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Design and Comparison of Different Controllers to Stabilize a Rotary Inverted Pendulum

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Abstract: ***This paper describes the design procedures and design of various controllers for stabilizing a Rotary Inverted Pendulum System (RIPS). A PV (Position-Velocity) controller, LQR (Linear Quadratic Regulator) controller with different weighing matrices and an observer-based controller are tried on RIPS in MATLAB Simulink. The outputs obtained with different weighing matrices are observed and compared for different conclusions. The controllers with the best values obtained in the simulation are tested on a test-bed of RIPS and are compared for various aspects. The controllers in Simulink are compared with the controllers in real time.***

Keywords: Rotary Inverted Pendulum, PV, LQR, Weighing Matrices, Observer-Based control

1.Introduction

A typical unstable non-linear Inverted Pendulum system is often used as a benchmark to study various control techniques in control engineering. Analysis of controllers on RIP illustrates the analysis in cases such as control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air-flow, stabilization of a cabin in a ship etc. RIP is a test bed for the study of various controllers like PID controller, LQR controller, Robust controllers, Fuzzy-Logic, AI techniques, GA techniques and any more. A normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright, this can be done by applying a torque at the pivot point, by moving the pivot point horizontally as part of a feedback system.

In this paper controllers are developed that keep the pendulum upright without any oscillations. The model is simulated using the MATLAB application. The paper is organized as follows. Section 2 deals with the modelling of the system, Section 3 discusses the control techniques PID, LQR, observer based controller, Section 4 gives the test bed results, and Section 5 discusses the conclusion drawn from the analysis of these controllers in Simulink and on test bed.

2.Modelling of Rotary Inverted Pendulum

The Rotary Inverted Pendulum mainly consists of a rotary arm, vertical pendulum, and a servo motor which drives the system. An encoder is attached to the arm shaft in order to measure the rotation angle of the arm and pendulum.

The rotary pendulum model shown in in Fig. the rotary arm in attached to the servo system and is actuated. The arm has a length of L_r, a moment of inertia J_r, and its angle, θ, increases positively when it rotates counter clockwise (CCW). The servo should turn in the CCW direction when the control voltage is positive, i.e. , $V_m > 0$.

The pendulum link is connected to the rotary arm. It has a total length of L_p and its center of mass is $\frac{L_p}{2}$. The moment of inertia about its center of mass is J_p. The inverted pendulum angle, α, is zero when it is hanging downwards and increases when rotated CCW.

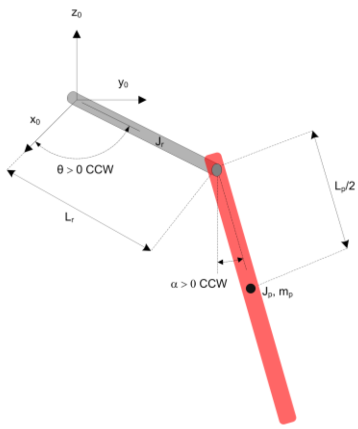

Figure 2.1: Free-body diagram of RIPS

The equations of motion (EOM) for the pendulum system were developed using the Euler-Lagrange method.More specifically, the equations that describe the motion of the rotary arm and the pendulum with respect to the servo motor voltage will be obtained using Euler-Lagrange equation

$$\frac{\partial^2 L}{\partial t\,\partial \dot{q}_i} - \frac{\partial L}{\partial \dot{q}_i} = Q_i$$

The Lagrangian of the system is described by

$$L = T - V$$

Where,

Tis the total kinetic energy of the system and V is the total potential energy of the system

The generalized forcesQ_i are used to describe the non-conservative forces (e.g., friction) applied on the system with respect to the generalized coordinates. In this case the generalized force acting on the rotary arm is

$$Q_1 = \tau - D_r\dot{\theta}$$

and acting on the pendulum is

$$Q_2 = -D_p\dot{\alpha}$$

The total potential energy of the system is

$$V = mg\frac{L_p}{2}\cos(\alpha)$$

and the total kinetic energy of the system is

$$T = \frac{1}{2}(J_r + m_p{L_r}^2)\dot{\theta}^2 + \frac{2}{3}m_p{L_p}^2\dot{\alpha}^2 - m_pL_pL_r\cos(\alpha)\dot{\theta}\dot{\alpha}$$

Solving the above two equations for the Lagrangian and the derivatives, the EOM of the system are

$$\left(m_p{L_r}^2 + \frac{1}{4}m_p{L_p}^2 - \frac{1}{4}m_p{L_p}^2\cos^2(\alpha) + J_r\right)\ddot{\theta} - \left(\frac{1}{2}m_pL_pL_r\cos(\alpha)\right)\ddot{\alpha} + \left(\frac{1}{2}m_p{L_p}^2\sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha} + \left(\frac{1}{2}m_pL_pL_r\sin(\alpha)\right)\dot{\alpha}^2 = \tau - D_r\dot{\theta}$$

$$\frac{1}{2}m_pL_rL_p\cos(\alpha)\ddot{\theta} + \left(J_p + \frac{1}{4}m_p{L_p}^2\right)\ddot{\alpha} - \frac{1}{4}m_p{L_p}^2\cos(\alpha)\sin(\alpha)\dot{\theta}^2 + \frac{1}{2}m_pL_pg\sin(\alpha) = -D_p\dot{\alpha}$$

The torque applied at the base of the rotary arm is described as

$$\tau = \frac{\eta_g\eta_m k_g k_t k_m(V_m - k_m\dot{\theta})}{R_m}$$

When the nonlinear equations are linearized about the operating point$(\theta,\alpha) = (0,0)$, the resultant EMO of the inverted pendulum are defined as:

$$(m_p{L_r}^2 + J_r)\ddot{\theta} - \frac{1}{2}m_pL_pL_r\ddot{\alpha} = \tau - D_r\dot{\theta}$$

and

$$\frac{1}{2}m_pL_rL_p\ddot{\theta} + \left(J_p + \frac{1}{4}m_p{L_p}^2\right)\ddot{\alpha} + \frac{1}{2}m_pL_pg\alpha = -D_p\dot{\alpha}$$

Solving the above equations for the acceleration terms yields

$$\ddot{\theta} = \frac{1}{J_T}\left\{-\left(J_p + \frac{1}{4}m_p{L_p}^2\right)D_r\dot{\theta} + \frac{1}{2}m_pL_pL_rD_p\dot{\alpha} + \frac{1}{4}{m_p}^2{L_p}^2L_rg\alpha + \left(J_p + \frac{1}{4}m_p{L_p}^2\right)\tau\right\}$$

and

$$\ddot{\alpha} = \frac{1}{J_T}\left\{\frac{1}{2}m_pL_pL_rD_r\dot{\theta} - (J_r + m_p{L_r}^2)D_p\dot{\alpha} - \frac{1}{2}m_pL_pg(J_r + m_p{L_r}^2)\alpha - \frac{1}{2}m_pL_pL_r\tau\right\}$$

Where

$$J_T = J_pm_p{L_r}^2 + J_rJ_p + \frac{1}{4}J_rm_p{L_p}^2$$

The A and B matrices for state-space representation can then be found as

$$A = \frac{1}{J_T}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4}{m_p}^2{L_p}^2L_rg & -\left(J_p + \frac{1}{4}m_p{L_p}^2\right)D_r & \frac{1}{2}m_pL_pL_rD_p \\ 0 & -\frac{1}{2}m_pL_pg(J_r + m_p{L_r}^2) & \frac{1}{2}m_pL_pL_rD_r & -(J_r + m_p{L_r}^2)D_p \end{bmatrix}$$

$$B = \frac{1}{J_T}\begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4}m_p{L_p}^2 \\ -\frac{1}{2}m_pL_pL_r \end{bmatrix}$$

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3. Design of Controllers for Rotary Inverted Pendulum

3.1 PV (Position-velocity) Control

In this paper we will find control strategies that balance the pendulum in the upright position while maintaining a desired position of the arm. When balancing the system, the pendulum angle α is small and balancing can be accomplished with a simple PD controller, as shown in Figure 3.1.1. The control law can then be expressed as

$$u = k_{p,\theta}(\theta_r - \theta) - k_{p,\alpha}\alpha - k_{d,\theta}\dot{\theta} - k_{d,\alpha}\dot{\alpha}$$

Where, $k_{p,\theta}$ is the arm angle proportional gain, $k_{p,\alpha}$ is the pendulum angle proportional gain, $k_{d,\theta}$ is the arm angle derivative gain, $k_{d,\alpha}$ is the pendulum angle derivative gain. The desired angle of the arm is denoted by θ_r and the reference for the pendulum position is zero (i.e. upright position).

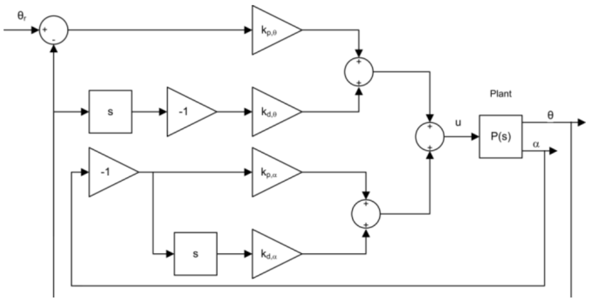

Figure 3.1.1: Block Diagram of PV Controller

As mentioned, the integral term is eliminated taking the constraints of noise and derivative control is used as a velocity feedback and only negative velocity is fed-back to the system. And in practical system we use a filter to suppress the noise generated by the derivative control. By trial and error method we obtain the gain values of the controller as $K_d = -2, K_p = 2$ for the control of the rotary arm and $K_d = 2.5, K_p = 30$ for the control of the pendulum.

3.2 LQR Controller

Linear Quadratic Regulator (LQR) theory is a technique that is ideally suited for finding the parameters of the pendulum balance controller. Given that the equations of motion of the system can be described in the form

$$\dot{x} = Ax + Bu$$

Where Aand Bare the state and input matrices, respectively, the LQR algorithm computes a control law u such that the performance criterion or cost function

$$J = \int_0^\infty \left(x_{ref} - x(t)\right)^T Q\left(x_{ref} - x(t)\right) + u(t)^T R u(t) dt$$

is minimized. The design matrices Q and R hold the penalties on the deviations of the state variables from their set-point and the control actions, respectively. When an element of Q is increased, therefore, the cost function increases the penalty associated with any deviations from the desired set-point of that state variable, and thus the specific control gain will be larger. When the values of the R matrix are increased, a larger penalty is applied to the aggressiveness of the control action and the control gains are uniformly decreased. In our case the state vector x is defined

$$x = [\theta\ \alpha\ \dot{\theta}\dot{\alpha}]^T$$

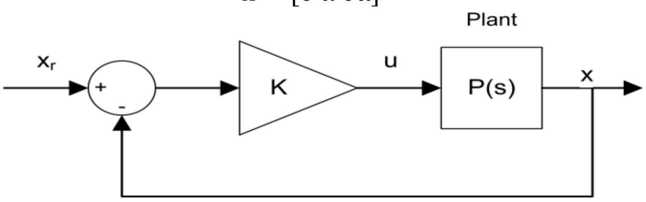

Figure 3.2.1: Block diagram of an LQR Controller

Since there is only one control variable, R is a scalar. The reference signal x_{ref} is set to $[\theta_r\ 0\ 0\ 0]$, and the control strategy used to minimize cost function J is thus given by

$$u = K(x_{ref} - x) = k_{p,\theta}(\theta_r - \theta) - k_{p,\alpha}\alpha - k_{d,\theta}\dot{\theta} - k_{d,\alpha}\dot{\alpha}$$

This control law is a state-feedback control and is illustrated in the above figure. It is equivalent to the PV control designed.

The LQR gain matrix K is obtained by using MATLAB software, using code “lqr(A,B,Q,R)”.

3.3 Observer-Based control

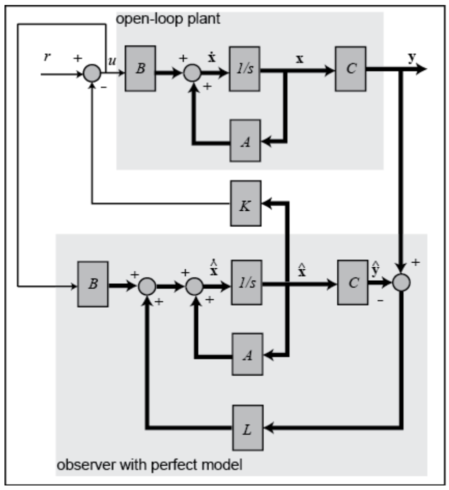

Figure 3.3.1: Block diagram of observer based control

In LQR, all the states are utilised which is an unnecessary action. Hence, here in observer control, we use only the states which are necessary for the control action. To address the situation where not all the state variables are measured, a state estimator must be designed. A schematic of the state estimator is shown below.

The condition to be met is that the system states are completely observable. The dynamics of the state estimate is described by the following equation.

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

The dynamics of the error in the state estimate is described by

$$\dot{e} = \dot{x} - \dot{\hat{x}} = (Ax + Bu) - (A\hat{x} + Bu + L(Cx - C\hat{x}))$$

$$\dot{e} = (A - LC)e$$

Combining the LQR with the state estimate gives us the full compensator and the state-space matrices are given by

$$\begin{bmatrix}\dot{x}\\ \dot{e}\end{bmatrix} = \begin{bmatrix}A - BK & BK\\ 0 & A - LC\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix} + \begin{bmatrix}B\tilde{N}\\ 0\end{bmatrix}r$$

$$y = \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix} + [0]r$$

4.Simulation & Results

The model parameters are shown in below table.

Table 1: Rotary Inverted pendulum Parameters

Motor	
$R_m = 8.4$	Resistance
$K_t = 0.042$	Current-torque (N-m/A)
$K_m = 0.042$	Back-emf constant (V-s/rad)
Rotary Arm	
$M_r = 0.095$	Mass (kg)
$L_r = 0.085$	Total length (m)
$J_r = M_r*L_r^2/12$	Moment of inertia about pivot (kg-m^2)
$D_r = 0.0015$	Equivalent Viscous Damping Coefficient (N-m-s/rad)
Pendulum Link	
$M_p = 0.024$	Mass (kg)
$L_p = 0.129$	Total length (m)
$J_p = M_p*L_p^2/12$	Moment of inertia about pivot (kg-m^2)
$D_p = 0.0005$	Equivalent Viscous Damping Coefficient (N-m-s/rad)
$g = 9.81$	Gravity Constant

After substituting RIP parameters in A and B matrices, we can get

$$A = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 149.2751 & -14.9183 & 49.1493\\ 0 & -261.6091 & 14.7448 & -86.1356\end{bmatrix}$$

$$B = \begin{bmatrix}0\\ 0\\ 49.7275\\ -49.1493\end{bmatrix}$$

4.1 PV Controller

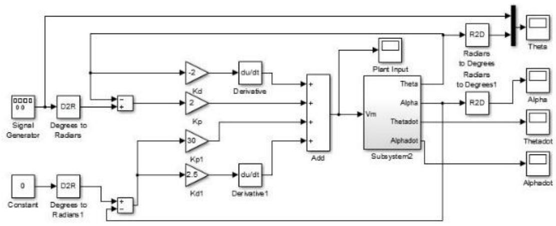

Figure 4.1.1: Simulink Model of PV Controller

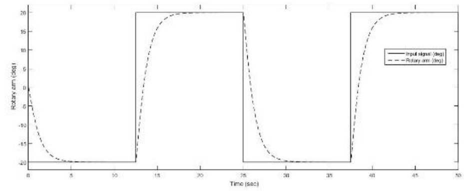

Figure 4.1.2: Response of arm for a PV Controller

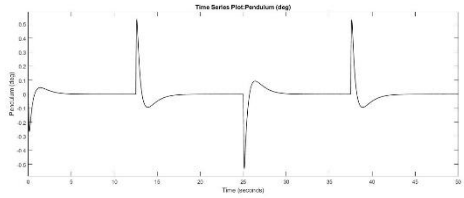

Figure 4.1.3: Response of Pendulum for a PV

4.2 LQR Controller

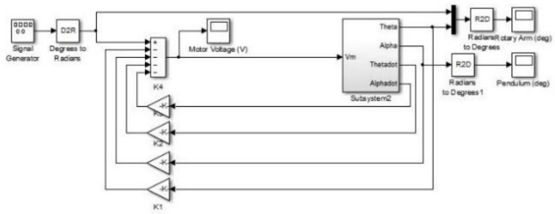

Figure 4.2.1: Simulink Model of an LQR Controller

For R=1; Q=[20 0 0 0; 0 5 0 0; 0 0 1 0; 0 0 0 1]:

K = [4.4721 1.3528 0.9364 0.4866]

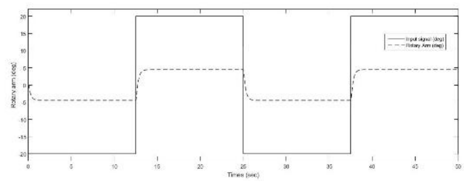

Figure 4.2.2: Response of arm

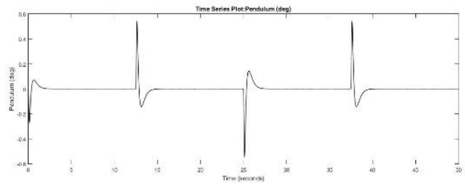

Figure 4.2.3: Response of pendulum

For R=15, Q=[20 0 0 0; 0 5 0 0; 0 0 1 0; 0 0 0 1]:

K = [1.1547 0.2361 0.2010 0.1185]

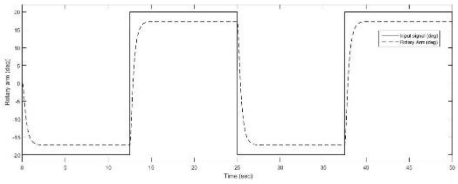

Figure 4.2.4: Response of arm

Figure 4.2.5: Response of pendulum

For R=25, Q=[20 0 0 0;0 5 0 0; 0 0 1 0; 0 0 0 1]:

K = [0.8944 0.1586 0.1499 0.0889]

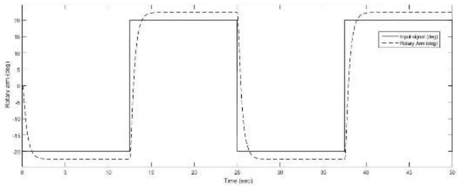

Figure 4.2.8: Response of arm

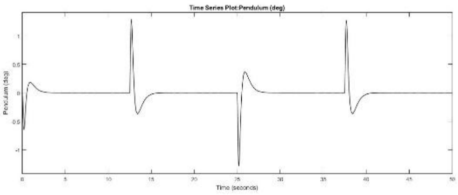

Figure 4.2.9: Response of pendulum

4.3 Observer-Based control

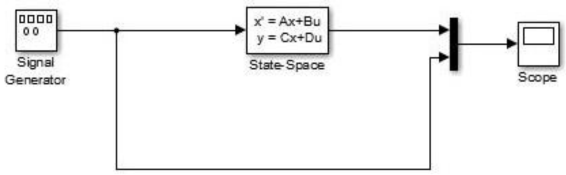

Figure 4.3.1: Simulink Model of observer-based control

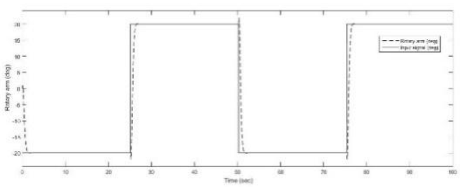

Figure 4.3.2: Response of Rotary arm

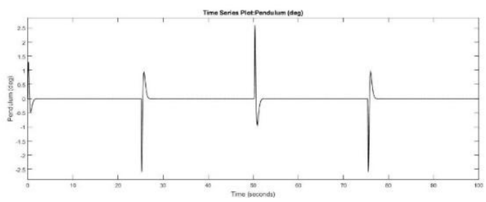

Figure 4.3.3: Response of pendulum

4.4 Comparison of PV and LQR controllers

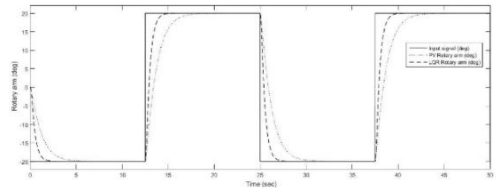

Figure 4.4.1: Response of Rotary arm

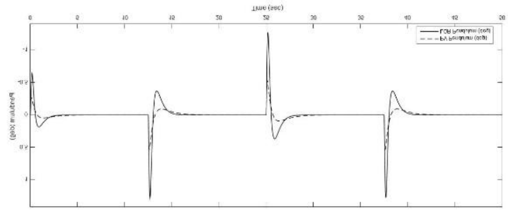

Figure 4.4.2: Response of Pendulum

5.Conclusions

From the simulation results, we can observe that the system can be stabilized by many controllers. But the necessity is for the controller with better response. Among PV and LQR controllers, LQR controller gives much better response. Using the Observer-based control, we can minimize the error and neglect the unnecessary states while controlling. Even though there wouldn't be any difference in the output, but coming to the application oriented it simplifies and reduces the cost required to build the application.

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