International Journal of Scientific Engineering and Research (IJSER)
ISSN (Online): 2347-3878
Index Copernicus Value (2015): 62.86 | Impact Factor (2015): 3.791

Designing of Single Sampling Plan on Sustainable Quality Region

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Abstract: ***This is a paper developing a single sampling plan based on MAPD and SQR, which are representative of quality interest of all involved in production. The sustainable quality will exceed AQL and limited by MAPD so that consumers interest is highly appreciated along with producers protection. Tables were presented and examples were illustrated.***

Keywords: SSP, MAPD, SQR, AQL, Operating Ratio

1. Introduction

There are many sampling plans developed in terms of percent defectives by Horsnell (1954), Cameron(1952),Dodge-Romig (1959).......Mandelson (1962) and Mayer (1967) are the pioneers suggested the significance of MAPD and Soundararajan (1975) Ramkumar (2011, 2013, 2017) developed interesting results on MAPD. Divya (2012) and Ramkumar (2012) had introduced the sampling plans on interval quality and explained how it is effective in what situations. Designing a Sampling plan with sustainable quality on consumer's point of view is to reject bad lots when it exceeds consumer's quality aspiration. To achieve this objective, the plan for designing SSP with MAPD is preferred so that consumer will be protected from getting lower quality product. The second quality level is fixed as SQR which is the difference of MAPD and AQL. SQR is the sustainable quality region under which the proportion of defective is exceeding acceptable quality level and limiting the proportions unto maximum allowable quality level. Thus (MAPD, SQR) is more consumer oriented but protection is assured to producer also. The properties of MAPD as well as AQL are significantly come under this model so that manufacturers, vendors, customers as well as the statisticians and quality controlling agencies were interested to use the sampling plan on this quality indices. When the exact AQL cannot be fixed due to lack of consistency of production information the SQR with MAPD can be taken in the production process confidently. Thus for the products at initial level of production SQR, MAPD based sampling plan is more effective than other designs.

2. Designing SSP with MAPD and SQR

Fix MAPD and SQR in a production process. The quality indices SQR and MAPD were used to design a new SSP by constructing an operating ratio

$$\mathrm{R} = \frac{\mathrm{MAPD}}{\mathrm{SQR}} = \frac{\mathrm{p}^*}{\mathrm{p}*-\mathrm{p}_1} = \frac{\mathrm{np}^*}{(\mathrm{np}*-\mathrm{np}_1)}, \text{ ....(1)}$$

The new design is efficient to contain the variability of quality that can be accommodated in terms of MAPD. For example$SQR = 2xMAPD - AQL,$ or $SQR = 0.5 \times MAPD - AQL$ etc will be a good measure for the producers as well as the consumers to identify their quality of the product.

There exist a monotonic increase sequence of operating ratio corresponding to the acceptance numbers using the Poisson unity values. Find appropriate c nearly less than or equal to the operating ratio and hence (MAPD,SQR) from the derived values of np^* or n*SQR using the Poisson unity values, then $n = \frac{c}{p^*}$ or $n = \frac{nSQR}{SQR}$.

2.1 Construction of the Plan

It is assumed that the number of defectives in large production follows Poisson distribution. Then the probability of acceptance of the lot with c defectives is

$$\mathrm{P_a(p)} = \sum_{\mathrm{r}=0}^{\mathrm{c}} \frac{\mathrm{e}^{-\mathrm{np}}\,(\mathrm{np})^{\mathrm{r}}}{\mathrm{r}!} \text{ ....(2)}$$

AQL (p_1) is decided from the expression

$$\mathrm{P_a(p_1)} = \sum_{\mathrm{r}=0}^{\mathrm{c}} \frac{\mathrm{e}^{-\mathrm{np}_1}(\mathrm{np}_1)^{\mathrm{r}}}{\mathrm{r}!} \geq (1-\propto) \text{..........(3)}$$

for specified values of n and c

The values of np_1 at 5% and 10% level are available in literature. Also from the definition of the point of inflection of a continuous function Pa(p), for specified n & c.

$$\text{MAPD} = p^* = c/n \text{..........(4)}$$

By appropriate value of R, for given MAPD and SQR by the search procedure one can detect c by comparing the operating ratio to the tabled value of R .Hence n is calculated approximating to the nearest integer.

2.2 Construction of Tables

Values of np_1, and np* is obtained from equation (2) &(3) and using this values $nSQR = (np^* - np_1)$, $nMAPD = np^* = c$ are found out and R is determined from equation(1) for c=1,2..........40 (Table:1). Table 2, represents some sampling plans corresponding to specified MAPD and SQR. The operating ratio for each pairs of (MAPD, SQR) is calculated and corresponding sampling plan is developed under the construction of sampling plan. Table 3 is a conversion table to identify other quality indices of the designed plan like LTPD, AQL, AOQL, TQR and MAAOQ, where nLTPD, n AQL, nAOQL, n TQR and nMAAOQ, were taken from Ramkumar (2003, 2006) conversion table . Table 4 shows (MAPD, SQR) for various combination of (n,c), it was constructed by finding $nSQR$, and $nMAPD$ from Table 1 and hence MAPD and SQR for the values of n.

Table 1: Operating ratio R

c	R	nSQR	c	R	nSQR	c	R	nSQR	c	R	nSQR
1	1.5504	0.645	11	2.6987	4.076	21	3.4392	6.106	31	4.0249	7.702
2	1.692	1.182	12	2.7842	4.31	22	3.5026	6.281	32	4.0775	7.848
3	1.836	1.634	13	2.866	4.536	23	3.5648	6.452	33	4.1302	7.99
4	1.9704	2.03	14	2.9449	4.754	24	3.6265	6.618	34	4.182	8.13
5	2.0947	2.387	15	3.0211	4.965	25	3.6862	6.782	35	4.2327	8.269
6	2.2108	2.714	16	3.0954	5.169	26	3.7453	6.942	36	4.2827	8.406
7	2.3186	3.019	17	3.1675	5.367	27	3.8028	7.1	37	4.3326	8.54
8	2.4206	3.305	18	3.2386	5.558	28	3.8599	7.254	38	4.3814	8.673
9	2.5182	3.574	19	3.3066	5.746	29	3.9157	7.406	39	4.4298	8.804
10	2.6103	3.831	20	3.3738	5.928	30	3.9704	7.556	40	4.4773	8.934

Table 2: SSP for specified values of MAPD and SQR

	SQR							
MAPD	0.005	0.01	0.02	0.025	0.03	0.035	0.04	0.045
0.01	(400,4)	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)
0.02	(1500,30)	(200,4)	(50,1)	(50,1)	(50,1)	(50,1)	(50,1)	(50,1)
0.03		(467,14)	(33,1)	(33,1)	(33,1)	(33,1)	(33,1)	(33,1)
0.04		(750,30)	(100,4)	(25,1)	(25,1)	(25,1)	(25,1)	(25,1)
0.05			(180,9)	(80,4)	(40,2)	(20,1)	(20,1)	(20,1)
0.06			(233,14)	(133,8)	(67,4)	(33,2)	(17,1)	(17,1)
0.07			(314,22)	(171,12)	(100,7)	(57,4)	(43,3)	(14,1)
0.08			(375,30)	(213,17)	(138,11)	(88,7)	(50,4)	(38,3)
0.09			(444,40)	(256,23)	(156,14)	(111,10)	(78,7)	(44,4)
0.1				(300,30)	(190,19)	(130,12)	(90,9)	(60,6)

SSP for specified values of MAPD and SQR (Continued)

	SQR								
MAPD	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085	0.09
0.01	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)	(100,1)
0.02	(50,1)	(50,1)	(50,1)	(50,1)	(50,1)	(50,1)	(50,1)	(50,1)	(50,1)
0.03	(33,1)	(33,1)	(33,1)	(33,1)	(33,1)	(33,1)	(33,1)	(33,1)	(33,1)
0.04	(25,1)	(25,1)	(25,1)	(25,1)	(25,1)	(25,1)	(25,1)	(25,1)	(25,1)
0.05	(20,1)	(20,1)	(20,1)	(20,1)	(20,1)	(20,1)	(20,1)	(20,1)	(20,1)
0.06	(17,1)	(17,1)	(17,1)	(17,1)	(17,1)	(17,1)	(17,1)	(17,1)	(17,1)
0.07	(14,1)	(14,1)	(14,1)	(14,1)	(14,1)	(14,1)	(14,1)	(14,1)	(14,1)
0.08	(13,1)	(13,1)	(13,1)	(13,1)	(13,1)	(13,1)	(13,1)	(13,1)	(13,1)
0.09	(33,3)	(11,1)	(11,1)	(11,1)	(11,1)	(11,1)	(11,1)	(11,1)	(11,1)
0.1	(40,4)	(30,3)	(20,2)	(10,1)	(10,1)	(10,1)	(10,1)	(10,1)	(10,1)

Table 3: MAPD (%) & SQR % for specified SSP (n,c)

	n									
	50		100		200		500		1000	
c	MAPD	SQR	MAPD	SQR	MAPD	SQR	MAPD	SQR	MAPD	SQR
1	0.02	0.0129	0.01	0.00645	0.005	0.003225	0.002	0.00129	0.001	0.00065
2	0.04	0.02364	0.02	0.01182	0.01	0.00591	0.004	0.002364	0.002	0.00118
3	0.06	0.03268	0.03	0.01634	0.015	0.00817	0.006	0.003268	0.003	0.00163
4	0.08	0.0406	0.04	0.0203	0.02	0.01015	0.008	0.00406	0.004	0.00203
5	0.1	0.04774	0.05	0.02387	0.025	0.011935	0.01	0.004774	0.005	0.00239
6	0.12	0.05428	0.06	0.02714	0.03	0.01357	0.012	0.005428	0.006	0.00271
7	0.14	0.06038	0.07	0.03019	0.035	0.015095	0.014	0.006038	0.007	0.00302
8	0.16	0.0661	0.08	0.03305	0.04	0.016525	0.016	0.00661	0.008	0.00331
9	0.18	0.07148	0.09	0.03574	0.045	0.01787	0.018	0.007148	0.009	0.00357
10	0.2	0.07662	0.1	0.03831	0.05	0.019155	0.02	0.007662	0.01	0.00383
11	0.22	0.08152	0.11	0.04076	0.055	0.02038	0.022	0.008152	0.011	0.00408
12	0.24	0.0862	0.12	0.0431	0.06	0.02155	0.024	0.00862	0.012	0.00431
13	0.26	0.09072	0.13	0.04536	0.065	0.02268	0.026	0.009072	0.013	0.00454
14	0.28	0.09508	0.14	0.04754	0.07	0.02377	0.028	0.009508	0.014	0.00475
15	0.3	0.0993	0.15	0.04965	0.075	0.024825	0.03	0.00993	0.015	0.00497
16	0.32	0.10338	0.16	0.05169	0.08	0.025845	0.032	0.010338	0.016	0.00517
17	0.34	0.10734	0.17	0.05367	0.085	0.026835	0.034	0.010734	0.017	0.00537

18	0.36	0.11116	0.18	0.05558	0.09	0.02779	0.036	0.011116	0.018	0.00556
19	0.38	0.11492	0.19	0.05746	0.095	0.02873	0.038	0.011492	0.019	0.00575
20	0.4	0.11856	0.2	0.05928	0.1	0.02964	0.04	0.011856	0.02	0.00593

Table 3: Conversion table for MAPD and SQR

c=np*	R	p_1/p^*	p_2/p^*	TQR/p*	AOQL/p*	MAAOQ/p*
1	1.5504	0.3550	3.8858	3.5308	0.8400	0.7360
2	1.6920	0.4090	2.6622	2.2532	0.6855	0.6765
3	1.8360	0.4553	2.2266	1.7712	0.6473	0.6473
4	1.9704	0.4925	1.9981	1.5056	0.6360	0.6288
5	2.0947	0.5226	1.8547	1.3321	0.6336	0.6160
6	2.2108	0.5477	1.7558	1.2082	0.6353	0.6063
7	2.3186	0.5687	1.6817	1.1130	0.6389	0.5987
8	2.4206	0.5869	1.6245	1.0376	0.6433	0.5925
9	2.5182	0.6029	1.5784	0.9755	0.6479	0.5874
10	2.6103	0.6169	1.5404	0.9235	0.6528	0.5830
11	2.6987	0.6295	1.5088	0.8793	0.6575	0.5793
12	2.7842	0.6408	1.4816	0.8408	0.6623	0.5760
13	2.8660	0.6511	1.4584	0.8073	0.6669	0.5731
14	2.9449	0.6604	1.4378	0.7773	0.6713	0.5704
15	3.0211	0.6690	1.4196	0.7506	0.6756	0.5681
16	3.0954	0.6769	1.4033	0.7264	0.6797	0.5659
17	3.1675	0.6843	1.3884	0.7041	0.6836	0.5640
18	3.2386	0.6912	1.3755	0.6843	0.6874	0.5622
19	3.3066	0.6976	1.3631	0.6655	0.6911	0.5606
20	3.3738	0.7036	1.3523	0.6487	0.6946	0.5591
21	3.4392	0.7092	1.3419	0.6326	0.6980	0.5577
22	3.5026	0.7145	1.3325	0.6180	0.7012	0.5564
23	3.5648	0.7195	1.3238	0.6044	0.7043	0.5551
24	3.6265	0.7243	1.3160	0.5917	0.7073	0.5540
25	3.6862	0.7287	1.3081	0.5793	0.7102	0.5529
26	3.7453	0.7330	1.3011	0.5681	0.7131	0.5519
27	3.8028	0.7370	1.2950	0.5579	0.7158	0.5509
28	3.8599	0.7409	1.2885	0.5475	0.7184	0.5500
29	3.9157	0.7446	1.2830	0.5384	0.7209	0.5492
30	3.9704	0.7481	1.2771	0.5289	0.7234	0.5484
31	4.0249	0.7515	1.2716	0.5201	0.7258	0.5476
32	4.0775	0.7548	1.2672	0.5125	0.7281	0.5468
33	4.1302	0.7579	1.2619	0.5040	0.7303	0.5461
34	4.1820	0.7609	1.2577	0.4969	0.7325	0.5454
35	4.2327	0.7637	1.2533	0.4896	0.7346	0.5447
36	4.2827	0.7665	1.2494	0.4829	0.7366	0.5442
37	4.3326	0.7692	1.2453	0.4761	0.7386	0.5436
38	4.3814	0.7718	1.2418	0.4700	0.7406	0.5430
39	4.4298	0.7743	1.2380	0.4638	0.7425	0.5424
40	4.4773	0.7767	1.2349	0.4582	0.7443	0.5419

Procedure with example how to use the tables for designing SSP ON SPECIFIED AQL AND SQR

1) Compute $\text{R} = \frac{\text{MAPD}}{\text{SQR}}$, where SQR=MAPD-AQL.
2) Compare value of R matching with nearest operating ratio in Table: 1 and locate acceptance number (c) correspondingly.
3) Compute $n = \frac{nMAPD}{MAPD}$ or $n = \frac{nSQR}{SQR}$ from Table 1
4) The required sampling plan (n, c) for a specified MAPD and SQR is obtained.

Example 1

For an electronic component MAPD= 6% where SQR = MAPD-AQL=1.5%

The quality indices are p*=.06 and $(p^* - p_1) = .032$

Then $R = \frac{0.06}{0.032} = 1.875$

From Table 1 approximate $R = 1.875$ (exceeding $R = 1.8360$), the corresponding $c = 4$

Then $n = \frac{np^*}{p^*} = \frac{4}{0.06} = 67$

The needed sampling plan to test the quality of the computer component is (67,4)

Using Table 4

Now $\frac{p_1}{p^*} = 0.4925$ so $p_1 = \frac{p_1}{p^*} \times p^* = 0.4925 \times 0.06 = 0.02955$

Now $\frac{p_2}{p^*} = 1.9981$ $p_2 = \frac{p_2}{p^*} \times p^* = 1.9981 \times 0.06 = 0.119886$

$$TQR = p_2 - p_1 = 0.119886 - 0.02955 = 0.09025$$

$\frac{AOQL}{p^*} = 0.6360$ so $AOQL = \frac{AOQL}{p^*} \times p^* = 0.6360 \times 0.06 = 0.03816$

$\frac{MAAOQ}{p^*} = 0.6288$ so $MAAOQ = \frac{MAAOQ}{p^*} \times p^* = 0.6288 \times 0.06 = 0.037728$

Example 2

A consumer friendly article is designed with $MAPD = 10\%$ defectives and $SQR = .5\, xMAPD - AQL) = 3.84\%$ defectives where AQL=1.16%

The operating ratio $R = \frac{MAPD}{SQR} = 2.604$

Appropriate R from Table: 1 is 2.6103 (exceeding 2.5182), $c = 10$, then

$n = \frac{10}{0.1} = 100$ .The required sampling plan is (100,10)

Example 3

The electronic device is qualitatively indexed by ($MAPD = 8.3\%$ & $SQR = 2.5\%$) . What is the sampling plan to set quality inspection?

From Table:3 the approximate combination of $(MAPD, SQR)$ is found by search procedure and it is $(0.085, 0.026835)$ .the corresponding sampling plan is (200,17)

Example 4

Application in A Simulated Data

The life length of a type of bulb in hours follows gamma distribution with parameters ($\alpha = 98,\ \beta = 8.5\ and\ \gamma = 1000$) , a sample of 25 bulbs show the following number of hours of burning where the bulbs were supplied in a lot size of 3000. As per the Quality Standard prescribed by Quality Control Agency , the life length of bulbs was expected a minimum life length of 1700hrs and $MAPD = 8\%$ with $SQR = 0.85\, MAPD - AQL$ where $AQL = 2.4\%$.

sample number	1	2	3	4	5	6	7	8	9	10
life length(hrs)	1823	1846	1863	1823	1822	**1693**	1880	1917	1774	1726
sample number	11	12	13	14	15	16	17	18	19	20
life length(hrs)	1876	1905	1923	1850	1824	1789	1766	1866	1759	1792
sample number	21	22	23	24	25	26	27	28	29	30
life length(hrs)	1825	**1648**	1803	1782	**1683**	1846	1800	1753	1906	1820
sample number	31	32	33	34	35	36	37			
life length(hrs)	1924	1722	1759	1818	1845	1896	1817			

Therefore

$$R = \frac{MAPD}{SQR} = \frac{8}{4.4} = 1.8181$$

From Table 1, the approximate $R = 1.8360$ (exceeding $R = 1.6920$), the corresponding $c = 3$ and $nSQR$ =1.634. then sample size

$$n = \frac{nSQR}{SQR} = \frac{1.634}{0.044} \approx 37$$

Therefore the optimum SSP is (37, 3). There are three items defective (sample number 6, 22 and 25) in a random sample of 37 units following Gamma distribution as mentioned above. Hence by using attribute SSP (37, 3) the lot of 3000 bulbs will be accepted for sale.

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