International Journal of Scientific Engineering and Research (IJSER)
ISSN (Online): 2347-3878
Index Copernicus Value (2015): 62.86 | Impact Factor (2015): 3.791

Effect of Angularity on Hydraulic Conductivity of Porous Media

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Abstract: ***Permeability, which is also known as Hydraulic Conductivity is considered one of the most important parameters in ground water resources and management. It is dependent on several factors such as grain size, particle size distribution, shape factor and porosity of the porous media. Thus, the problem of estimating permeability becomes an arduous task. To investigate the variation of permeability with respect to shape factor (angularity), the present study has been conducted with the fabrication of an experimental set up consisting of permeameter with appurtenances to measure discharge and pore pressure difference. The data reflects that the shape of the particles is mainly responsible for these variations which affect the hydraulic conductivity of the aquifer material. The experiments have been conducted in laminar range. All the results have been corrected to a constant porosity of 40% using Kozeny's equation. On the basis of experimental results a positive correlation between permeability (K), angularity (η) and size of particles (dg) has been developed. The equation developed can be used for the estimation of permeability involving porous media with similar shaped particles with reasonable accuracy. It has been observed that as angularity of the material increases the K value decreases.***

Keywords: Hydraulic Conductivity, Angularity, Shape factor, Permeameter, Porosity, Kozeny's Equation

1. Introduction

In concrete mix design, the workability or the compaction factor depends upon the angularity of its aggregate. Thus angularity is governed not only by the shape of particles, but also by the size of the particle.

While characterizing the shape of particles, most of the research workers have embraced sphere as the standard shape and proposed shape factors which have been obtained by comparing certain experimental data for the given material with the given data for the sphere. Krumbein and Heywood have developed shape factors which are based on observations and measurements on individual particles. Most of these works were related to sedimentation contemplates [9, 10].

2. Formulation of the Problem

In the present study an attempt is made to develop the relationship between the permeability and angularity of granular materials for linear regime of flow based on dimensional analysis. For this, experiment was conducted on various materials to study the following relationships between- Friction factor and Reynold's number; and Size of particles and Angularity. A brief discussion regarding the formulation of the problem is given below:

2.1 Relationship between Friction Factor and Reynold's Number

The permeation of fluid through porous media depends on a number of variables. According to Rose [2], head loss across the porous bed, H may be expressed as,

$$H = F_1 (v, l, d, D, \rho, \mu, g, e, n, Z, U) \quad (1)$$

Where, H is the head loss across porous bed; v, superficialvelocity of flow; l, the length of bed; d, size of particle; Diameter of the permeameter; ρ, mass density of fluid; μ,viscosity of fluid; g, acceleration due to gravity; e, surfaceroughness of particles; n, porosity of the bed; Z,dimensionless shape factor; U, dimensionless factor for sizedistribution.

By dimensional analysis equation 1 reduces to,

$$\frac{H}{d} = F_2\left(\frac{\rho v d}{\mu}, \frac{gd}{v^2}, \frac{L}{D}, \frac{D}{d}, n, Z, \frac{e}{d}, U\right) \quad (2)$$

If beds composed of uniform sizes are studied and it is assumed that surface roughness will have a comparatively small effect on porous media flow, equation (2) can be written as,

$$\frac{H}{d} = F_3\left(\frac{\rho v d}{\mu}, \frac{gd}{v^2}, \frac{L}{D}, n, Z,\right) \quad (3)$$

Parameters H/d, L/d, and gd/v2 can be combined to give a single dimensionless factor, conventionally known as the friction factor i.e.

$$F_3\left(\frac{H}{d}, \frac{gd}{v^2}, \frac{L}{D}, \frac{D}{d}, n, Z, \frac{\rho v d}{\mu}\right) = 0 \quad (4)$$

Equation 4 may be further simplified as;

$$F_4\left(\frac{igd}{v^2}, \frac{\rho v d}{\mu}, \frac{D}{d}, n, Z,\right) = 0 \quad (5)$$

Where, i=H/L is the hydraulic gradient, $\frac{\text{igd}}{\text{v2}}$ is the friction factor and $\frac{\rho v d}{\mu}$ is the Reynold's no.

If all the experiments are conducted at a constant porosity and it is assumed that the D/d ratios involved in this study will have negligible effect on percolation. Equation 5 simplifies to

$$F_5\left(\frac{igd}{v^2}, \frac{\rho v d}{\mu}, n, Z,\right) = 0 \quad (6)$$

2.2 Relationship between the Size of Particles and their Angularity

The angularity of particles, defined as the porosity of the material when it is compacted in a standard manner prescribed in BS: 812-1967, will depend on the shape of particles as well as on their size. Mathematically, this can be expressed as,

$$\eta = F_6 (Z, d) \quad (7)$$

Where η is the porosity when the material is compacted in a standard manner by prescribed BS: 812-1967 [3]. Other terms have been defined earlier. For a material belonging to the same shape group, Z will be constant in Equation7. Therefore, Equation (7) can be modified as

$$\eta = F_7 (d) \quad (8)$$

To study the functional relationship between the angularity and size of particles, tests were conducted on various sizes of different materials and the results analysed. An attempt has been made to correlate equation (6) and (7) [4].

3. Experimental Work

Figure 1 shows the experimental set up.

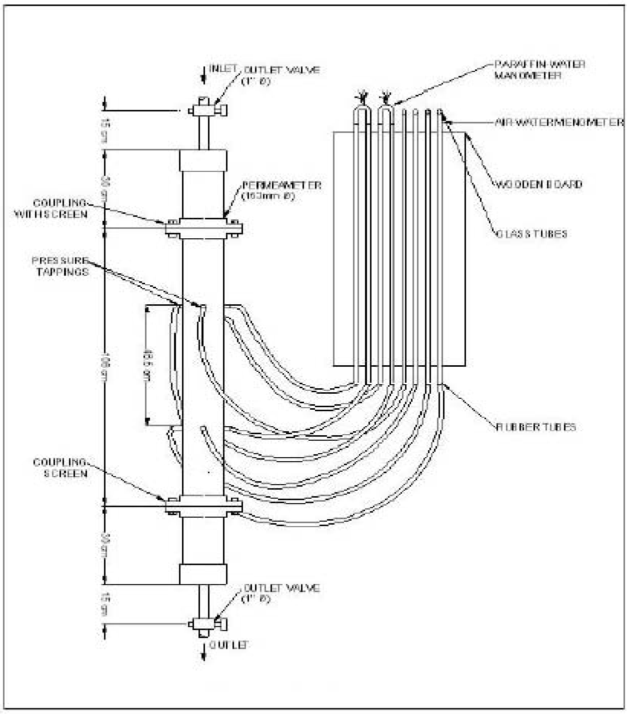

Figure 1: Details of permeameter set-up

3.1 Materials

The various materials used in the present study have been listed below:
i) Dolomite: 2.24 cm, 2.64 cm uniform sizes were used.
ii) Marble Chips: 1.12 cm, 1.32 cm uniform sizes were used.
iii) River Gravel: 2.24 cm, 2.5 cm uniform sizes were used.
iv) Sand: 0.03 cm, 0.085 cm uniform sizes were used.
v) Crushed Quartzite: 0.4 cm, 0.63 cm sizes were used.

3.2 Experimental Equipment

The experimental equipment comprised
(a) Source of supply
(b) Permeameter
(c) Discharge measuring device
(d) Manometers
(e) Pycnometer

(a) Source of supply
The permeameter receives its water supply from an overheadtank at a height of 2.65m above the permeameter outlet. Thetank receives its supply from a re-circulating tank so that aconstant head is maintained in the overhead tank.

(b) Permeameter
The constant head vertical flow type permeameter was usedfor hydraulic tests in this work. The main permeametersection consisted of a 10.16 cm internal diameter GI tubewith a total length of 1.06 meter with a test length of 46.5cm. Four pressures tapping making an angle of $90^{o}$to each other were provided along the circumference of permeameter at the starting and ending points of the test length. This arrangement of tapping points was adopted to ensure the mean pressure at the section under consideration. The inlet to the permeameter was regulated with the help of an outlet sluice valve of 25.42mm diameter. I.S. 2.0 mm mesh screen was used in the filter for resting the porous media. For filling and removing of the material, the permeameter was detached from its supports each time.

(c) Discharge measurement
The discharge was measured by volumetric method. The water was collected in a bucket for a certain period, which was recorded with a stopwatch and collected water was then measured with the help of a 2000 cc capacity glass jar. Volume of water collected at a particular duration will give the discharge.

(d) Manometer
To cover the desired range of flow, two types of manometer were used

- Air-water manometer
  Simple piezometers were used to measure the head losses of about 0.1 cm to 35.0 cm of water.
- Paraffin water manometer
  Inverted U-tube paraffin water differential manometers were used to measure low head losses. The manometers were supported on a wooden board with a graduated scale in cm, giving a correct reading of manometer up to one mm.

(e) Pycnometer
I.S. pycnometer is used in the specific gravity test.

3.3 Experimental Procedure

The various tests conducted during the course of this study can be divided into three categories.

(a) Preparation of samples
(b) Hydraulic tests
(c) Angularity Tests

(a) Preparation of samples: The materials used in this work were graded into various samples of known proportion and size with the help of various tests such as the Sieve analysis test, Specific gravity test and Angularity test.
(b) Hydraulic tests: The hydraulic tests were conducted tostudy the effect of resistance to flow of water in a givensample of material. The method of carrying out thesetests was as follows.
(c) Angularity tests:The size of the angular materials used in the present investigation was 0.3 mm minimum to 1.6 cm maximum. The size of the cylinder used was 15.0 cm in diameter and 30.5 cm deep. This size fulfilled the requirement that $D/dg \geq 8.0$. As Recommended by B.S. 812:1967. (Here D is the diameter of the cylinder and dg is the geometric mean size of the material.) All tests were performed in accordance with B.S. 812:1967.

Angularity No (A.N.) = η– 33

Where, η = porosity or percentage voids when the material is compacted in a standard manner prescribed by B.S 812:1967, Wc = weight of the aggregate in the cylinder, Ww = weight of the water required to fill the cylinder Gs = specific gravity of the aggregates

Preparation of the Bed

Before filling the permeameter with the material to be tested, the inlet portion of the permeameter was taken off. It was proposed in the present study to keep the porosity constant for all runs of the materials. Therefore, the weight of the material needed to fill the permeameter was calculated as:-

$$W_S = (1-n)\, V_T G_S \gamma_W \qquad (9)$$

Where W_S is the weight of the material; n, porosity; V_T, volume of the tube; G_S, specific gravity of the material; γ_W,specific weight of the water.

In order to get a uniformly packed bed throughout the length of the permeameter, the necessary quantity of material was divided into ten equal parts. Each part was thrown gently over the whole cross-section of the tube, and compacted by steel rod, after the surface was levelled. The packing method was same for all the runs. The number of blows required for compacting the material varied for various materials.

After packing the tube and levelling the top of the material, the coupling was fitted. After fixing the permeameter in the vertical position and connecting to the water supply system, the outlet and inlet valves were completely opened to make the material saturated for 5 to 6 minutes. Then the outlet valve was slowly closed so that the water entered the manometric tubes. Before starting the test, air in the permeameter tube as well as in manometer tubes was removed. After removing the air, the outlet valve was completely opened to start the test with maximum discharge.

3.4 Test Run

This involved three main operations:

a) Measuring the discharge through the permeameter.
b) Reading the pressure drop across the test length of the material.
c) Reading the temperature of water

In all the runs, observations were taken in receding order of magnitude of discharge and hydraulic gradient. The test was started with maximum discharge so that if there was any settlement of the bed due to the impact of the jet of incoming water, it was secured in the beginning. The bed was then checked at the end of each run for settlement, if any. The porosity of the bed was determined after making due allowances for settlement. The discharge was measured by collecting the water in a bucket for a certain period, which was recorded with a stopwatch. The flow was then reduced for the next observation of the run. This operation was continued till the discharge was decreased from maximum to a certain measurable minimum discharge. For each discharge two observations were recorded to make sure that flow was steady for each set of observation.

For higher discharge as the pressure drop was higher, the air water manometer was used. And for low discharge paraffin water manometer was used. The manometer readings from paraffin-water manometer were converted to head of water. The temperature of water was recorded at the beginning and the end of each run. After data was collected for one run, the procedure was repeated.

4. Analysis of Results

The present study investigates a relationship between the permeability (K) of the materials used, their angularity (η) and size of the particles (dg- geometrical diameter of particle). To study the relationship between angularity (η) and size of particles (dg), curve of logarithm of angularity (log η) and logarithm of size (log dg) of particles is drawn. The results of the experimental investigation on the different materials used in the present study are presented as Friction factor (Fr) vs Reynold's number (Re) graph on log-log scale. A discussion of results in relation to the different aspects of the problem studied is given below.

4.1 Relationship between angularity and size of particles

The angularity of the particles can be used a measure of the shape of the particles; but this definition of shape factor applies to irregular shaped only. In addition, the angularity of any material depends on its size. The angularity tests, were therefore conducted in irregular material of three sizes apart from one regular material. Experimental results for angularity tests for these materials are shown below in Table 1.

Table 1: Variation of angularity with size of particles

S.No	*Material*	*Size (mm)*	*Angularity (%)*
1	Dolomite	2.24	41.095
		2.64	40.224
2	Marble Chips	1.12	43.03
		1.32	42.446
3	River Gravel	2.24	45.799
		2.5	45.742
4	Crushed Quartzite	0.4	45.512
		0.63	44.755
5	Sand	0.03	43.031
		0.085	42.462

From a study of these results, it is seen that the angularity of any material decreases with an increase in its size. These results are plotted using log of angularity (log η) against the corresponding log of geometric mean diameter (log dg) of the material (Figure 1). All the data appear to follow a straight-line law given by

$$\log \eta = n \log d_g + \log C' \quad (10)$$

Equation 10 can be simplified as,

$$\eta = C' d_g^{\,n} \quad (11)$$

The value of index n in this equation was found to beconstant at -0.0402 (from Figure1) for all the materials. Thevalue of C' varies with shape of particles and its value,obtained from Figure 1, for various materials is shown in Table 2.

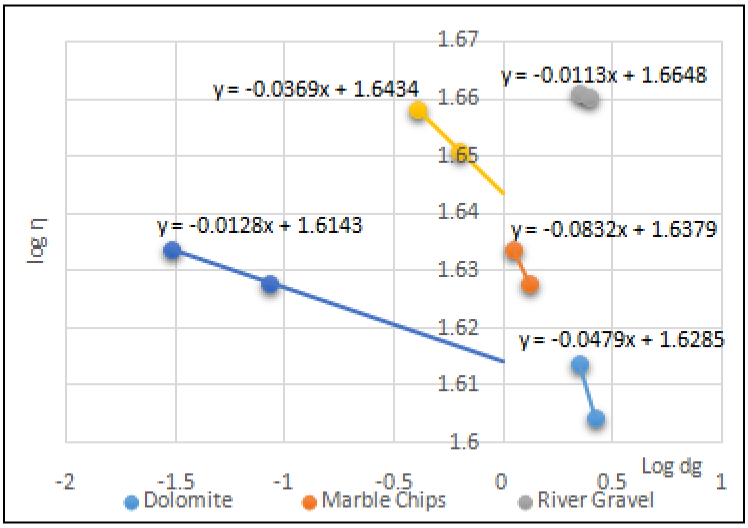

Figure 1: Curve of log η against log d_g

The Equation 11 can therefore, be written as:

$$C' = \eta\, d^{0.0402} \quad (12)$$

Where,
C' = parameter defining the angularity and depends on shape of the particles.

4.2 Relationship between permeability and angularity of granular material

The permeation of fluids through porous masses depends on a number of variables as mentioned in Equation 1. For uniform granular media at a constant porosity, the law governing the seepage phenomenon is given by Equation 5 given earlier. The results of experimental investigation for different materials can therefore be plotted on log-log graph as Friction factor ($Fr = 2igd_g/v^2$) vs. Reynold's number ($Re = \rho dg/v$) as shown in Figure 2-6.

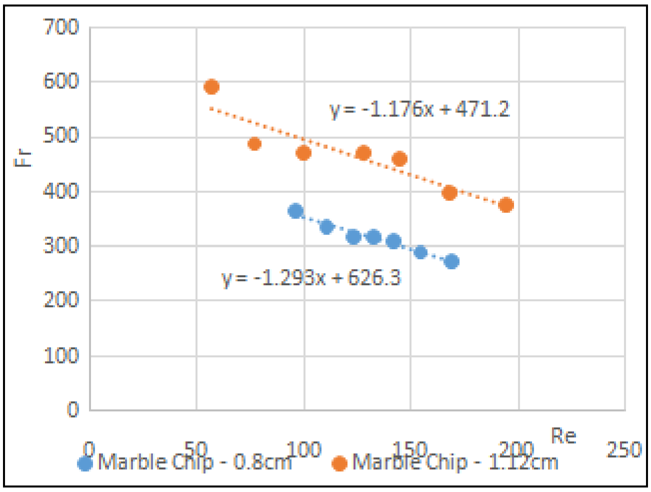

Figure 3: Fr-Re Curve for Marble Chips

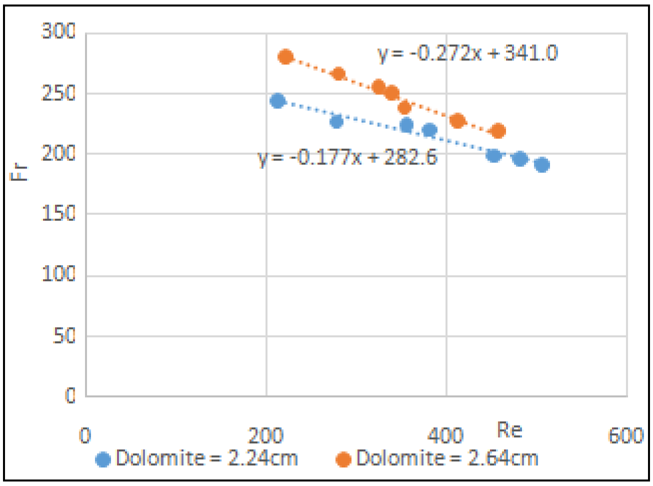

Figure 2: Fr-Re Curve for Dolomite

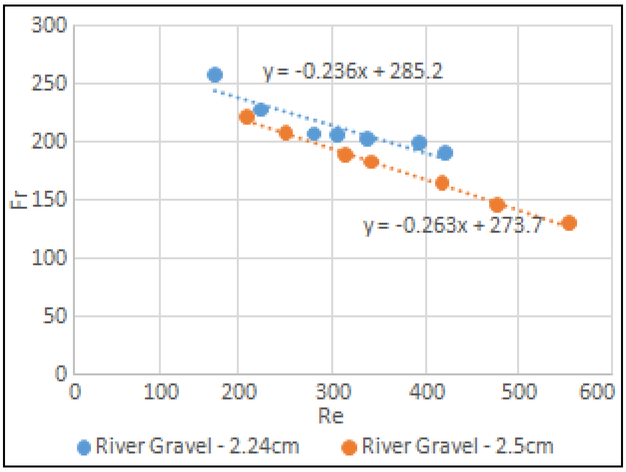

Figure 4: Fr-Re Curve for River Gravel

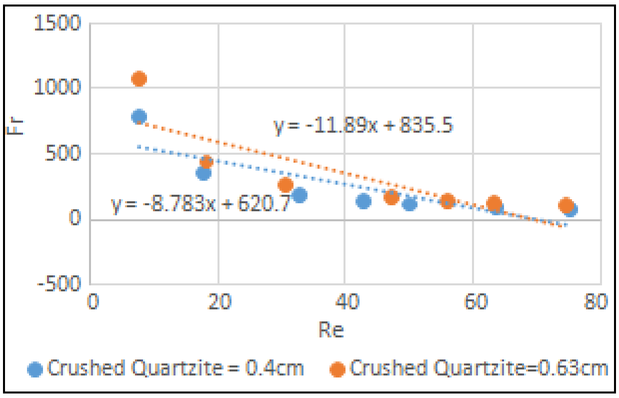

Figure 5: Fr-Re Curve for Crushed Quartzite

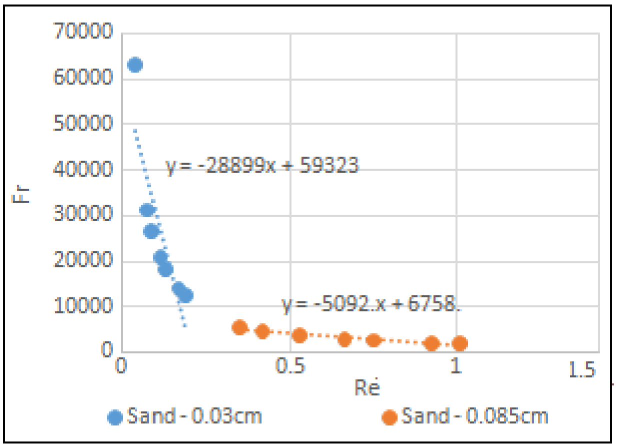

Figure 6: Fr-Re Curve for Sand

The variation was found to be linear. Within linear regime of flow, these results follow an equation of the form:

$$Fr = C/Re \qquad (13)$$

On substituting $F = 2igd/V^2$, $Re = vd/\nu$ and V=Ki
In equation 13,

$$\frac{K}{d_g^{\ 2}} = \frac{2\rho g}{\mu C_1} or, \frac{K}{d_g^{\ 2}} = \frac{2g}{\nu C_1} = C \quad (14)$$

Where, C_1 is the constant which for a given porosity depends only on the shape of particles; C, parameter defining the permeability of the material; d_g, geometric mean diameter of particles;ν, kinematic viscosity of the fluid, other parameters being as mentioned earlier.

For a constant porosity, the value of 'C' varies with shape of particles. The values of C_1 obtained from Figure 2-6 are shown in Table 2.

Table 2: Values of C and C_1

S. No	*MATERIALS*	*C_1*	*C*	*LOG C*	*LOG C'*
1	Dolomite	1619.9	136.86	2.136	1.628
2	Marble Chips	5624.6	39.42	1.596	1.635
3	River Gravel	1571.7	141.05	2.149	1.684
4	Sand	2176.4	101.86	2.008	1.581
5	Crushed Quartzite	7370.7	30.07	1.478	1.642

From equation 13 it is obvious that the coefficient C' depends on the shape of particles. Similarly, factor C in equation 14, which defines the permeability of materials, is also dependent on shape alone provided that the porosity and other factors are constant, it is therefore concluded that C' is a function of C. Mathematically, this can be expressed as:

$$C' = f(C) \qquad (15)$$

To study the relationship between C' and C, values of C' and C obtained from experimental results are plotted as log C vs log C' (Figure 7).

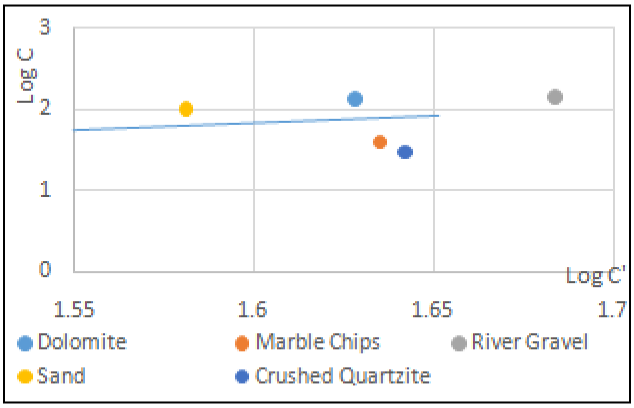

Figure 7: Log C vs Log C' at 40% porosity

To make C' independent of porosity, it was proposed to conduct all these tests at same porosity, i.e. 40% in each case. To achieve this it was found necessary to compact the material by crushing it with a heavy rammer, which could have crushed the material. No attempt was therefore made to compact the material with a heavy rammer to get the proposed porosity. Alternatively the tests had to be conducted at different suitable porosity for each material and by using Kozeny's formula, [5, 8] all the values of C obtained from equation 13, have been corrected to those at 40% porosity. All the points in Figure 7 followed a straight line given by

$$\text{Log } C' = \log B + m \log C \qquad (16)$$

Equation 16 can be simplified as

$$C' = BC^m \qquad (17)$$

The values of B and m in the above equation were obtained with the help of Figure 7

The final equation can be written as,

$$C' = 59.5937\, C^{-0.0859} \qquad (18)$$

Or

$$C^{+0.0859} = \frac{59.5937}{C'} \quad (19)$$

Substituting the values of C' and C from equations 12 and 14, respectively, the above equation can be expressed as:

$$\left[\frac{K}{d_g{}^2}\right]^{-0.3134} = \frac{22.99}{\eta d_g{}^{0.0439}}$$

$$K = \frac{4.5 \times 10^{-5} \times d_g{}^{2.14}}{\eta^{-3.19}} \quad (20)$$

Where K is the permeability of soil; dg is geometric mean diameter of particles (cm); and η is the angularity of particles (%).

From the above equation, it is obvious that that by conducting angularity number tests and sieve analysis, an idea about the permeability of a given material can be obtained. Since Equation 20 is based on experimental results at 40% porosity, the above value of permeability is applicable to this porosity only. For other porosity a suitable porosity function can be used.It has been observed that as angularity of the material increases the K value decreases as shown in figure 7.

Table 3: Comparison of Angularity & Hydraulic Conductivity

S. No	*Material*	*Size (Cm)*	*Angularity (%)*	*Hydraulic Conductivity (cm/s)*
1	Dolomite	2.24	41.095	21.3709
		2.64	40.224	59.1475
2	Marble Chips	1.12	43.03	15.227
		1.32	42.446	25.016
3	River Gravel	2.24	45.799	56.748
		2.5	45.742	67.99
4	Crushed Quartzite	0.4	45.512	14.139
		0.63	44.755	30.827

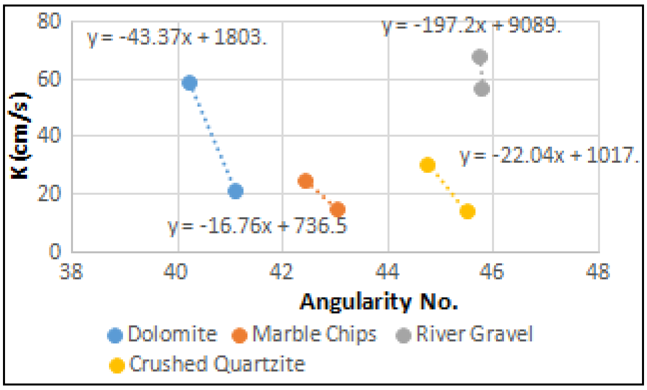

Figure 7: Angularity vs Hydraulic Conductivity

5. Conclusion

From the discussion related to the analysis of results, an important conclusion drawn on the basis of this study reflects that the experimental results agreed closely with Hazen's formula $K = C\,D_{10}{}^2$(where C varies from 41 to 146) [6, 7]. It was found that the main cause of variation of the coefficient C in Hazen's formula is the shape of particles. It shows that the hydraulic conductivity is affected by the shape of the particle besides other factors related to it. The proposed formula (equation 20) is applicable for 40% porosity and 25°C temperature. It can be used to calculate the hydraulic conductivity of natural materials by conducting angularity tests and sieve analysis.It has also been observed that as angularity of the material increases the K value decreases.

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