ) is b-T2.

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