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Heat transfer Analyses of Boundary Layer Fluid Flow in Presence of Variable Thermal Conductivity and Space Dependent Heat Generation /Absorption

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Abstract: ***The effects of space dependent heat generation/absorption in presence variable thermal conductivity on the boundary layer Visco-elastic fluid flow over stretching surface are analyzed. The governing boundary layer equations are reduced to ordinary non-linear differential equations using similarity transformation. Solutions of heat transfer equations are obtained numerically by shooting technique usingfourth order Runge kutta method. The effects of different physical parameters such as Magnetic field, Prandtl number etc on temperature profile are thoroughly discussed.***

Keywords: Thermal conductivity, space dependent heat generation/absorption. visco-elasticity, porosity

1. Introduction

The boundary layer flow over continuously stretching surface moving with a certain velocity in an otherwise quiescent fluid medium is an often-encountered flow in many engineering processes. Aerodynamic extrusion of plastic sheet, cooling of an infinite metallic plate in cooling bath, in condensation process of liquid film and polymer sheet etc are the practical applications of moving surface and also there are lots of applications in industries such as the hot rolling, wire drawing, glass fiber production and so on. In these processes it is very important to control the drag and the heat flux at the stretching surface in order to obtain good quality of products.

The pioneering work in this area was done by Sakiadis [1], in his work, he described the boundary layer assumptions and governing equations of the problem by considering the boundary layer viscous fluid flow over a continuous solid surface moving with constant velocity. It was then extended to that of stretching of boundary sheet with linear velocity by Crane [2]. His work was further verified by Tsou et.al [3] experimentally, at the same time, the thermal boundary layer for this flow configuration with constant wall temperature was discussed. Erickson et.al. [4] extended the work of sakiadis to account for mass transfer at the stretching sheet surface. Following these works the boundary conditions on the surface were extended by other researchers [5]-[10].

There are various applications in which significant temperature differences between the body surface and the ambient fluid exist. it is usually assumed that the sheet is inextensible, but in some different situations like in the polymer industries where it is necessary to deal with stretching of plastic sheet as mentioned by Crane[2].Chen and Char [7] have examined the heat transfer behaviors in this flow, considering the effect of suction and injection where the boundary surface is maintained with variable temperature. Considering the effect of temperature difference between the surface and the ambient fluid, some work have been carried out by Vajravelu and Rollins [11], Vajravelu and Nayfeh [12] on the flow and heat transfer introducing temperature dependent heat source/sink.

All the above investigations are restricted to the flow of Newtonian fluids. However, in recent past, the study of non-Newtonian fluid flow had shown immense interest because of ever increasing application of plastic films and artificial fibers in industry. In view of this the above study of boundary layer flow problem has been further channelised to the non-Newtonian fluid flow. Considering the survey of literature, it is noticed that Rajgopal et.al [13] considered the study of visco-elastic second order fluid flow over a stretching sheet by solving the boundary layer equation numerically. Battacharya et .al [14] and Nataraja et.al [15] have presented the problem of heat transfer in a visco-elastic fluid over a stretching sheet. Abel et.al [16] studied the effect of magnetic field on visco-elastic fluid flow and heat transfer over non- isothermal stretching sheet with internal heat generation. The study of heat generation or absorption effect is important in view of several physical problems, such fluids undergoing exothermic or endothermic chemical reactions Although, exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can express its average behavior for most of physical situations, Crepeau and Clark Sean [17] have used a space dependent exponentially decaying heat generation or absorption in their work on flow and heat transfer from a vertical plate. Postelnieu et al [18,19] have presented their work on convective flows with internal heat generation/absorption for both viscous fluid and fluid saturated porous media. Abo-Eldahab and Md.A.El.Aziz [20] analyzed the mixed convection boundary layer flow over an inclined continuously stretching surface with internal heat generation /absorption in presence of magnetic field with suction /blowing effect. Keeping this in view, we contemplate in present work to study the effect of space dependent internal heat generation/absorption with variable thermal conductivity on MHD boundary layer flow over non-isothermal stretching surface.

2. Mathematical Analysis

Basic boundary layer equations governing the flow of Walters liquid B can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma B_o^2}{\rho}u-\frac{\gamma}{k'}u \tag{2}$$

The boundary conditions governing the flow are:

$$u = b\,x \quad v = 0 \qquad \text{at} \qquad y = 0$$

$$u \to 0, \quad \frac{\partial u}{\partial y}\to\infty \qquad \text{as} \qquad y\to\infty \tag{3}$$

With b>0, Here *u* and *v* are velocity components along *x* and *y* direction σ is the electrical conductivity, B_0 is the applied magnetic field, k_0 is the visco-elastic parameter of the Walter's liquid *B*. *k'* permeability of porous medium, The other quantities have their usual meanings.

Flow analysis

The flow is caused slowly by the stretching of the sheet, the free stream velocity being zero, in order to obtain the mathematical form of the velocity field; we introduce the following similarity transformations as:

$$u=bxf'(\eta),\quad v=-\sqrt{b\nu}\,f(\eta)$$

$$\text{Where}\quad \eta=\sqrt{\frac{b}{\nu}}.y \tag{4}$$

With these changes of variables, equation (1) is identically satisfied and equation (2) is transformed into the following non-linear ordinary differential equation.

$$f'^2-ff''=f'''-k_1\{2f'f'''-ff''''-f''^2\}-M_nf'-k_2f' \tag{5}$$

Where

$$k_1=\frac{k_ob}{\gamma},\quad M_n=\frac{\sigma B_o^2}{b\rho},\quad k_2=\frac{\gamma}{k'b} \tag{6}$$

are non-dimensional visco-elastic parameter, Magnetic parameter and porosity parameter respectively and the boundary conditions takes the form

$$f=0 \quad f'=1 \qquad \text{at} \qquad \eta=0$$

$$f'\to 0 \quad f''\to 0 \quad \text{as} \qquad \eta\to\infty \tag{7}$$

Where prime denotes differentiation w.r.t η. The exact solution of equation (5) corresponding to the boundary conditions (7) is obtained as:

$$f=\frac{1}{\alpha}(1-e^{-\alpha\eta})$$

$$\text{Where } \alpha=\sqrt{\frac{1+Mn+k_2}{1-k_1}} \tag{8}$$

The solution for velocity field is obtained as:

$$u=bxe^{-\alpha\eta}\ ,\qquad v=-\sqrt{b\gamma}\,\frac{1-e^{-\alpha\eta}}{\alpha} \tag{9}$$

our result (9) is in good agreement with the result of Anderson [4] for the limiting case *k=0*

3. Heat Transfer Analysis

The energy equation by assuming thermal conductivity as function of temperature with space dependent internal heat generation /absorption for the two dimensional flow is:

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+q''' \tag{10}$$

Where ρ is the density, c_p is the specific heat at constant pressure, k is the thermal conductivity, which is assumed to vary with temperature and is given by

$$k=k_\infty(1+\varepsilon\theta)$$

$$\text{Where } \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ ,\qquad \text{(PSTCASE)}$$

$$k=k_\infty(1+\varepsilon g)$$

$$\text{Where } g(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ ,\qquad \text{(PHFCASE)} \tag{11}$$

where $\varepsilon=\frac{k_w-k_\infty}{k_\infty}$ is very small parameter which depends on the nature of the fluid and k_∞ is the thermal conductivity of the fluid far away from the sheet and k_w is thermal conductivity at the wall. The internal heat generation /absorption term q''' is modeled as

$$q'''=\frac{ku_{w(x)}}{x\upsilon}\left[A^*(T_w-T_\infty)e^{-\alpha\eta}\right] \tag{12}$$

Where A^* is coefficient of space dependent internal heat generation /absorption, and $A^*>0$ corresponds to internal heat generation while $A^*<0$ corresponds to internal heat absorption. Thermal boundary conditions depend upon the type of the heating processes. Here we consider two different types of heating process namely:

1) Prescribed surface temperature
2) Prescribed power law heat flux

Case (1): Prescribed Surface Temperature:

In this case we consider the boundary conditions a

$$T=T_w=T_\infty+A\,x^\lambda \qquad at \quad y=0.$$

$$T\to T_\infty \qquad as \qquad y\to\infty \tag{13}$$

T_w is variable wall temperature, A is a constant and λ is wall temperature parameter. When λ =0 the thermal boundary condition becomes isothermal. Using similarity transformation (11) and (12) ,equation (10) and (13) takes the form

$$(1+\varepsilon\theta)\,\theta'' + Pr\, f\,\theta' - Pr\,\lambda\, f'\,\theta + \varepsilon\,\theta'^2 + (1+\varepsilon\theta)A^* e^{-\alpha\eta} = 0$$

$\theta(\eta) = 1 \quad at \quad \eta = 0$ (14)

$\theta \to 0 \quad as \quad \eta \to \infty$ (15)

Where prime denotes differentiation w.r.t η

and $\Pr = \dfrac{\mu c_p}{k}$

Case 2: Prescribed Power Law Heat Flux

For this heating process, the boundary conditions are

$$-k\frac{\partial T}{\partial y} = Bx^{\lambda} \quad \text{at} \quad y = 0$$

$$T \to T_\infty \quad as \quad y \to \infty \qquad (16)$$

Where λ is the wall heat flux parameter, *B* is constant.

Defining

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

Where $T_w - T_\infty = \dfrac{Bx^\lambda}{k}\sqrt{\dfrac{b}{\upsilon}}$ (17)

and $k = k_\infty(1 + \varepsilon g)$,

With this change of variable equation (10) and corresponding boundary conditions takes the form

$$(1+\varepsilon g)\, g'' + \varepsilon g'^2 + \Pr f g' - \Pr s f' g + (1+\varepsilon g)A^* e^{-\alpha\eta} = 0 \qquad (18)$$

The boundary conditions are

$g'(\eta) = -1 \quad at \quad \eta = 0$

$g(\eta) \to 0 \quad as \quad \eta \to \infty$ (19)

We solve equations (14) and (18) subject to the boundary conditions (15) and (1 9) respectively.

Our interest lies in investigation of the flow behavior and heat transfer characteristics by analyzing the non-dimensional Nusselt number (Nu). This non-dimensional parameter is defined as:

$$Nu = \frac{-h}{T_w - T_\infty} T_y = -\theta'(0) \qquad \text{(PSTcase)}$$

$$Nu = \frac{-h}{T_w - T_\infty} T_y = \frac{1}{g(0)} \qquad \text{(PHF case)}$$

4. Numerical Solution

Since equation (14) and (18) are non-linear ordinary differential equations and exact solution do not seems to be feasible, therefore we solve equation (14) and (18) numerically, by using most efficient numerical shooting technique with fourth order Runge kutta algorithm to solve them [16].

In shooting technique we apply initial value method for which one more initial condition $\theta'(0)$ is necessary. Since $\theta'(0)$ is not prescribed. We start with an initial approximation to one unknown $\theta'(0)$ = α and approximate the value of η_∞, we consider an initial approximation by choosing α = α_0 as α_0 =1.0 then we solve the problem using Runge Kutta fourth order method, Selection of an appropriate finite value of η_∞ is most important aspect in this method. To select η_∞, we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $\theta_k(0)$ ($\theta_k = \theta'(0)$ in PST case and $\theta_k = \theta(0)$ in PHF case). The solution process is repeated with another larger (or smaller, as the case may be) value of η_∞. The values of $\theta_k(0)$ compared to their respective previous values, if they agreed to about six significant digits, the last value of η_∞ is used as the appropriate value for that particular set of parameters; otherwise the procedure was repeated until further changes in η_∞. which did not lead to any more change in the values of $\theta_k(0)$. The initial step size employed was h=0.01. The convergence criterion largely depends on fairly good guesses of the initial conditions in the shooting technique, and is based on the relative difference between the current and the previous iterations used, when this difference reaches 10^{-5} the solution is assumed to have converged and the iterative process is terminated. Then we integrate the resultant ordinary differential equations using standard fourth order Runge-Kutta method [16].

5. Results and Discussion

Numerical computation of the model has been carried out for different physical parameters like Visco-elastic parameter (k_1), Magnetic field parameter (*Mn*), space dependent term *(A*)* and Prandtl number *(pr)*. For detailed discussion of the results, the numerical values are plotted graphically in fig.1-8.Results for prescribed surface temperature (PST) are drawn in fig.1-4. It is noticed from these figures that the temperature distribution is unchanged at the wall (unity) with change of physical parameter, where it tends to zero in the free stream. Results for prescribed power law heat flux (PHF) are drawn in fig.5-8, here we observe that the wall temperature is not unity, it is changed at the wall with the change of physical parameters. Table-I shows a comparison of the present results for visco-elastic fluid in presence of variable thermal conductivity for the temperature gradient - $\theta'(0)$ with A.Postelnieu [18] and in our case we found that the effect of internal heat generation is more pronouncing for both positive and negative values of λ, and also temperature gradient increases when λ increases. This is logical because the internal heat generation results in an increase of the heat transfer close to the plate and this will induce more flow along the plate. Table-II illustrates the comparison of - $\theta'(0)$ values with those of Abo-Eldahab and Md.Aziz [20], we found qualitatively that our result is similar but quantitatively the values of temperature gradient - $\theta'(0)$ are less in our case.

Fig1 gives the graphical representation of the temperature profile $\theta(\eta)$ vs. η for different values of Visco-elastic parameter (k_1), temperature profile increases when k1 increases, this is due to fact that the thickening of thermal boundary layer occurs due to the increase of Visco-elastic normal stress. Fig2 illustrates the influence of magnetic field parameter (Mn) on temperature profile; the presence of magnetic field in an electrically conducting fluid gives rise to a resistive type of force called Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer and to increase the temperature. Also the effect on the flow and thermal fields become more, so as the strength of the magnetic field increases. The influences of the presence of space dependent internal heat generation (A*>0) or absorption (A*<0) in the boundary layer on temperature field is presented in fig3. it is clear from this figure that there is increase in temperature distribution of the fluid, when A^* increases. This is expected since the presence of heat source (A*>0) in the boundary layer generates energy, which causes the temperature of the fluid to increase. However as the heat source effect becomes large (A*=2) a distinctive peak in the temperature profile occurs in the fluid adjacent to the wall. This means that temperature of the fluid near the sheet is higher than the sheet temperature and consequently, heat is expected to transfer to the wall. Fig 4 represents the temperature profile $\theta(\eta)$ vs. η for different values of Pr. we infer from this figure that the temperature profile decreases with increase in Prandtl number (Pr). This is because of the fact that the thermal boundary layer thickness decreases with increase in Prandtl number (Pr).

Table I: Comparison of the values of -$\theta'(0)$ for different values of λ

λ	A. Postelnieu work [18]	Present Work for visco-elastic fluid with ε =. 1
-.35	3.6570	0.108458
-.3	1.51211	0.125055
-.2	0.72532	0.158076
-.1	0.40463	0.190254
0	0.20020	0.221668
.5	-0.39361	0.370349
.1	-0.79298	0.506899
1.5	-1.16250	0.633080
2	-1.42250	0.751243

The graphs for the situation when the boundary has been prescribed with heat flux (PHF) are shown in fig.5-8. It is noticed from these figures that the wall temperature is not unity, it is changed at the wall with the change of physical parameters like Visco-elastic parameter (k_1), magnetic field parameter (Mn), space dependent term (A*) and Prandtl number (Pr).we observe that these parameters have same qualitative effects which we found in PST case but quantitatively wall temperature is more in PHF case.

Over all qualitatively our results are in good agreement with Abo-Eldahab, Md.Aziz [20] but quantitatively it is less.

6. Summary and Conclusions

The governing equations for steady laminar flow of an in compressible and electrically conducting visco-elastic fluid over non- isothermal stretching surface in presence of space dependent internal heat generation/absorption with variable thermal conductivity was formulated. The resulting partial differential equations are transformed to ordinary non-linear differential equations by using similarity transformations. Because of its non-linearity, numerical evaluations are performed and graphical results are obtained to illustrate the details of flow and heat transfer characteristics and their dependence on some of the physical parameters.

The important findings of our study are as follows.
1) The effect of visco-elastic parameter is to increase the temperature profile in both the cases of prescribed surface temperature and prescribed wall heat flux.
2) The effect of magnetic parameter is to increase the temperature profile in both the cases of prescribed surface temperature and prescribed wall heat flux.
3) Temperature profile increases as the coefficient of space dependent internal heat generation/absorption increased.
4) Increase of Prandtl number results in decrease of wall temperature in the case of PST as well as PHF.

Table II: Comparison of the values of -$\theta'(0)$ for different values of Pr

Pr	Abo Eldahab Md. Aziz [20]	Present Work for visco-elastic fluid with ε =. 1	Present Work for visco-elastic fluid with ε = 0
0.7	0.3497589	0.252078	0.274618
1	0.443779	0.373032	0.401523

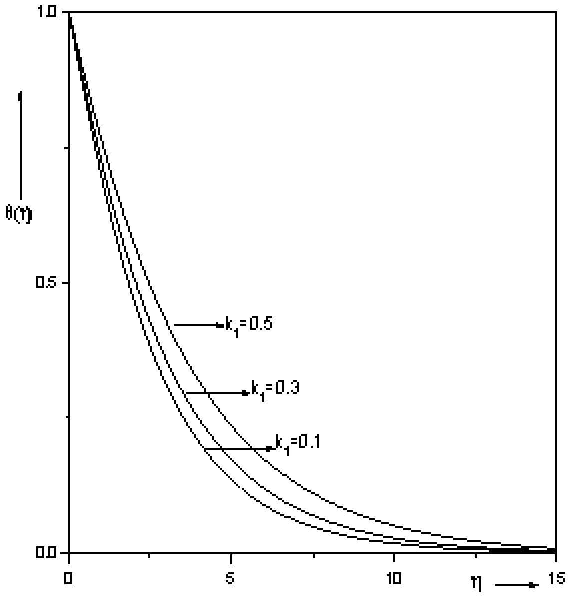

Fig1 variation of temperature profile $\theta(\eta)$ Vs η for different values of visco-elastic parameter k_1 when Mn=λ=ε=.1,k_2.05,$\bar{A}$=.01 and Pr=1(PST Case)

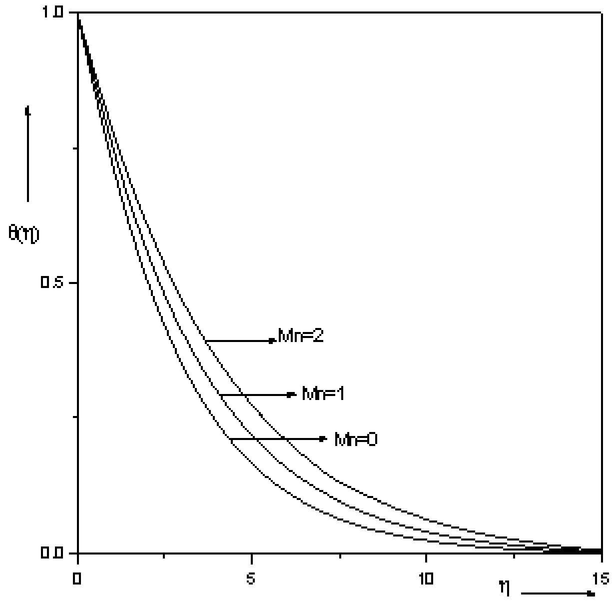

Fig2:variation of temperature profile $\theta(\eta)$ Vs η for different values of Mn when k_1=.1,λ=ε=.1,k_2.05,$\bar{A}$=.01and Pr=1

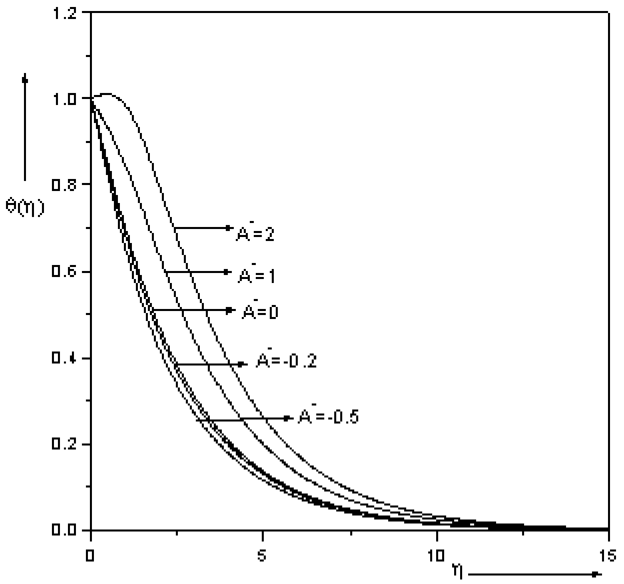

Fig3 :Variation temperature distribution $\theta(\eta)$ for different values of $\bar{A}$ when k_1=Mn=ε=λ=0.1,k_2=0.05 and Pr=1 (Pst case)

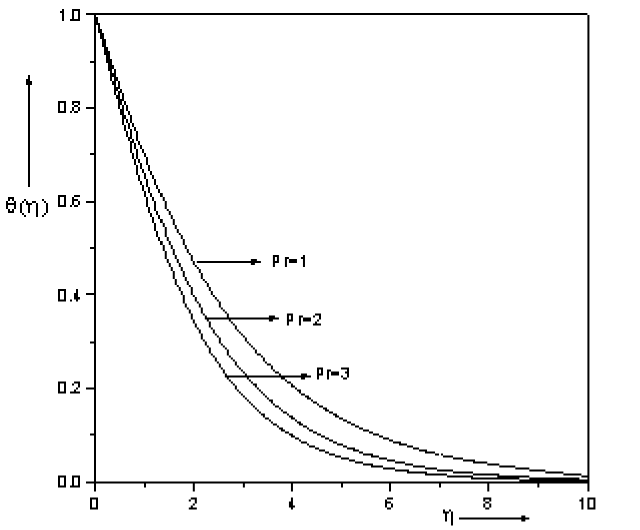

Fig4:variation of temperature profile $\theta(\eta)$ Vs η for different values of Pr,when k_1=.1,Mn=λ=ε=.1,k_2=.05 and $\bar{A}$=.01(PSTcase)

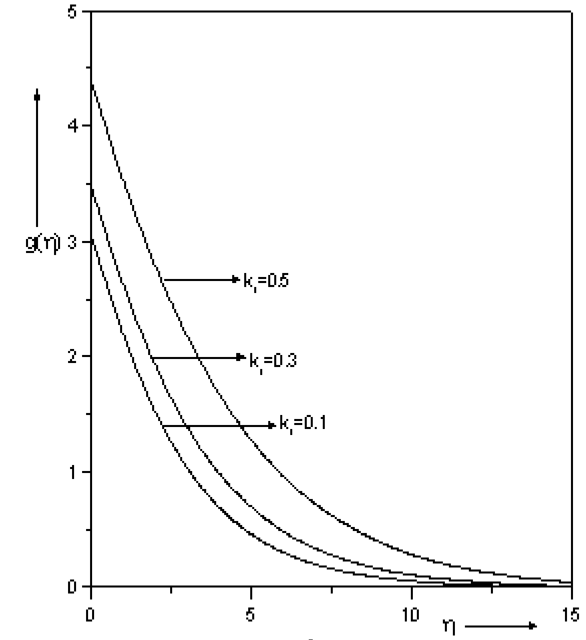

Fig5:variation of temperature profile θ(η) Vs η for different values of visco-elastic parameter k_1, when Mn=λ=ε=.1, k_2=0.05, A^*=.01 and Pr=1(PHFCase)

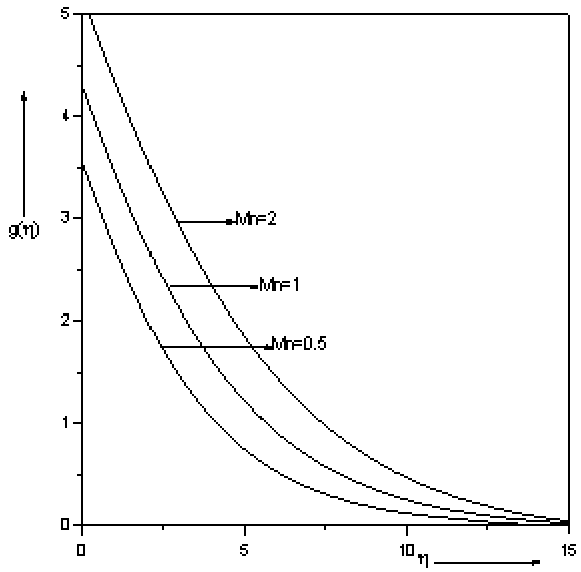

Fig6:Variation of temperature profile g(η) Vs η for different values of Mn when k_1=λ=ε=.1, k_2=.05, A^*=.01 and Pr=1(PHFcase)

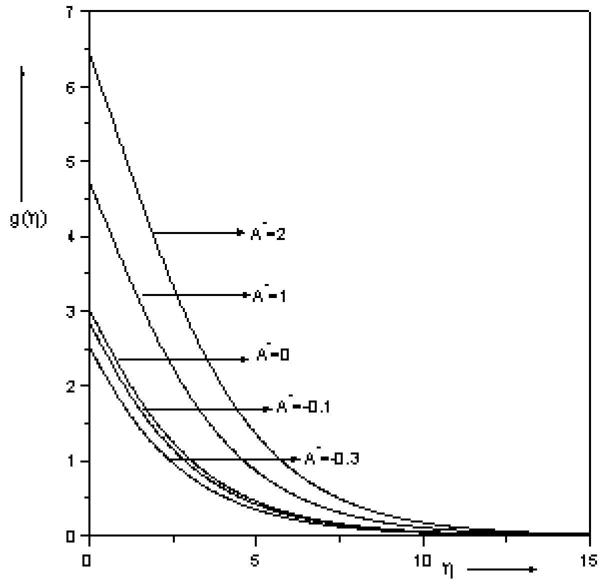

Fig7 :variation of temperature profile θ(η) Vs η for different values of A^* when k_1=Mn=λ=ε=.1, k_2=0.05 and Pr=1 (PHFcase)

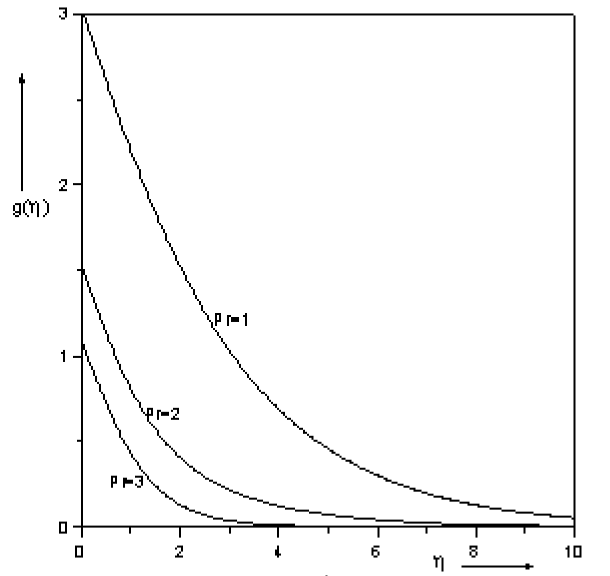

Fig8 :variation of temperature profile θ(η) Vs η for different values of Pr when k_1=Mn=λ=ε=.1, k_2=0.05 and A^*=.01 (PHFcase)

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