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An Evaluation Of Inverse Reflectivity $\frac{1}{|r|^2}$ as a Foundation of (w/w_p) for $\left(\frac{k}{k_F}\right)$ = 0.005, 0.05 and 0.1 with $r_s = 0.3w_p$ and 0.15 w_p.

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Abstract: ***Nonlocal effects in metal optics lead to rapidly varying longitudinal field near the surface but far from the surface only transverse electromagnetic field survive. This is true even at and above the plasma frequency, since the damping plasma wave is typically by a factor c/v_p longer than that of the transverse waves. Within the very successful classical longer than that of the transverse waves. Within the very successful classical Fresnel optics, which consider only transverse field, all the optics properties of a clean metal surface are determined by the bulk dielectric function of the metal (and the adjacent medium) which is a function of frequency. It seems desirable to have a similar description of the nonlocal surface effect in term of one or two general function which depend only on frequency and allow to calculate all the optical properties.***

Keywords: Non local, invers reflectivity$\frac{1}{|r|^2}$, long wave length of surface response properties

1. Introduction

FEIBELMAN1-3 has shown from microscopic consideration that this can induced be achieved in the long wavelength limit (LWL) i.e. if the scale of the spatial variation of the transverse electromagnetic field is much long than the width of the surface region, in which deviation from the asymptotic transverse field are important with typical metals the condition for the LWL are not below and if a realistic damping is taken into account, also at and mean well above the plasma frequency and the nonlocal surface effects on the reflection amplitude and other measurable quantities can be expressed in terms of two surface response function $d_{\perp}$ (ω) and $d_{||}$(ω) which depends only on the frequency ω. From a microscopic point of view, these surface response function involve integrals over the surface region and they can not be used to calculate the surface field. But they can be used to calculate the microscopic response properties of clean surface and also of surface covered with this field, and they can, in term, be evaluated from experimental results. They offer a general, meaningful and economical way to present experimental and also theoretical result. One optical properties of metal surfaces. A very transparent method to derive the surface parameter $d_{\perp}$ (ω) and $d_{||}$(ω) which also clarifies their physical meaning, has been proposed by Apell[4] and is presented in a slightly generalized form. The idea is old[5-7] and has been extended by plieth and Naegele[8]. One extrapolate the asymptotic transverse field toward the surface and drives boundary condition for this filed by an integration of Maxwell's equation in the surface region. These boundary conditions contain certain moment of the derivation of the exact fields from the extrapolated fields. Mukhopadhayay and Lundqvist9 have obtained the reflection coefficient in term of these moments. One shows that the expressing random in the LWL to FEIBELEMAN'S result and to equivalent expression given by Bagchi. et. al[10]. One consider the single local three layer model, which has been discussed by Macintyre and Asphs'' and is frequently used to present experimental data. In the LWL such a model can be used to express the surface parameter $d_{\perp}$ (ω) and $d_{||}$(ω) in terms of thickness d of the surface layer of the dielectric constant of the surface layer and metal substrate, provide the surface layer has a reduced symmetry $\left(\varepsilon^s_{xx} = \varepsilon^s_{yy} \neq \varepsilon^s_{zz}\right)$. But it is not possible to determine these political constants and the thickness of the surface layer uniquely from the values of $d_{\perp}$ (ω) and $d_{||}$(ω) or from political measurement. Moreover, the nonlocal calculation who that it does not increase the insight into the physics of the problem if one expresses the surface response function $d_{\perp}$ (ω) and $d_{||}$(ω) interms of parameter of local mode, even if this if formally possible. Furthermore, it terms one that within the LWL nonlocal effects can be simulated by a local three layer model so that there is no good reason to express experimental data in terms of dielectric function of such a model. Then, one consider within the hydrodynamic approximation a three layer model in which both the surface layer and the bulk metal sustain longitudinal fields. Within the LWL, one present explicit analytic results for $d_{\perp}$ (ω) and $d_{||}$(ω) and for the ellipsometry parameter. These forms are useful for the interpretation of experimental data on metal films absorbed on metallic substrate.

In this chapter, we have evaluated the inverse reflectivity $\frac{1}{|r|^2}$ as a function of (ω/$ω_p$). These inverse reflectivity has been calculated by taking the $d_{\perp}$ (ω) values of chapter IV and by taking different values of k_x/k_y.

2. Mathematical formula used in the evaluation

In the long wavelength limit (LWL) the surface response properties can be expressed in terms of two function $d_{\perp}$ (ω) and $d_{||}$(ω) which hare realted to mean values of the nonlocal dielectric tensor and its inverse. For optical frequency the wavelength (or decay length) of the transverse field both

inside and outside the metal is much longer than the typical length scale on which longitudinal fields and induced charge densities at the surface vary. Even at and above the plasma frequency the longitudinal fields decay (due to damping effects) much faster than transverse fields. This is easily visualized with the hydrodynamic approximation which yields for the z component of the wave vector of the longitudinal field p_l, and of the transverse field p_t.

$$p_l^2 = \frac{5}{3v_F^2}\left[\omega(\omega + ir) - \frac{\omega_n^2}{\varepsilon_b}\right] - k_x^2 \quad (5.1)$$

and

$$p_t^2 = \frac{\omega\varepsilon_b}{(\omega+ir)}\frac{1}{c^2}\left[\omega(\omega + ir) - \frac{\omega_n^2}{\varepsilon_b}\right] - k_x^2 \quad (5.2)$$

Where V_F is the Fermi velocity, r is damping, ε_b is dielectric constant bond electron ω_n is the frequency of plasma wave in a polarisable, screening bulk grad. $\omega_p^2 = \omega_n^2/\varepsilon_b(\omega_p)$. In the case of s-polarization and in long wavelength limit. The reflection coefficient and transverse coefficients are given by

$$r_s = \frac{p_t - p_a}{p_t + p_a}\left[1 + 2ip_a d_{||}(\omega)\right] \quad (5.3)$$

$$t_s = \frac{2p_a}{p_t + p_a}\left[1 + i(p_t - p_a)\, d_{||}(\omega)\right] \quad (5.4)$$

Where

$$p_t - p_a = (\varepsilon_t - \varepsilon_a)\omega^2/\, c^2 \; and \; d_{||} = \frac{\delta_y^{(1)}(a)}{\varepsilon_a - \varepsilon_t} + a \quad (5.5)$$

Where local dielectric constant ε_a in the half space z < a and ε_t in the metallic half space z >a. $k_x^2 + p_a^2 = \varepsilon_a\omega^2 / c^2$ and $k_x^2 + p_t^2 = \varepsilon_t\omega^2 - c^2$. $\delta_y^{(1)}(a)$ is the difference between exact and reference filed in case of p-polarization the situation is little more complicated and the response function $d_\perp$ (ω) and $d_{||}$(ω) and reflection and transmission amplitude are given by

$$d_{\perp}(\omega) = \left(\frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_t}\right)^{-1} \int_{-\infty}^{+\infty} dz \left\{\varepsilon_{zz}^{-1}(z) - \left[\frac{1}{\varepsilon_a}\theta(-z) + \frac{1}{\varepsilon_t}\theta(z)\right]\right\} \quad (5.6)$$

an

$$d_{||}(\omega) = (\varepsilon_a - \varepsilon_t)^{-1} \int_{-\infty}^{+\infty} dz\{\varepsilon_{zz}(z) - \varepsilon a\theta - z + \varepsilon t\theta(z) \quad (5.7)$$

$$r_p = \frac{\epsilon_t p_a - \epsilon_a p_t - i(\epsilon_a - \epsilon_t)(p_a p_t d_{||} - k_x^2 d_\perp)}{\epsilon_t p_a + \epsilon_a p_t - i(\epsilon_a - \epsilon_t)(p_a p_t d_{||} - k_x^2 d_\perp)} \quad (5.8)$$

and

$$t_p = \frac{2\epsilon_a p_a}{\epsilon_t p_a + \epsilon_a p_t - i(\epsilon_a - \epsilon_t)(p_a p_t d_{||} - k_x^2 d_\perp)} \quad (5.9)$$

Now when $d_\perp$ (ω) become large even within LWL for large electric field induced in the surface region. This is possible under certain condition together with then so called multipole surface plasmon[12]. A part from this situation $d_\perp$ is small $|d_{||}|, |d_\perp| \sim \tilde{d} \ll c/\omega$ where $\tilde{d} = (\varepsilon_2 - \varepsilon_1)$ one can expand the denominator of equation (5.8).

$$r_p = \frac{\epsilon_t p_a - \epsilon_a p_t}{\epsilon_t p_a + \epsilon_a p_t}\left[1 + 2ip_a \frac{\varepsilon_a p_t^2 d_{||} - \varepsilon_t k_x^2 d_\perp}{\varepsilon_a p_t^2 - \varepsilon_t k_x^2}\right] \quad (5.9)$$

Where $\varepsilon_t^2 p_a^2 - \varepsilon_a^2 p_t^2 = (\epsilon_t\text{-}\varepsilon_a)(\varepsilon_a p_t^2 - \varepsilon_t k_x^2)$ (5.10)

Now consider a metallic surface layer (0 < z < d) separating a metallic halfspace (z>d) from the adjacent dielectric (z<0) with dielectric constant ε_a. Both metallic layer are allowed to sustain longitudinal field by the choice of model dielectric function. The longitudinal dielectric function in the surface layer is written as

$$\varepsilon_\mu(q, \omega) = \varepsilon_{bs}(\omega\ ) - \frac{\omega_{ns}^2}{\omega(\omega + ir_s) - \beta_s q^2} \quad (5.11)$$

Including spatial dispersion through a Dude-like free electron term in addition to a non dispersive local term which takes bond electron into account. The corresponding transverse dielectric function is

$$\varepsilon_{ts} = \varepsilon_\mu(0, \omega) \quad (5.12)$$

The electric field in the dielectric (z < 0), the transverse field in the bulk metal (z > d) and E^t=$E^>$. The total field $\bar{E} = \overset{-t}{E} + \overset{-l}{E}$ in the metallic halfspace containing in addition to $\overset{-t}{E}$, the longitudinal field

$$E_x^l(z) = E_x^l e^{isp_l}, E_z^l(z) = \frac{p_l}{k_x} E_x^l(z) \quad (5.13)$$

Where $k_x^2 + p_l^2 = q_l^2$ where $q_l(\omega)$ is defined by $\varepsilon_1(q_{1,}\omega) = 0$ and both the transverse wave number p_t and the longitudinal wave number p_t have none-negative real and imaginary part. In the surface layer (0 < z < d) the transverse field is written as

$$E_{\{x\}}^{ts}(z) = \left\{\begin{matrix}1\\ -k_{x/p_{ts}}\end{matrix}\right\}(E_{x+}^{ts} e^{iz\, p_{ts}} \pm E_{x-}^{ts} e^{-iz\, p_{ts}}) \quad (5.14)$$

With and upper (lower) symbol refer to the x(z) component. Similarly the longitudinal field in the layer is given by

$$E_{\{x\}}^{ls}(z) = \left\{\begin{matrix}1\\ p_{ls\,/k_x}\end{matrix}\right\}(E_{x+}^{ls} e^{iz\, p_{ts}} \pm E_{x-}^{ls} e^{-iz\, p_{ts}}) \quad (5.15)$$

With $k_x^2 + p_{ls}^2 = q_{ls}^2$ and $\varepsilon_{ls}(q_{ls,}\omega) = 0$

The displacement field is determine by the transverse field and $D^2\varepsilon_{ts}E^{ts}$ in the surface layer (0 < z < d), D=$\varepsilon_t E^t$ in the metallic halfsapce (z > d) and D=$\varepsilon_a E^<$ in the dielectric (z < 0). At the metal-metal interface z=d, the two standard boundary condition ε_x and D_z are continuous and the two additional boundary condition are $\varepsilon_b E_z$ and QE_x^l are continuous. Where E_x^l stands for tangential component of the longitudinal electric field and $Q = \varepsilon_t/(\varepsilon_b\text{-}\varepsilon_t)$. Using these four boundary conditions, one can express the coefficients determine the field in the surface layer in term of the corresponding coefficient in the metallic half space one obtains

$$E_{x\tau}^{ts} e^{i\tau p_{ts}^d} = \frac{1}{2}\left[\left(1 + \tau\frac{\varepsilon_t p_{ts}}{\varepsilon_{ts}\, p_t}\right) E_x^t e^{ip_t^d} + \left(1 - \frac{Q}{Q_s}\right) E_x^l e^{ip_t^d}\right] \quad (5.16)$$

$$E_{x\tau}^{ls} e^{i\tau p_{ls}^d} = \frac{1}{2}\left[\tau\frac{k_x^2}{p_t p_{ls}}\left(\frac{\varepsilon_t}{\varepsilon_{ts}} - \frac{\varepsilon_b}{\varepsilon_{bs}}\right) E_x^t e^{ip_t^d} + \left(\frac{Q}{Q_s} + \frac{\varepsilon_b p_l}{\varepsilon_{bs} p_{ls}}\right) E_x^l e^{ip_t^d}\right] \quad (5.17)$$

Now the surface integral reduce to

$$\delta_x^{(n)}(0) = \int_o^d dz\, z^{n-1}(\varepsilon_{ts} E_x^{ts}(z) - \varepsilon_t E_x^t e^{izp_t})/E_x^t \quad (5.18)$$

and

$$\eta_z^{(1)}(0) = \left\{\int_o^d dz\, [E_z^{ts}(z) + E_z^{\ell s}(z) - E_z^>(z)] + \int_o^d dz\, E_z^\ell(z)\right\} / [E_t E_z^>(0)] \quad (5.19)$$

The reflection amplitude r_p is obtained in the form

$$r_p = (A - B)/(A + B) \quad (5.20)$$

where

$$A = \varepsilon_t p_a \left\{\cos\ \ p_{ts}^d - \frac{\varepsilon_{ts} p_t}{\varepsilon_t p_{ts}}\left[1 + \left(1 - \frac{Q}{Q_s}\right)\lambda\right] \mathrm{i}\sin p_{ts}^d\right\} \quad (5.21a)$$

$$B = \varepsilon_a p_t \left\{ \left[1 + \left(1 - \frac{Q}{Q_s}\right)\lambda\right] \cos \quad p_{ts}^d - \frac{\varepsilon_t p_{ts}}{\varepsilon_{ts} p_t} \mathrm{i} \sin \mathrm{p}_{\mathrm{ts}}^{\mathrm{d}} \right.$$
$$\left. - \frac{\mathrm{k}_{\mathrm{x}}^2}{p_t p_{\ell_s}} \left(\frac{\varepsilon_t}{\varepsilon_{ts}} - \frac{\varepsilon_b}{\varepsilon_{bs}}\right) \mathrm{i} \sin \mathrm{p}_{\ell_s}^{\mathrm{d}} + \lambda \left(\frac{Q}{Q_s} \cos \mathrm{p}_{\ell_s}^{\mathrm{d}} - \frac{\varepsilon_b p_l}{\varepsilon_{bs} p_{\ell_s}} \mathrm{i} \sin \mathrm{p}_{\ell s}^{\mathrm{d}}\right) \right\} \quad (5.21b)$$

λ is free electron current and is given by

$$\lambda = \left(\mathrm{E}_{\mathrm{x}}^{\ell} / \mathrm{E}_{\mathrm{x}}^{\mathrm{t}}\right) e^{i(p_\ell - p_t)d} \quad (5.22)$$

Surface integrals are reduced to

$$\delta_{\mathrm{x}}^{(1)}(0) = \mathrm{i} \frac{\varepsilon_{\mathrm{t}}}{\mathrm{p}_{\mathrm{t}}} \left\{ \left[\cos \mathrm{p}_{\mathrm{ts}}^{\mathrm{d}} - \frac{\varepsilon_{ts} p_t}{\varepsilon_t p_{ts}} \left(1 + \left(1 - \frac{Q}{Q_s}\right)\lambda\right) \mathrm{i} \sin \mathrm{p}_{\mathrm{ts}}^{\mathrm{d}} \right] e^{\mathrm{ip}_{\mathrm{t}}^{\mathrm{d}}} - 1 \right\} \quad (5.23)$$

$$\delta_{\mathrm{x}}^{(2)}(0) = \frac{\varepsilon_{\mathrm{ts}}}{\mathrm{p}_{\mathrm{ts}}^2} \left\{ \left[1 + \left(1 - \frac{Q}{Q_s}\right)\lambda\right] \quad (1 - \cos \mathrm{ptsd}) + \varepsilon t p t s \varepsilon t s p t i \sin \mathrm{ptsd} e^{\mathrm{iptd}} - \varepsilon \mathrm{tpt2}(e^{\mathrm{iptd}} - 1) \right. \quad (5.24)$$

and

$$\eta_{\mathrm{z}}^{(1)}(0) = \frac{\mathrm{e}^{\mathrm{ip}_{\mathrm{t}}^{\mathrm{d}}}}{\varepsilon_t} \left\{ \frac{\mathrm{p}_{\mathrm{t}}}{\mathrm{ip}_{\mathrm{ts}}^2} \left[1 - \cos \mathrm{p}_{\mathrm{ts}}^{\mathrm{d}} + \frac{\varepsilon_t p_{ts}}{\varepsilon_{ts} p_t} \mathrm{i} \sin \mathrm{p}_{\mathrm{ts}}^{\mathrm{d}} \right] \right.$$
$$\left. - \left(\frac{\varepsilon_t}{\varepsilon_{ts}} - \frac{\varepsilon_b}{\varepsilon_{bs}}\right) \frac{\sin \mathrm{p}_{\mathrm{ts}}^{\mathrm{d}}}{\mathrm{p}_{\ell s}} - \frac{1}{\mathrm{ip}_{\mathrm{t}}} \left(1 - \mathrm{e}^{-\mathrm{ip}_{\mathrm{t}}^{\mathrm{d}}}\right) + \lambda \frac{\mathrm{p}_{\mathrm{t}}}{\mathrm{ik}_{\mathrm{x}}^2} \left[\left(1 - \frac{\mathrm{Q}}{\mathrm{Q}_{\mathrm{s}}}\right) (1 - \cos \mathrm{ptsd} - \mathrm{QQs1} - \cos p \ell \mathrm{sd} + 1 - \varepsilon b p \ell \varepsilon b s p \ell s \mathrm{i} \sin p \ell \mathrm{sd} \right. \quad (5.25)$$

There results are exact. For thick metallic layer on metal substrates, it is necessary to work with the exact (within HD) formula but for thin surface layer and clean surfaces simplification are possible.

Local Limits

The general formulas (5.16-5.25) apply a also to be special cases that the dielectric response of the surface layer and / or the metallic half space is local, but it requires some care to pass from the nonlocal case to the local limit.

If one wants to neglect nonlocal effects in the surface layer, one has to omit spatial dispersion, i.e. to set β_s=0 in (5.11), and thereby, to longitudinal fields in the layer. But if we naively perform the limit $\beta_s \to 0$ which implies, $q_{ls} \to \infty, p_{ls} -> \infty$, we get wrong results which are different from those of a calculation which neglects longitudinal fields in the layer from the beginning. Similarly, if we want to retain nonlocality in the surface layer but not in the metallic halfspace, we get a wrong answer if we take the limit $\beta -> 0, p_t -> \infty$. The difficulty is related to the second ABC ($\mathrm{QE}^{\mathrm{y}}{}_{\mathrm{x}}$ continuous) at the metal-metal interface, which couples the induced charges on both sides of the interface. If there are longitudinal fields only on one side of the interface, this ABC cannot be satisfied. But in the limit $\beta -> 0$, which leaves the value $Q = \varepsilon_t / (\varepsilon_b - \varepsilon_t)$ unchanged, this ABC is not abandoned.

Fortunately there is a simple alternative way to swith off the spatial dispersion in (5.11). One may, for 0 < x <1, replace $\omega^2{}_{ns}$ in (5.11) by $\mathrm{x}\omega^2{}_{ns}$ and include at the same time an additive contribution $-(1-\mathrm{x})\omega^2{}_{ns}/[\omega(\omega + \mathrm{i}\gamma_s)]$ in the nondispersive part $\varepsilon_{bs}(\omega)$. this procedure leaves thetransverse dielectric constant ε_{ts} unchanged and leads in the limit $\mathrm{x} \to 0$ to the local case.

With the redefined ε_{bs}, the correct prescription for taking the local limit of the surface layer ($\mathrm{x} \to 0$), is to set $\varepsilon_{bs} = \varepsilon_{ts}$ and $Q_s = \infty$. Then (5.22) reduces to

$$\lambda = \frac{\mathrm{k}_{\mathrm{x}}^2}{p_1 p_2} \left(1 - \frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{b}}}\right) \quad (5.26)$$

And (5.7) yields $\mathrm{E}_{\mathrm{x}+}^{1\,\mathrm{s}} = \mathrm{E}_{\mathrm{x}+}^{1\,\mathrm{s}} 0$, the absence of longitudinal fields in the surface layer, Furthermore, (5.25) reduces to

$$n_s(1)_{(0)} = \frac{1}{i p_{ts} \varepsilon_{ts}} \left[\frac{\varepsilon_{ts} p_t}{\varepsilon_t p_{ts}} (1 - \cos p_{ts} d) + i \sin p_{ts} d\right] e^{ip_t d} \frac{1}{ip_t \varepsilon_t} \left(e^{ip_t d} - 1\right) + \frac{1}{ip_\ell \varepsilon_t} \left(1 - \frac{\varepsilon_{\mathrm{t}}}{\varepsilon_{\mathrm{b}}}\right) \left[1 + \frac{\mathrm{k}_{\mathrm{x}}^2}{\mathrm{p}_{\mathrm{ts}}^2} (1 - \cos p_{ts} d)\right] e^{ip_t d} \quad (5.27)$$

independent of P_{ls}.

If we set in addition $\varepsilon_b - \varepsilon_t$, we neglect nonlocal effects in the metallic half space, λ=0, and $\delta_{\mathrm{x}}^{(n)}(o), n_{\mathrm{z}}^{(1)}(o)$reduce to the local values with an isotropic surface dielectric tensor $\varepsilon_{\mathrm{xx}}^{s} = \varepsilon_{\mathrm{zz}}^{s} = \varepsilon_{ts}$. (Note that the last step, $\varepsilon_b \to \varepsilon_t, Q \to \infty$. Then $\mathrm{E}_{\mathrm{x}}^1$vanishes according to, but $\lambda Q / Q_s$ remains finite and becomes independent of Q_s, the quantity which occurs only through the second ABC. It is straightforward to evaluate this limit, but the resulting formulas remain rather lengthy. We give explicit results only for the important LWL.

Long Wavelength Limit

One now assumed that the thickness d of the surface layer and the wavelength and / or decay length of the plasma waves in both the surface layer and the bulk metal are much smaller than the wavelength and/or decay length of transverse electric fields in any of the space regions, so that $\mathrm{p_{ts}d}$, $\mathrm{p_t d}$, $\mathrm{k_x d}$, $\mathrm{k_x/p_{ls}}$, $\mathrm{k_x/p_l}$, etc. are much smaller than unity, and Pls and Pl are effectively independent of the angle of incidence $\theta_a [K_x = (\omega / c)\, \varepsilon_a^{1/2} \sin \theta_a]$. For free electron metals, but also for noble metals, this is a good approximation for all frequencies of interest.

In this long wave length limit reduces to

$$\lambda = \frac{\mathrm{k}_{\mathrm{x}}^2}{\mathrm{p_t}} \frac{\varepsilon_{\mathrm{t}} [\mu_{\mathrm{s}} + (\mu - \mu_{\mathrm{s}}) \cos \mathrm{p_{ls}} d}{\mathrm{p}_{\ell} \varepsilon_{\mathrm{b}} \cos \mathrm{p}_{\ell \mathrm{s}} \varepsilon_{\mathrm{bs}} \left(\frac{\mathrm{Q}}{\mathrm{Q_s})^{\mathrm{i}}}\right) \sin \mu_{\ell \mathrm{s}} d} \quad (5.28)$$

Where

$$\mu_{\mathrm{s}} = (\varepsilon_{\mathrm{bs}} - \varepsilon_{\mathrm{ts}}) / \varepsilon_{\mathrm{ts}} \quad (5.29)$$

and similar μ, has been introduced for brevily, According to Q=1/μ for our ABC. But we retain Q explicitly in order to see the possible effect of ABC on the final result (e.g. Q=q_ℓ^2 *or* $Q = \beta q_1^2$

To leading order in $\mathrm{P_{ts}d}$ the nonlocal correction to $\delta_{\mathrm{x}}^{(n)}(0)$ in (5.4) is negligible.

$$\delta_{\mathrm{x}}^{(1)}(0) = (\varepsilon_{\mathrm{ts}} - \varepsilon_{\mathrm{t}}) \mathrm{d}\, \delta_{\mathrm{x}}^{(2)}(0) = 0 \quad (5.30)$$

and (5.22)reduces after some algebra to(5.31)

$$\mathrm{n}_{\mathrm{z}}^{(1)}(0) = \left(\frac{1}{\varepsilon_{\mathrm{ts}}} - \frac{1}{\varepsilon_{\mathrm{t}}}\right) \mathrm{d} + \mathrm{i} \left[\frac{\left[(2\mu_{\mathrm{s}} - \mu)\frac{\mathrm{Q}}{\mathrm{Q_s}} - \mu_{\mathrm{s}}\right](1 - \cos \mathrm{p_{lsd}})}{\mathrm{P}_1 \varepsilon_{\mathrm{b}} \cos \mathrm{P_{ls}} d - \mathrm{P_{ls}} \varepsilon_{\mathrm{bs}} \left(\frac{\mathrm{Q}}{\mathrm{Q_s}}\right) \mathrm{i} \sin \mathrm{P_{ls}} d} \right.$$
$$\left. + \frac{-\mu \cos \mathrm{P_{ls}} d + \mu_{\mathrm{s}} \frac{\mathrm{P}_1 \varepsilon_{\mathrm{b}}}{\mathrm{P_{ls}} \varepsilon_{\mathrm{bs}}} \mathrm{i} \sin \mathrm{P_{ls}} d}{\mathrm{P}_{\mathrm{l}\varepsilon_{\mathrm{b}}} \cos \mathrm{P_{ls}} d - \mathrm{P_{ls}} \varepsilon_{\mathrm{bs}} \left(\frac{\mathrm{Q}}{\mathrm{Q_s}}\right) \mathrm{i} \sin \mathrm{P_{ls}} d} \right.$$

Which includes nonlocal effects in both the surface layer and the metallic half space, and reduces to the local result for $\mu = \mu_s = 0$.

If we set $\varepsilon_{\mathrm{ts}} = \varepsilon_{\mathrm{t}}, \varepsilon_{\mathrm{bs}} = \varepsilon_{\mathrm{b}}$ and $\mathrm{Q_s} = \mathrm{Q}$. etc., one recover (5.22) the result for a homogeneous metallic half space in z>0.

For local response of the surface layer, $\mu_s \to 0, Q_s \to \infty$ (5.31) reduces to

$$n_z^{(1)}(0) = \left(\frac{1}{\varepsilon_{ts}} - \frac{1}{\varepsilon_t}\right) d + \frac{i}{P_1}\left(\frac{1}{\varepsilon_b} - \frac{1}{\varepsilon_t}\right), \quad (5.32)$$

The LWL of (5.84)

If we retain non locality only in the surface layer but not in the bulk, $\mu \to 0, Q \to = \infty$, we obtain from (5.31)

$$n_z^{(1)}(0) = \left(\frac{1}{\varepsilon_{ts}} - \frac{1}{\varepsilon_t}\right) d + \frac{2}{P_{ls}}\left(\frac{1}{\varepsilon_{ts}} - \frac{1}{\varepsilon_{bs}}\right) \tan\left(\frac{1}{2} P_{ls} d\right), \quad (5.33)$$

Which leads with to

$$= d\frac{\varepsilon_a}{\varepsilon_t - \varepsilon_a}\left[(\varepsilon_t - \varepsilon_{ts})\left(\frac{1}{\varepsilon_{ts}} - \frac{1}{\varepsilon_t}\right) - \varepsilon_t\left(\frac{1}{\varepsilon_{ts}} - \frac{1}{\varepsilon_{bs}}\right)\frac{\tan\left(\frac{1}{2} P_{ls} d\right)}{\frac{1}{2} P_{ls} d}\right] \quad (5.34)$$

Inserting this into the ellipsometry formula one obtained a result which has first been published and discussed by ABELES and LOPEZ[1]. From it becomes obvious that nonlocal effects in the surface layer are most important near its plasma frequency ω_{ps}. for $\omega = \omega_{ps}$, the value $n_z^{(1)}(0) = (\varepsilon_{ts}^{-1} - \varepsilon_t^{-1})d$ of the local approximation becomes large and P_{ls} becomes small (in the absence of damping : $P_{ts} = 0, P_{ls} = 0$ for $\omega = \omega_{ps}$). then tan $(P_{ls} d/2)$ reduces to $P_{ls} d/2$ and (5.33) yields $z^{(1)}(0) = (\varepsilon_{ts}^{-1} - \varepsilon_t^{-1})d$, i.e. the nonlocal effects remove the large structure in the ω dependence of $n_z^{(1)}(0)$ predicted by the local approximation. The same mechanism is responsible for the large discrepancy between the local and the nonlocal calculation of the reflection coefficient neat the plasma frequency of the surface layer. Well below ω_{ps} the nonlocal effects in (5.34) field only a small correction to the local results, since $P_{ls} \approx i|P_{ls}|$ and $|P_{ls}|d \to$ for typical metals, except for very thin surface layers (of a few Angstroms). For $\omega > \omega_{ps}$ and small damping, p_{ls} is real and (5.33.34) predict strong plasma wave effects for such frequencies for which $P_{ls} d/2$ becomes an odd multiple of $\pi/2$. In this case d is an odd multiple of the half plasma wavelength $\lambda_{ls}=2\pi/P_{ls}$ of the layer, and sanding plasma waves can be excited in the layer. These are the plasma resonance of or thin metallic film prodicted by MELNKY and HARRISON and observed by NILSON[2].

The important differences between the nonlocal formula (5.31) and the local approximation for the interpretation of ellipsometry experiments near the plasma frequency of the substrate have recently been emphasized in a numerical study by KEMPA and GERHARDTS[3].

Our discussed the interesting question whether standing plasma waves can also be optically excited in a thing metal film on a metallic substrate. Within the present context the answer is given by (5.31). Plasma resonances in the surface layer will occur, if the denominator becomes small i.e. if

$$tan(Plsd) \approx P_1 \varepsilon_b Q_s / (iP_{ls} \varepsilon_{bs} Q). \quad (5.35)$$

If one neglect damping and assume the bulk plasma frequency ω_{ps}, for $\omega_{ps} < \omega < \omega_p, P_{ls}$ is real whereas $P_l = i[P_l]$ is purely imaginary, so that he right hand side of (5.35) is a smoothly varying, real function of ω. Thus, if the values of $_{Pls}d$ varies in the interval $\omega_{ps} < \omega < \omega_p$ by more than π, (5.35) is satisfied in this interval at least once, whereas for $\omega < \omega_{ps}$ and $\omega > \omega_p$(5.35) can not be satisfied (if we assume both ε_b and ε_{bs} positive. If damping effects are not too large, one expects therefore in the frequency interval between the plasma frequency of the surface layer and that of the bulk metal resonances due to excitation of standing plasma waves in the layer, provided the layer is thick enough and the difference between the plasma frequencies , $\omega_p - \omega_{ps} > 0$, is large enough. This qualitative discussion applies to the FORSTMANN-STENSCHKE ABC[4]$[Q = \varepsilon_t/(\varepsilon_b - \varepsilon_t)]$ and to the BOARDMAN-RUPPIN ABC[5]$(Q = q_1^2)$ as well characteristic differences become apparent, if one assumes the bulk plasma frequency ω_p to be much larger than that the surface layer, ω_{ps}. Then for $\omega_{ps} < \omega_< \omega_p$ one finds $P_{ls} < |P_l|$ and (5.35) is satisfied for cot $(P_{ls} d) \approx 0$. i.e., if $P_{ls} d$ is to close to an odd multiple of $\pi/2$, which means that the surface layer contains an odd multiple of one quarter of a plasma wavelength. If the BAC in changed (Q=q_l^2 *or* $Q = \beta q_1^2$) (5.35) is satisfied for tan $(P_{ls} d) \approx 0$. i.e. $P_{ls} d$ is close to an integer multiple of π, which means that the surface layer accommodates an integer (non-zero) multiple of a half plasma wavelength[6]. This demonstrates that the frequency dependence of $n_z^{(1)}(0)$or, equivalently, of $d_\perp$contains the information about excitation modes in the surface layer.

If one interpret the surface layer as the selvedge region of the metal half space, differing from the bulk metal only by a reduced density of free electrons, we deal with the hydrodynamic model used to discuss surface electro magnetic fields. The "local Plasmon" excitation discussed in that context is physically the same thing as a standing plasma wave in the surface layer. Surface resonances of this type are closely related to the "multiple surface plasmons".[7-9]

In the preceding discussion, excitation of standing plasma waves in the surface layer was related to pole type singularities of $d_\perp$ (ω) -$d_{||}$(ω) or, equivalently, of $n_z^{(1)}(0)$ given by (5.31). It must understood that a genuine singularity of $n_z^{(1)}(0)$ as a function of ω can occur only in the LWL. Such a singularity results from vanishing denominator of the LWL of λ. Given by (5.17). From the general definition of λ, we understand the meaning of this singularity. The ratio E_x^l/E_x^t of the x component of longitudinal and transverse electric field being usually of the order of $k_x^2/P_t P_1 \ll 1$, is now of order unity, since a strong longitudinal field is accompanied with the standing wave. As a consequence, the value of $n_z^{(1)}(0)$given by (5.22) is, at the renance frequency, enhanced by a factor of $P_t P_1/k_x^2$. In the LWL this enhancement appears as a singularity.

References

[1] K.Kempa, R.R. Gerhardts : Solid Srare Commun 53, 579 (1985)
[2] C. Schwatz, W.L. Schaich : Phys. Rev B30, 1059 (1984)
[3] A. Bagchi, R.G. Barrera, A.K. Rajagopal:Phys. Rev B20, 4824 (1979)

[4] J.D.E. McIntyre, D.E. Asphes : Surf. Sci, 24, 417 (1971)
[5] J.E. Slipe : Phys. Rev. B22, 1589 (1980)
[6] A.J. Bennett : Phys. Rev. B1.203 (1970)
[7] L. Genzel, T.P. Martin, U. Kreibig : Z. Phys. B21, 339 (1975)
[8] J.C. Maxwell-Garnett : Phil. Trans. Roy. Soc. 203, 385 (1904) : 205, 237 (1906)
[9] A. Leibsely, B.N.J. Persson : J. Phys C : Solid State Phys 16, 5375 (1983)
[10] R. Rupin : Phys. Rev B11, 2871 (1975)
[11] A.D. Boardmann, B.V. Prajape Y.O. Nakanura : Phys. State. Sol (b) 75, 347 (1976)
[12] J.E. Inglesfield, EL. Wikborg : J. Phys F5, 1706 (1975)