Efficient Estimators of Population Variance Using Known Population Mode & Variance of Auxiliary Variable

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Abstract: ***In this paper, using known information of population mode (M_x) along with population variance (S_x^2) of the auxiliary variable x, we have proposed the ratio-type and product-type estimators of population variance (S_y^2) of the main variable y. Also we have proposed the class of estimators of S_y^2 whose proposed estimators are the members. Up to terms of order n^{-1}, expressions for biases, MSEs and minimum MSEs are obtained and compared with each other. Theoretical results are supported by the empirical study by taking five populations from the literature.***

Keywords: Correlation coefficient, Coefficient of kurtosis, Coefficient of skewness, Population variance, Population mode, Product-type estimator, Ratio-type estimator, Regularity conditions, Taylor series.

1. Introduction

It is well known that the use of the auxiliary information increases the efficiency of the estimator at the estimation stage, which is developed to estimate the population parameter of the study variable. Rather than the population mean, population variance is also one of that main parameter in which researchers have keen interest to estimate. In some areas such as laboratories and industries, producer wants to know the effect of variations on their products. In literature so many researchers Srivastava and Jhajj (1980),Isaki (1983),Upadhyaya and Singh (1999),Kadilar and Cingi (2006),Subramani and Kumarapandiyan (2012a,b, 2013,2015), have done work in this direction by using different information of auxiliary variable such as $C_x, \beta_{1x}, \beta_{2x}, S_x, \rho, M_d, Q_1, Q_3, Q_r, Q_d, Q_a$ & their combinations.

Recently Sharma et al. (2016a) suggested the new parametric relationship for population mode (M_y) as

$$M_y = \bar{Y} - k\frac{\mu_{30}}{S_y^2}$$

where k is unknown constant to be determined. They obtained the value of k by minimizing the MSE (up to terms of order n^{-1}) of the conventional consistent estimators of M_y as

$$\hat{M}_{o1} = \bar{y} - k_1\frac{m_{30}}{s_y^2}.$$

In this study, we first propose ratio-type and product-type estimator for population variance S_y^2 by using the known information of population mode along with population variance of the auxiliary variable x. The expressions for their minimum mean squared error are derived up to the first order of approximation. Then we propose generalized class of estimator of S_y^2 and obtained minimum mean squared error, up to the first order of approximation. Theoretically and numerically we show that proposed estimators are more efficient than the existing estimators.

2. Notations and Results

Let y be the variable of interest and x is the auxiliary variable. The observations on both the variables y and x are taken out from the sample of the size n, which has been drawn from the finite population of size N with the technique of SRSWOR. On the i^{th} unit, Y_i and X_i denote the values of the variables y and x respectively and corresponding small letters y_i and x_i denote the sample values.

Taking,

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \qquad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2, \qquad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$

$$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(Y_i - \bar{Y})^r (X_i - \bar{X})^s, \qquad \lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$$

Obviously

$\rho_{xy} = \lambda_{11} = \rho$(Correlation between x and y),

$\beta_{1y} = \lambda_{30}$(Coefficient of skewness of y),

$\beta_{1x} = \lambda_{03}$(Coefficient of skewness of x),

$\beta_{2x} = \lambda_{04}$(Coefficient of kurtosis of x).

Defining,

$$\delta = \frac{s_y^2}{S_y^2} - 1, \qquad \epsilon = \frac{\bar{x}}{\bar{X}} - 1,$$

$$\eta = \frac{s_x^2}{S_x^2} - 1, \qquad \eta' = \frac{m_{03}}{\mu_{03}} - 1.$$

For the sake of simplicity, assume that N is large enough as compares to n so that finite population correction (fpc) terms are ignored throughout.

For the given SRSWOR, we have the following expectations,

$$E(\delta) = E(\epsilon) = E(\eta) = E(\eta') = 0,\ E(\epsilon^2) = \frac{1}{n}C_x^2,$$

$$E(\epsilon\eta) = \frac{1}{n}\lambda_{03}C_x = \frac{1}{n}\beta_{1x}C_x, \qquad E(\epsilon\delta) = \frac{1}{n}\lambda_{21}C_x,$$

and up to terms of order n^{-1}

$$E(\delta^2) = \frac{1}{n}(\lambda_{40} - 1) = \frac{1}{n}(\beta_{2y} - 1),$$

$$E(\eta^2) = \frac{1}{n}(\lambda_{04} - 1) = \frac{1}{n}(\beta_{2x} - 1),$$

$$E(\eta'^2) = \frac{1}{n}\frac{(\lambda_{06} - 6\lambda_{04} - \lambda_{03}^2 + 9)}{\lambda_{03}^2} = \frac{1}{n}\frac{(\lambda_{06} - 6\beta_{2x} - \beta_{1x}^2 + 9)}{\beta_{1x}^2}$$

$$E(\epsilon\eta') = \frac{1}{n}\frac{(\lambda_{04} - 3)}{\lambda_{03}}C_x = \frac{1}{n}\frac{(\beta_{2x} - 3)}{\beta_{1x}}C_x,$$

$$E(\delta\eta) = \frac{1}{n}(\lambda_{22} - 1),$$

$$E(\delta\eta') = \frac{1}{n}\frac{(\lambda_{23} - 3\lambda_{21} - \lambda_{03})}{\lambda_{03}} = \frac{1}{n}\frac{(\lambda_{23} - 3\lambda_{21} - \beta_{1x})}{\beta_{1x}},$$

$$E(\eta\eta') = \frac{1}{n}\frac{(\lambda_{05} - 4\lambda_{03})}{\lambda_{03}} = \frac{1}{n}\frac{(\lambda_{05} - 4\beta_{1x})}{\beta_{1x}}.$$

3. Proposed Estimators

If population mode $M_x (= \bar{X} - K_x \frac{\mu_{03}}{S_x^2})$ of the auxiliary variable x is known then we here propose the ratio-type and product-type estimators of S_y^2 as

$$\hat{S}_{yoR}^2 = s_y^2 \frac{S_x^2 + M_x}{s_x^2 + \widehat{M}_x} \tag{3.1}$$

$$\hat{S}_{yoP}^2 = s_y^2 \frac{s_x^2 + \widehat{M}_x}{S_x^2 + M_x} \tag{3.2}$$

where the constants K_{xR} and K_{xp} involved in (3.1) & (3.2) are determined by minimizing the MSEs of the respective estimators.

Re-write the estimators in terms of $\delta, \varepsilon, \eta, \eta'$ and expanding them up to second degree of approximation and taking the expectations as given in section second, upto terms of order n^{-1}, we get,

$$B(\hat{S}_{yoR}^2) = \frac{1}{n}\frac{S_y^2}{S_x^2 + M_x}[\delta_p\{(\beta_{2x} - 1)S_x^2 + 2S_x\beta_{1x} - 2K_x\beta$$
$$+ 2K_xS_x(\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22}) - K_xS_x$$

$$B(\hat{S}_{yop}^2) = \frac{1}{n}\frac{S_y^2}{S_x^2 + M_x}[K_xS_x\{\lambda_{05} - \beta_{1x}(\beta_{2x} + 3)\} + (\lambda_{22}$$

and

$$MSE(\hat{S}_{yoR}^2) = \frac{1}{n}S_y^4\Big[(\beta_{2y} - 1) + (\beta_{2x} - 1)\delta_p^2 - 2(\lambda_{22} -$$
$$+ \frac{1}{(S_x^2 + M_x)}\{\delta_p + 2\delta_pS_x\beta_{1x} - 2S_x\lambda_{21}$$
$$- 2K_{xr}\delta_pS_x(\lambda_{05} - \beta_{1x}(\beta_{2x} + 3)) - 2K$$

$$MSE(\hat{S}_{yoP}^2) = \frac{1}{n}S_y^4\Big[(\beta_{2y} - 1) + (\beta_{2x} - 1)\delta_p^2 + 2(\lambda_{22} -$$
$$+ \frac{1}{(S_x^2 + M_x)}\{\delta_p + 2\delta_pS_x\beta_{1x} + 2S_x\lambda_{21}$$
$$- 2K_{xr}\delta_pS_x(\lambda_{05} - \beta_{1x}(\beta_{2x} + 3)) - 2K$$

Note that $B(\hat{S}_{yoR}^2)\ \&\ B(\hat{S}_{yoP}^2)$ are of order n^{-1} and hence their contribution to the MSEs will be the order of n^{-2}.

Above $MSE(\hat{S}_{yoR}^2)$ and $MSE(\hat{S}_{yoP}^2)$ are respectively minimized for

$$K_{xR} = \frac{\{R_{1s}S_x\beta_x + R_{1s}S_x^2(\lambda_{05} - \beta_{1x}(\beta_{2x} + 3)) - S_y^2(\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22})\}}{\lambda_x}$$

$$K_{xP} = \frac{\{R_{1s}S_x\beta_x + R_{1s}S_x^2(\lambda_{05} - \beta_{1x}(\beta_{2x} + 3)) + S_y^2(\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22})\}}{\lambda_x}$$

andup to terms of order n^{-1}their minimum MSEs are given by

$$MSE_{min}(\hat{S}_{yoR}^2) = \frac{1}{n}S_y^4\Big[(\beta_{2y} - 1) + ((\beta_{2x} - 1)\delta_p^2 - 2(\lambda_{22} -$$
$$- \frac{\{S_x\beta_x/(S_x^2 + M_x) + \delta_p(\lambda_{05} - \beta_{1x}(\beta_{2x} +}{\lambda_x}$$

$$MSE_{min}(\hat{S}_{yoP}^2) = \frac{1}{n}S_y^4\Big[(\beta_{2y} - 1) + ((\beta_{2x} - 1)\delta_p^2 + 2(\lambda_{22} -$$
$$- \frac{\{S_x\beta_x/(S_x^2 + M_x) + \delta_p(\lambda_{05} - \beta_{1x}(\beta_{2x} +}{\lambda_x}$$

where

$$\delta_p = \frac{S_x^2}{S_x^2 + M_x}$$
$$\lambda_x = \lambda_{06} - 6\beta_x + \beta_{ox}$$
$$\beta_x = \beta_{2x} - \beta_{1x}^2 - 3$$
$$\beta_{ox} = \beta_{1x}^2\beta_{2x} - 2\beta_{1x}\lambda_{05} - 9$$
$$B_x = \lambda_{13} - 3\rho - \beta_{1x}\lambda_{12}$$

4. Proposed Class of Estimators

Whatever be the sample chosen, let (u, v) assume values in a bounded closed convex subset R of two-dimensional real space containing the point $(1, 1)$, we here propose the class of estimators of population variance S_y^2 by using the known information of population variance (S_x^2) and population mode (M_x) of auxiliary variable x as

$$\hat{S}_{yg}^2 = s_y^2 t(u, v) \tag{4.1}$$

where $t(u, v)$ be the function of $u(= s_x^2/S_x^2)$ and $v(= \widehat{M}_x/M_x)$ which is continuous and bounded in R.The first and second partial derivatives of $t(u, v)$ exist and are continuous and bounded in R and such that $t(1,1) = 1$.

Since there is finite number of samples, therefore MSE of the class of estimators exist.

Expanding the function $t(u, v)$ about the point $(1,1)$ in second order Taylor's series, we obtain,

$$\hat{S}^2_{yg} = s_y^2\Big[t(1,1) + (u-1)t_1(1,1) + (v-1)t_2(1,1) + \frac{1}{2}\{(u-1)^2 t_{11}(1,1) + 2(u-1)(v-1)t_1$$

where $t_1(1,1) = t_1(say)$ & $t_2(1,1) = t_2(say)$ denotes the first order partial derivatives and $t_{11}(1,1), t_{12}(1,1)$&$t_{22}(1,1)$ denotes the second order partial derivatives.

Now re-write this equation in $\delta, \varepsilon, \eta, \eta'$ form and then taking expectations up to order n^{-1} we get,

$$MSE(\hat{S}^2_{yg}) = \frac{1}{n}S_y^4\Big[(\beta_{2y}-1) + t_1^2(\beta_{2x}-1) + 2t_1(\lambda_{22}-1) - 2t_2K_{xg}\frac{S_x}{M_x}(\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22}) + 2t_1t - 2t_2^2K_{xg}\frac{S_x^2}{M_x^2}\beta_x\Big]$$

Where t_1, t_2 and K_{xg} are constant terms, whose values are obtained by minimizing the mean square error of $\hat{S}^2_{yg}$.

The $MSE(\hat{S}^2_{yg})$ is minimized for

$$t_1 = \frac{\{\lambda_{05} - \beta_{1x}(\beta_{2x}-1)\}C - (\lambda_{22}-1)B - \beta_{1x}A}{(\beta_{2x}-1)B}$$

$$t_2 = \frac{M_xA}{S_xB}$$

$$K_{xg} = \frac{C}{A}$$

and

$$MSE_{min}(\hat{S}^2_{yg}) = \frac{1}{n}S_y^4\Bigg[(\beta_{2y}-1) - \frac{\{\{\lambda_{05} - \beta_{1x}(\beta_{2x}-1)\}C -}{(\beta_{2x} -} - \frac{\{2(BC(\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22}) + AC\beta_x -}{B^2}$$

where,

$a = \beta_x(\beta_{2x}-1) - \beta_{1x}\{\lambda_{05} - \beta_{1x}(\beta_{2x}-1)\}$

$b = (\beta_{2x}-1)(\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22}) - (\lambda_{22}-1)\{\lambda_{05} - \beta_{1x}($

$c = (\beta_{2x}-1)\lambda_x - \{\lambda_{05} - \beta_{1x}(\beta_{2x}-1)\}^2$

$A = c\{(\beta_{2x}-1)\lambda_{21} - \beta_{1x}(\lambda_{22}-1)\} - ab$

$B = a^2 + c(\beta_{1x}^2 - \beta_{2x} + 1)$

$C = a\{(\beta_{2x}-1)\lambda_{21} - \beta_{1x}(\lambda_{22}-1)\} + b(\beta_{1x}^2 - \beta_{2x} + 1)$

5. Comparison

For comparison purpose, we consider the following existing modified ratio-type estimators of S_y^2 from the literature (we exclude the estimators using known information of deciles and percentiles), which are listed and proposed by Subramani and kumarapandiyan (2015).

Table 5.1: Existing modified ratio-type estimators for S_y^2 with their MSE

Estimator	Mean squared error (MSE)
$\hat{S}_1^2 = s_y^2\frac{S_x^2}{s_x^2}$ Isaki (1983)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_1^2(\beta_{2x}-1) - 2\delta_1(\lambda_{22}-1)]$
$\hat{S}_2^2 = s_y^2\frac{S_x^2 + C_x}{s_x^2 + C_x}$ Kadilar &Cingi (2006)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_2^2(\beta_{2x}-1) - 2\delta_2(\lambda_{22}-1)]$
$\hat{S}_3^2 = s_y^2\frac{S_x^2 + \beta_{2x}}{s_x^2 + \beta_{2x}}$ Upadhyaya & Singh (1999)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_3^2(\beta_{2x}-1) - 2\delta_3(\lambda_{22}-1)]$
$\hat{S}_4^2 = s_y^2\frac{S_x^2 + \beta_{1x}}{s_x^2 + \beta_{1x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_4^2(\beta_{2x}-1) - 2\delta_4(\lambda_{22}-1)]$
$\hat{S}_5^2 = s_y^2\frac{S_x^2 + \rho}{s_x^2 + \rho}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_5^2(\beta_{2x}-1) - 2\delta_5(\lambda_{22}-1)]$
$\hat{S}_6^2 = s_y^2\frac{S_x^2 + S_x}{s_x^2 + S_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_6^2(\beta_{2x}-1) - 2\delta_6(\lambda_{22}-1)]$
$\hat{S}_7^2 = s_y^2\frac{S_x^2 + M_d}{s_x^2 + M_d}$ Subramani & Kumarapandiyan (2012a)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_7^2(\beta_{2x}-1) - 2\delta_7(\lambda_{22}-1)]$
$\hat{S}_8^2 = s_y^2\frac{S_x^2 + Q_1}{s_x^2 + Q_1}$ Subramani & Kumarapandiyan (2012b)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_8^2(\beta_{2x}-1) - 2\delta_8(\lambda_{22}-1)]$
$\hat{S}_9^2 = s_y^2\frac{S_x^2 + Q_3}{s_x^2 + Q_3}$ Subramani & Kumarapandiyan (2012b)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_9^2(\beta_{2x}-1) - 2\delta_9(\lambda_{22}-1)]$
$\hat{S}_{10}^2 = s_y^2\frac{S_x^2 + Q_r}{s_x^2 + Q_r}$ Subramani & Kumarapandiyan (2012b)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_{10}^2(\beta_{2x}-1) - 2\delta_{10}(\lambda_{22}-1)]$
$\hat{S}_{11}^2 = s_y^2\frac{S_x^2 + Q_d}{s_x^2 + Q_d}$ Subramani & Kumarapandiyan (2012b)	$\frac{1}{n}S_y^4[(\beta_{2y}-1) + \delta_{11}^2(\beta_{2x}-1) - 2\delta_{11}(\lambda_{22}-1)]$

$\hat{S}_{12}^2 = s_y^2\frac{S_x^2+Q_a}{s_x^2+Q_a}$ Subramani & Kumarapandiyan (2012b)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{12}^2(\beta_{2x}-1)-2\delta_{12}(\lambda_{22}-1)]$
$\hat{S}_{13}^2 = s_y^2\frac{\beta_{2x}S_x^2+C_x}{\beta_{2x}s_x^2+C_x}$ Kadilar &Cingi (2006)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{13}^2(\beta_{2x}-1)-2\delta_{13}(\lambda_{22}-1)]$
$\hat{S}_{14}^2 = s_y^2\frac{C_xS_x^2+\beta_{2x}}{C_xs_x^2+\beta_{2x}}$ Kadilar & Cingi (2006)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{14}^2(\beta_{2x}-1)-2\delta_{14}(\lambda_{22}-1)]$
$\hat{S}_{15}^2 = s_y^2\frac{\beta_{1x}S_x^2+C_x}{\beta_{1x}s_x^2+C_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{15}^2(\beta_{2x}-1)-2\delta_{15}(\lambda_{22}-1)]$
$\hat{S}_{16}^2 = s_y^2\frac{C_xS_x^2+\beta_{1x}}{C_xs_x^2+\beta_{1x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{16}^2(\beta_{2x}-1)-2\delta_{16}(\lambda_{22}-1)]$
$\hat{S}_{17}^2 = s_y^2\frac{\rho S_x^2+C_x}{\rho s_x^2+C_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{17}^2(\beta_{2x}-1)-2\delta_{17}(\lambda_{22}-1)]$
$\hat{S}_{18}^2 = s_y^2\frac{C_xS_x^2+\rho}{C_xs_x^2+\rho}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{18}^2(\beta_{2x}-1)-2\delta_{18}(\lambda_{22}-1)]$
$\hat{S}_{19}^2 = s_y^2\frac{S_xS_x^2+C_x}{S_xs_x^2+C_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{19}^2(\beta_{2x}-1)-2\delta_{19}(\lambda_{22}-1)]$
$\hat{S}_{20}^2 = s_y^2\frac{C_xS_x^2+S_x}{C_xs_x^2+S_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{20}^2(\beta_{2x}-1)-2\delta_{20}(\lambda_{22}-1)]$
$\hat{S}_{21}^2 = s_y^2\frac{M_dS_x^2+C_x}{M_ds_x^2+C_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{21}^2(\beta_{2x}-1)-2\delta_{21}(\lambda_{22}-1)]$
$\hat{S}_{22}^2 = s_y^2\frac{C_xS_x^2+M_d}{C_xs_x^2+M_d}$ Subramani & Kumarapandiyan (2013)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{22}^2(\beta_{2x}-1)-2\delta_{22}(\lambda_{22}-1)]$
$\hat{S}_{23}^2 = s_y^2\frac{\beta_{1x}S_x^2+\beta_{2x}}{\beta_{1x}s_x^2+\beta_{2x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{23}^2(\beta_{2x}-1)-2\delta_{23}(\lambda_{22}-1)]$
$\hat{S}_{24}^2 = s_y^2\frac{\beta_{2x}S_x^2+\beta_{1x}}{\beta_{2x}s_x^2+\beta_{1x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{24}^2(\beta_{2x}-1)-2\delta_{24}(\lambda_{22}-1)]$
$\hat{S}_{25}^2 = s_y^2\frac{\rho S_x^2+\beta_{2x}}{\rho s_x^2+\beta_{2x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{25}^2(\beta_{2x}-1)-2\delta_{25}(\lambda_{22}-1)]$
$\hat{S}_{26}^2 = s_y^2\frac{\beta_{2x}S_x^2+\rho}{\beta_{2x}s_x^2+\rho}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{26}^2(\beta_{2x}-1)-2\delta_{26}(\lambda_{22}-1)]$
$\hat{S}_{27}^2 = s_y^2\frac{S_xS_x^2+\beta_{2x}}{S_xs_x^2+\beta_{2x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{27}^2(\beta_{2x}-1)-2\delta_{27}(\lambda_{22}-1)]$
$\hat{S}_{28}^2 = s_y^2\frac{\beta_{2x}S_x^2+S_x}{\beta_{2x}s_x^2+S_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{28}^2(\beta_{2x}-1)-2\delta_{28}(\lambda_{22}-1)]$
$\hat{S}_{29}^2 = s_y^2\frac{M_dS_x^2+\beta_{2x}}{M_ds_x^2+\beta_{2x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{29}^2(\beta_{2x}-1)-2\delta_{29}(\lambda_{22}-1)]$
$\hat{S}_{30}^2 = s_y^2\frac{\beta_{2x}S_x^2+M_d}{\beta_{2x}s_x^2+M_d}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{30}^2(\beta_{2x}-1)-2\delta_{30}(\lambda_{22}-1)]$
$\hat{S}_{31}^2 = s_y^2\frac{\rho S_x^2+\beta_{1x}}{\rho s_x^2+\beta_{1x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{31}^2(\beta_{2x}-1)-2\delta_{31}(\lambda_{22}-1)]$
$\hat{S}_{32}^2 = s_y^2\frac{\beta_{1x}S_x^2+\rho}{\beta_{1x}s_x^2+\rho}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n}S_y^4[(\beta_{2y}-1)+\delta_{32}^2(\beta_{2x}-1)-2\delta_{32}(\lambda_{22}-1)]$

$\hat{S}^2_{33} = s_y^2 \frac{S_x S_x^2 + \beta_{1x}}{S_x s_x^2 + \beta_{1x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{33}^2(\beta_{2x}-1) - 2\delta_{33}(\lambda_{22}-1)]$
$\hat{S}^2_{34} = s_y^2 \frac{\beta_{1x} S_x^2 + S_x}{\beta_{1x} s_x^2 + S_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{34}^2(\beta_{2x}-1) - 2\delta_{34}(\lambda_{22}-1)]$
$\hat{S}^2_{35} = s_y^2 \frac{M_d S_x^2 + \beta_{1x}}{M_d s_x^2 + \beta_{1x}}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{35}^2(\beta_{2x}-1) - 2\delta_{35}(\lambda_{22}-1)]$
$\hat{S}^2_{36} = s_y^2 \frac{\beta_{1x} S_x^2 + M_d}{\beta_{1x} s_x^2 + M_d}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{36}^2(\beta_{2x}-1) - 2\delta_{36}(\lambda_{22}-1)]$
$\hat{S}^2_{37} = s_y^2 \frac{S_x S_x^2 + \rho}{S_x s_x^2 + \rho}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{37}^2(\beta_{2x}-1) - 2\delta_{37}(\lambda_{22}-1)]$
$\hat{S}^2_{38} = s_y^2 \frac{\rho S_x^2 + S_x}{\rho s_x^2 + S_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{38}^2(\beta_{2x}-1) - 2\delta_{38}(\lambda_{22}-1)]$
$\hat{S}^2_{39} = s_y^2 \frac{M_d S_x^2 + \rho}{M_d s_x^2 + \rho}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{39}^2(\beta_{2x}-1) - 2\delta_{39}(\lambda_{22}-1)]$
$\hat{S}^2_{40} = s_y^2 \frac{\rho S_x^2 + M_d}{\rho s_x^2 + M_d}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{40}^2(\beta_{2x}-1) - 2\delta_{40}(\lambda_{22}-1)]$
$\hat{S}^2_{41} = s_y^2 \frac{M_d S_x^2 + S_x}{M_d s_x^2 + S_x}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{41}^2(\beta_{2x}-1) - 2\delta_{41}(\lambda_{22}-1)]$
$\hat{S}^2_{42} = s_y^2 \frac{S_x S_x^2 + M_d}{S_x s_x^2 + M_d}$ Subramani & Kumarapandiyan (2015)	$\frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_{41}^2(\beta_{2x}-1) - 2\delta_{42}(\lambda_{22}-1)]$

where $\delta_i = \frac{S_x^2}{S_x^2 + w_i}$; $i - 1,2,\dots,42;\ and\ w_1 = 1, w_2 = C_x, w_3 = \beta_{2x}, w_4 = \beta_{1x}, w_5 = \rho, w_6 = S_x, w_7 = M_d, w_8 = Q_1, w_9 = Q_3, w_{10} = Q_r, w_{11} = Q_d, w_{12} = Q_d, w_{13} = \frac{C_x}{\beta_{2x}}, w_{14} = \frac{\beta_{2x}}{C_x}, w_{15} = \frac{C_x}{\beta_{1x}}, w_{16} = \frac{\beta_{1x}}{C_x}, w_{17} = \frac{C_x}{\rho}, w_{18} = \frac{\rho}{C_x}, w_{19} = \frac{C_x}{S_x}, w_{20} = \frac{S_x}{C_x}, w_{21} = \frac{C_x}{M_d}, w_{22} = \frac{M_d}{C_x}, w_{23} = \frac{\beta_{2x}}{\beta_{1x}}, w_{24} = \frac{\beta_{1x}}{\beta_{2x}}, w_{25} = \frac{\beta_{2x}}{\rho}$,
$w_{26} = \frac{\rho}{\beta_{2x}}, w_{27} = \frac{\beta_{2x}}{S_x}, w_{28} = \frac{S_x}{\beta_{2x}}, w_{29} = \frac{\beta_{2x}}{M_d}, w_{30} = \frac{M_d}{\beta_{2x}}, w_{31} = \frac{\beta_{1x}}{\rho}$,
$w_{32} = \frac{\rho}{\beta_{1x}}, w_{33} = \frac{\beta_{1x}}{S_x}, w_{34} = \frac{S_x}{\beta_{1x}}, w_{35} = \frac{\beta_{1x}}{M_d}, w_{36} = \frac{M_d}{\beta_{1x}}, w_{37} = \frac{\rho}{S_x}, w_{38} = \frac{S_x}{\rho}, w_{39} = \frac{\rho}{M_d}, w_{40} = \frac{M_d}{\rho}, w_{34} = \frac{S_x}{M_d}, w_{33} = \frac{M_d}{S_x}$.

We hereintroducedthe corresponding product-type estimators by using the same amount of known information $w_i; i = 1,2,\dots,42$ as used in above modified ratio-type estimators as

$$\hat{S}^2_{ip} = s_y^2 \frac{s_x^2 + w_i}{S_x^2 + w_i} (i = 1,2,\dots,42)$$

andup to terms of order n^{-1}their MSE's are given as

$$MSE(\hat{S}^2_{ip}) = \frac{1}{n} S_y^4[(\beta_{2y}-1) + \delta_i^2(\beta_{2x}-1) + 2\delta_i(\lambda_{22}-1)]$$

where$\delta_i = \frac{S_x^2}{S_x^2 + w_i}$; $i = 1,2,\dots,42$.

Srivasthava and Jhajj (1980) defined a class of estimator for population variance using the known information of population mean and population variance of auxiliary variable as

$$T_{h43} = s_y^2 h(u, v)$$

where $h(u,v)$ is a function of $u = \frac{\bar{x}}{\bar{X}}$ and $v = \frac{s_x^2}{S_x^2}$ and satisfy certain regularity conditions. Up to the terms of order n^{-1}, minimum MSE of this class of estimators is

$$MSE_{min}(T_{h43}) = \frac{1}{n} S_y^4 \left[(\beta_{2y}-1) - \lambda_{21}^2 - \frac{(\lambda_{22} - \lambda_{21}\beta_{1x} - 1)^2}{\beta_{2x} - \beta_{1x}^2 - 1}\right]$$

We compare proposed estimators with the existing estimators in form of theorems as:

Theorem 1: Up to the terms of order n^{-1} $MSE(\hat{S}^2_{yoR}) < MSE(\hat{S}^2_{iR}), i = 1,2,\dots,42$.

if

$$(\lambda_{22} - 1) = \begin{cases} > \frac{1}{2}\left\{(\delta_p + \delta_i)(\beta_{2x} - 1) - \frac{\theta_1}{(\delta_p - \delta_i)}\right\} for\ \delta_p > \delta_i\ \ i.e. M_x < w_i \\ < \frac{1}{2}\left\{(\delta_p + \delta_i)(\beta_{2x} - 1) + \frac{\theta_1}{(\delta_i - \delta_p)}\right\} for\ \delta_p < \delta_i\ \ i.e. M_x > w_i \end{cases}$$

where $\theta_1 = -\frac{1}{(S_x^2+M_x)}\{2S_x\lambda_{21} - \delta_p - 2\delta_p\beta_{1x}\}$
$$-\frac{\{S_x\beta_x/(S_x^2+M_x) + \delta_p(\lambda_{05} - \beta_{1x}(\beta_{2x}+3)) - (\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22})\}^2}{\lambda_x}$$

Theorem 2: Up to the terms of order n^{-1}
$MSE(\hat{S}^2_{yoP}) < MSE(S^2_{iP}), i = 1,2,\dots,42.$
If
$$(\lambda_{22}-1) = \begin{cases} < -\frac{1}{2}\left\{(\delta_p+\delta_i)(\beta_{2x}-1) - \frac{\theta_2}{(\delta_p-\delta_i)}\right\} \text{ for } \delta_p > \delta_i \text{ i.e. } M_x < w_i \\ > -\frac{1}{2}\left\{(\delta_p+\delta_i)(\beta_{2x}-1) + \frac{\theta_2}{(\delta_i-\delta_p)}\right\} \text{ for } \delta_p < \delta_i \text{ i.e. } M_x > w_i \end{cases}$$
where $\theta_2 = \frac{1}{(S_x^2+M_x)}\{2S_x\lambda_{21} + \delta_p + 2\delta_p\beta_{1x}\}$
$$-\frac{\{S_x\beta_x/(S_x^2+M_x) + \delta_p(\lambda_{05} - \beta_{1x}(\beta_{2x}+3)) + (\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22})\}^2}{\lambda_x}$$

Theorem 3: Up to the terms of order n^{-1}
$MSE(\hat{S}^2_{yg}) < MSE(T_{h43})$
if
$$\frac{\{\{\lambda_{05} - \beta_{1x}(\beta_{2x}-1)\}C - (\lambda_{22}-1)B - \beta_{1x}A\}^2}{(\beta_{2x}-1)B^2} + \frac{\{2(BC(\lambda_{23} - 3\lambda_{21} - \beta_{1x}\lambda_{22}) + AC\beta_x - AB\lambda_{21}) - A^2 - C^2\lambda_x\}}{B^2} > \lambda_{21}^2 + \frac{(\lambda_{22} - \lambda_{21}\beta_{1x} - 1)^2}{\beta_{2x} - \beta_{1x}^2 - 1}.$$

6. Empirical Study

We have taken 5 natural populations (where first 4 populations have positive correlation and 5th population has negative correlation) to show the efficiency of the proposed estimators over the existing estimators in Table 2.The descriptions of the populations are described below.

Pop. I: Murthy (1967), p-398
y= Number of absentees &x= Number of workers
$\bar{Y} = 9.6512, \lambda_{22} = 2.7094, \beta_{2y} = 6.5387, \rho = 0.6608.$

Pop. II: Chakravarty et al.(1967), p-207
y= Weight (kg) of female&x= Height (cm) of female
$\bar{Y} = 28.5313, \lambda_{22} = 1.3731, \beta_{2y} = 2.2575, \rho = 0.2306.$

Pop. III: Chochran (1999), p-325
y= Total number of persons&x= Average persons per room
$\bar{Y} = 101.1000, \lambda_{22} = 1.5433, \beta_{2y} = 2.3523, \rho = 0.6515.$

Pop. IV: Singh (2003), p-1126
y= Crude Death Rate&x= Total Fertility Rate
$\bar{Y} = 10.8716, \lambda_{22} = 1.4154, \beta_{2y} = 3.6527, \rho = 0.5493.$

Pop. V: Maddala&Lahiri(1992),p-96
y= Deflated prices of veal&x= Consumption per capital of veal
$\bar{Y} = 7.6375, \lambda_{22} = 0.8697, \beta_{2y} = 1.4348, \rho = -0.6823.$

Table 2: $\frac{1}{n} \times MSE$and Efficiencies for existing & proposed ratio-type estimators, induced product-type estimators and class of estimators of S_y^2

Ratio-type Estimator	Pop. I	Pop.II	Pop.III	Pop.IV	Product-type Estimator	Pop.V
$\hat{S}_y^2$	9832.8371	4.1239	62330.1456	1874.5592	$\hat{S}_y^2$	2708.1634
	100	100	100	100		100
$\hat{S}_{1R}^2$	6625.5930	11.5193	69348.4784	1828.3506	$\hat{S}_{1P}^2$	4579.5978
	148.4069	35.7999	89.8796	102.5273		59.1354
$\hat{S}_{2R}^2$	6625.7138	11.4880	69204.3612	1770.8799	$\hat{S}_{2P}^2$	4366.3591
	148.4042	35.8975	90.0668	105.8547		62.0234
$\hat{S}_{3R}^2$	6626.2970	8.5750	66997.7417	1721.3209	$\hat{S}_{3P}^2$	3690.3141
	148.3911	48.0921	93.0332	108.9024		73.3857
$\hat{S}_{4R}^2$	6625.7176	10.8328	69083.6625	1791.3313	$\hat{S}_{4P}^2$	4573.0726
	148.4041	38.0686	90.2241	104.6461		59.2198
$\hat{S}_{5R}^2$	6625.7692	11.2885	68628.2375	1768.2670	$\hat{S}_{5P}^2$	5659.1956
	148.4030	36.5319	90.8229	106.0111		47.8542
$\hat{S}_{6R}^2$	6636.9948	8.5208	62619.4275	1720.7463	$\hat{S}_{6P}^2$	3609.0639
	148.1519	48.3980	99.5380	108.9387		75.0378
$\hat{S}_{7R}^2$	6652.0510	3.9733	51528.6804	1715.2867	$\hat{S}_{7P}^2$	2985.4082
	147.8166	103.7903	120.9620	109.2855		90.7133

$\hat{S}^2_{8R}$	6643.3136	3.9731	51701.8940	1720.4807	$\hat{S}^2_{8P}$	3050.1771
	148.0110	103.7955	120.5568	108.9555		88.7871
$\hat{S}^2_{9R}$	6662.6405	3.9735	51447.2112	1730.6888	$\hat{S}^2_{9P}$	2933.1902
	147.5817	103.7851	121.1536	108.3129		92.3283
$\hat{S}^2_{10R}$	6639.5861	9.4574	61966.1378	1717.9022	$\hat{S}^2_{10P}$	3276.0327
	148.0941	43.6050	100.5874	109.1191		82.6659
$\hat{S}^2_{11R}$	6631.9694	10.3764	65159.1944	1720.5651	$\hat{S}^2_{11P}$	3612.0264
	148.2642	39.7431	95.6583	108.9502		74.9763
$\hat{S}^2_{12R}$	6652.5003	3.9733	51550.1126	1717.9347	$\hat{S}^2_{12P}$	2980.2724
	147.8066	103.7903	120.9118	109.1170		90.8697
$\hat{S}^2_{13R}$	6625.6392	11.5115	69283.9533	1791.3311	$\hat{S}^2_{13P}$	4426.4097
	148.4059	35.8242	89.9633	104.6462		61.1819
$\hat{S}^2_{14R}$	6627.1717	3.9727	57420.2897	1716.8799	$\hat{S}^2_{14P}$	3029.2716
	148.3715	103.8060	108.5507	109.1841		89.3998
$\hat{S}^2_{15R}$	6625.8521	11.4757	68746.8467	1721.3208	$\hat{S}^2_{15P}$	2765.4831
	148.4011	35.9359	90.6662	108.9024		97.9273
$\hat{S}^2_{16R}$	6625.8684	4.8297	67387.4631	1766.9741	$\hat{S}^2_{16P}$	4550.3258
	148.4007	85.3863	92.4952	106.0887		59.5158
$\hat{S}^2_{17R}$	6625.7761	11.3848	69127.7771	1744.7859	$\hat{S}^2_{17P}$	4989.8752
	148.4028	36.2229	90.1666	107.4378		54.2732
$\hat{S}^2_{18R}$	6625.9831	7.1434	64462.8835	1739.3270	$\hat{S}^2_{18P}$	140502.2139
	148.3982	57.7302	96.6915	107.7750		1.9275
$\hat{S}^2_{19R}$	6625.5964	11.5117	69329.2784	1792.0142	$\hat{S}^2_{19P}$	4447.7655
	148.4068	35.8236	89.9045	104.6063		60.8882
$\hat{S}^2_{20R}$	6654.9915	3.9732	51507.4314	1717.2306	$\hat{S}^2_{20P}$	2984.1072
	147.7513	103.7929	121.0119	109.9230		90.7529
$\hat{S}^2_{21R}$	6625.5947	11.5191	69345.9831	1803.7968	$\hat{S}^2_{21P}$	4548.9315
	148.4069	35.8005	89.8828	103.9230		59.5341
$\hat{S}^2_{22R}$	6698.3923	4.1146	57459.2257	1731.1195	$\hat{S}^2_{22P}$	2768.3310
	146.7940	100.2260	108.4772	108.2860		97.8266
$\hat{S}^2_{23R}$	6627.1232	7.8402	61345.0421	1735.2572	$\hat{S}^2_{23P}$	2716.7043
	148.3726	52.5994	101.6058	108.0277		99.6856
$\hat{S}^2_{24R}$	6625.6406	11.3392	69229.6779	1805.7199	$\hat{S}^2_{24P}$	4575.0458
	148.4058	36.3685	90.0339	103.8123		59.1942
$\hat{S}^2_{25R}$	6626.6663	5.3083	65874.5118	1716.0757	$\hat{S}^2_{25P}$	22828.1889
	148.3829	77.6878	94.6195	109.2352		11.8632
$\hat{S}^2_{26R}$	6625.6603	11.4607	69022.9144	1789.3581	$\hat{S}^2_{26P}$	5239.0876
	148.4054	35.9830	90.3036	104.7615		51.6915
$\hat{S}^2_{27R}$	6625.6123	10.6068	69016.1049	1743.0071	$\hat{S}^2_{27P}$	3940.0135
	148.4065	38.8798	90.3125	107.5474		68.7349
$\hat{S}^2_{28R}$	6629.5359	10.5627	65938.7927	1741.0640	$\hat{S}^2_{28P}$	3789.7640
	148.3186	39.0421	94.5273	107.6674		71.4599
$\hat{S}^2_{29R}$	6625.6025	11.4889	69304.9324	1763.0768	$\hat{S}^2_{29P}$	4394.4801
	148.4067	35.8946	89.9361	106.3232		61.6265
$\hat{S}^2_{30R}$	6634.1287	4.4081	54862.9531	1723.8978	$\hat{S}^2_{30P}$	3095.3165
	148.2160	93.5528	113.6106	108.7396		87.4923
$\hat{S}^2_{31R}$	6625.7817	9.0747	68943.7130	1769.6135	$\hat{S}^2_{31P}$	4589.2316
	148.4027	45.4439	90.4073	105.9304		59.0113
$\hat{S}^2_{32R}$	6625.9714	11.2000	66500.6409	1719.7693	$\hat{S}^2_{32P}$	2690.6724
	148.3984	36.8205	93.7286	109.0006		100.6501
$\hat{S}^2_{33R}$	6625.5965	11.3437	69313.0884	1806.1728	$\hat{S}^2_{33P}$	4575.7202
	148.4068	36.3541	89.9255	103.7863		59.1855
$\hat{S}^2_{34R}$	6653.9033	7.7805	53673.6322	1736.3317	$\hat{S}^2_{34P}$	2715.4288
	147.7755	53.0030	116.1281	107.9609		99.7324
$\hat{S}^2_{35R}$	6625.5947	11.5139	69343.8772	1813.7519	$\hat{S}^2_{35P}$	4578.7370
	148.4069	35.8167	89.8856	103.3526		59.1465
$\hat{S}^2_{36R}$	6695.5519	3.9888	54938.9042	1761.3520	$\hat{S}^2_{36P}$	2709.7658
	148.8563	103.3870	113.4536	106.4273		99.9409
$\hat{S}^2_{37R}$	6625.5979	11.4622	69251.0749	1790.0687	$\hat{S}^2_{37P}$	5118.1444
	148.4068	35.9783	90.0060	104.7200		52.9130
$\hat{S}^2_{38R}$	6644.1464	5.2619	60132.4907	1716.3309	$\hat{S}^2_{38P}$	77211.6763
	147.9925	78.3728	103.6547	109.2190		3.5075
$\hat{S}^2_{39R}$	6625.5954	11.5176	69335.7944	1802.3832	$\hat{S}^2_{39P}$	4680.1845

	148.4068	35.8052	89.8961	104.0045		57.8645
$\hat{S}^2_{40R}$	6670.2155	4.0627	51487.4549	1728.4844	$\hat{S}^2_{40P}$	2665.1590
	147.4141	101.5064	121.0589	108.4510		101.6136
$\hat{S}^2_{41R}$	6625.7246	11.4881	69202.3819	1762.1034	$\hat{S}^2_{41P}$	4365.4111
	148.4040	35.8971	90.0694	106.3819		62.0368
$\hat{S}^2_{42R}$	6626.1371	4.4323	62505.3518	1724.5890	$\hat{S}^2_{42P}$	3153.5927
	148.3947	93.0420	99.7197	108.6960		85.8755
$\hat{S}^2_{yoR}$ **(Proposed)**	6091.8894	3.8199	49386.5649	1647.0581	$\hat{S}^2_{yoP}$**(Proposed)**	2629.1413
	161.4087	**107.9583**	**126.2087**	**113.8126**		**103.0056**
T_{h43}	6559.2363	3.6959	48053.8622	1550.6839	T_{h43}	2259.4322
	149.9083	111.5804	129.7089	120.8860		119.8604
$\hat{S}^2_{yg}$ **(Proposed)**	5167.7832	3.5061	34790.0686	1548.5232	$\hat{S}^2_{yg}$**(Proposed)**	2191.2309
	190.2719	**117.6207**	**179.1607**	**121.0546**		**123.5910**

For graphical representation, we have considered the most efficient estimator among the ratio-type estimators taken by Subramani and kumarapandiyan (2015)&induced product-type estimators, optimum estimator ofT_{h43} and the optimum estimator of the proposed class of estimators' $\hat{S}^2_{yg}$.

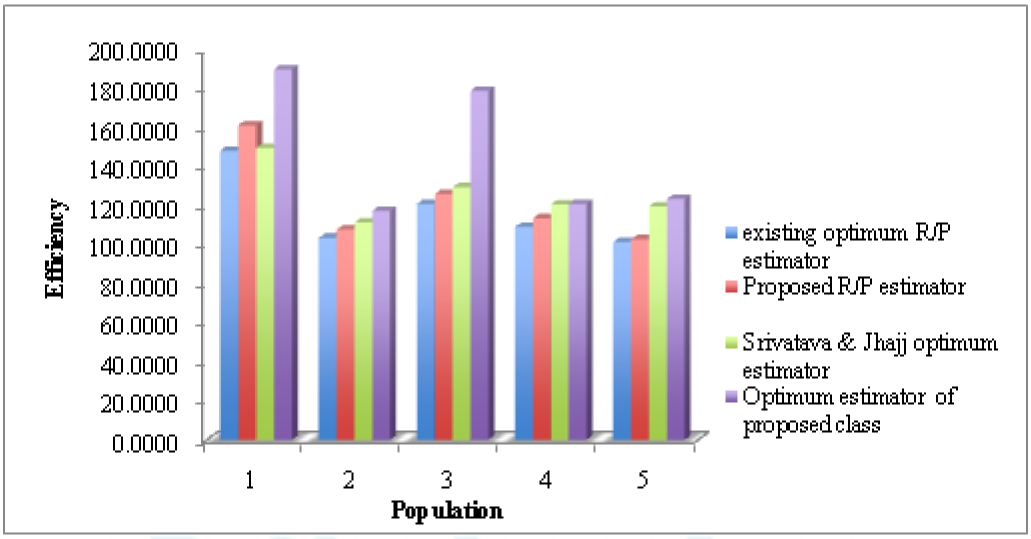

Figure 1: The bar graph shows the performance of proposed estimators and exciting optimum estimators

7. Conclusion

From Table 2 and bar chart, we conclude that the performance of proposed ratio-type $(\hat{S}^2_{yoR})$ and product-type $(\hat{S}^2_{yoP})$estimators isbetter than the existing modified ratio-type (S^2_{iR}) and induced product-type (S^2_{iP}) estimators for $i = 1,2,\dots,42$ respectively. Also the proposedoptimumestimator of class$(\hat{S}^2_{yg})$ is more efficient than all the existing estimators and even from the class of estimators of Srivastava and Jhajj (1980) (T_{h43}),in which they have used the known information of population mean and population variance of auxiliary variable x.From this study, we conclude that to estimate population variance (S^2_y) the known information of population variance (S^2_x) along with population mode (M_x) is optimum rather than use of known S^2_x along with any other population parameter such that $C_x, \beta_{1x}, \beta_{2x}, \rho, M_d, Q_1, Q_3$, etc.

References

[1] Chakravarty, I. M., Laha, R. G., & Roy, J. (1967). *Handbook of Methods of Applied Statistics: Techniques of Computation, Descriptive Methods, and Statistical Inference*. John Wiley & Sons.

[2] Cochran, W. G. (1999). *Sampling Techniques (*Vol.3). John Wiley & Sons.

[3] Gujarati, D. N. (2004). *Basic Econometrics.*Mc. Graw Hills Pub. Co, New York.

[4] Isaki, C.T. 1983. Variance estimation using auxiliary information. *Journal of the American Statistical Association,* 78: 117-123.

[5] Kadilar, C. and Cingi, H. 2006. Ratio estimators for population variance in simple and stratified sampling. *Applied Mathematics and Computation*, 173: 1047-1058.

[6] Kumar S. (2015). An estimator of the mean estimation of study variable using median of auxiliary variable. *Sri Lankan Journal of Applied Statistics,*Vol (16-2),107-115.

[7] Maddala, G. S., &Lahiri, K. (1992). *Introduction to econometrics* (Vol. 2). New York: Macmillan.

[8] Murthy, M. N. (1967). *Sampling theory and methods.* Statistical Publishing Society, Calcutta.

[9] Sharma, M.K., Brar, S.S. &Kaur, H. (2016). Estimators of population mode using new parametric relationship mode. *International Journal of Probability and Statistics,* 5(1), 18-24.

[10] Singh, S. (2003). *Advanced Sampling Theory With Applications: How Michael"" Selected"" Amy* (Vol. 2). Springer Science & Business Media.

[11] Srivastava, S. K. (1980). A class of estimator using auxiliary information in sample survey. *Canad. Jour. Stat. N, pp.*232-254.

[12] Srivastava, S. K., &Jhajj, H. S. (1980). A class of estimators using auxiliary information for estimating finite population variance. *Sankhya, C*, *42*(1-2), 87-96.

[13] Subramani J. &Kumarapandiyan G. (2012a). Variance estimation using median of the auxiliary variable. *International Journal of Probability and Statistics,* Vol. 1(3),36-40.

[14] Subramani J. &Kumarapandiyan G. (2012b). Variance estimation using quartiles and their functions of an auxiliary variable. *Internation Journal of Statistics and Applications,* Vol. 2(5), 67-42.

[15] Subramani J. &Kumarapandiyan G. (2013). New modified ratio estimator for estimation of population mean when median of the auxiliary variable is known. *Pak. J. Stat. Oper. Res.*Vol.IX No.2, pp.137-145.

[16] Subramani J. &Kumarapandiyan G. (2015). A class of modified ratio estimators for estimation of population variance.

[17] Upadhyaya, L.N. and Singh, H.P. 1999. An estimator for population variance that utilizes the kurtosis of an auxiliary variable in sample surveys. *Vikram Mathematical Journal*, 19, 14-17.

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