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# * Derivations in Prime Gamma Nearrings

**Dr. D. Bharathi[1], K. Sreenivasulu[2]**

[1]Professor, Department of Mathematics, S.V.U, Tirupathi, A.P., India
[2]Research scholar, Department of Mathematics, S.V.U, Tirupathi, A.P., India
ksreenugkce [at]gmail.com

**Abstract:** ***Let N be a prime Γ-nearring of a 2-torsion free and let $D_1$ and $D_2$ be a * derivations with the condition $D_1(a)\alpha D_2(b)=D_2(b)\alpha D_1(a)$ for all a, bϵN and αϵΓ on N. Then $D_1D_2$ is a * derivation on N if and only if either $D_1$=0 or $D_2$=0.***

**Keywords:** * derivation, Γ-nearring, 2-torsion free

## 1. Introduction

The notion of derivations in nearrings introduced by Bell [1] and Mason. They obtained some basic properties of derivations in nearrings and some commutativity conditions for a Γ- * nearrings with derivations. Cho [2] studied some characterization of Γ- * nearrings and some regularity conditions.

In classical ring theory, Posner [3], Herstein [4], Bergen [5], Bell amd Daif [6] worked on derivations in prime and semiprime rings and gets some commutativity properties of prime rings with derivations. In nearring theory, Bell and Mason [1] and also Cho [7] worked on derivations in prime and semiprime nearrings.

This paper, we slightly extended the results of Cho [7] in prime Γ-* nearrings with certain conditions by using derivations.

## 2. Main Theorems

**Lemma 1:** Let D be an arbitrary additive endomorphism of N. Then D(αb)= aαD(b)+ D(a)αb* if and only if D(aαb)= D(a)αb*+ aαD(b) for all a, bϵN and αϵΓ.

**Proof:** Assume that D(aαb)= D(a)αb*+aαD(b) for all a, bϵN and αϵΓ. For αϵΓ and from
aα(b+b)=aαb+aαb
and N satisfies left distributive law,
D(aα(b+b))=aαD(b+b)+D(a)α(b*+b*)
= aα(D(b)+D(b))+D(a)αb*+ D(a)αb*
= aαD(b)+aαD(b)+D(a)αb*+ D(a)αb*
and D(aαb+aαb)= D(aαb)+D(aαb)
=D(a)αb*+ aαD(b)+ D(a)αb*+ aαD(b)
By comparing these two equations, we have
aαD(b)+D(a)αb*= D(a)αb*+ aαD(b)
Hence D(aαb)= D(a)αb*+ aαD(b) for all a, bϵN and αϵΓ.
Conversely, assume that D(aαb)= D(a)αb*+ aαD(b) for all a, bϵN and αϵΓ
Then form D(aα(b+b))= D(aαb+aαb) and we can induce that, the above calculate of this equality D(aαb)= aαD(b)+ D(a)αb* for all a, bϵN and αϵΓ. ◊

**Lemma 2:** Let D be a * derivation on N. Then N satisfies the following right distributive laws, a, b, cϵN and α, βϵΓ.
{aαD(b)+D(a)αb*}βc = aαD(b)βc+D(a)αb*βc,
{D(a)αb*+ aαD(b)}βc= D(a)αb*βc+aαD(b)βc

**Proof:** From the calculation for D((aαb)βc)=D(aα(bβc)) for all a, b, cϵN and α, βϵΓ and Lemma 1, we can find our result.

**Lemma 3:** Let N be a prime Γ-nearring and U be a nonzero subset of N. If 'a' be an element of N such that UΓa={0}, then a=0.

**Proof:** Since U≠{0}, there exists an element uϵU such that u≠0.Consider that uΓNΓa⊂UΓa={0}.Since u≠0 and N is a prime Γ-nearring, we have that a=0. ◊

**Lemma 4:** Let N be a prime Γ-nearring and U a nonzero N-subset of N.If D is a nonzero * derivation on N then
(i) If a, bϵN and aΓUΓb={0}, then a=0 or b=0
(ii) If aϵN and D(U)Γa={0}, then a=0
(iii) If aϵN and aΓD(U)={0}, then a=0.

**Proof:** (i) Let a, bϵN and aΓUΓb={0}
Then aΓUΓNΓb⊂ aΓUΓb={0}.
Since N is a prime Γ-nearring, aΓU=0 or b=0
If b=0, then we are done.
So, if b≠0, then aΓU=0
By the lemma 3, then a=0
(ii) Suppose D(U)Γa={0}, for aϵN.Then for all uϵU and bϵN, by the lemma, we have
for all a, bϵN and α, βϵΓ, 0=D(bαu)βa= (bαD(u)+ D(b)αu*)βa
= bαD(u)βa+ D(b)αu*βa
So D(b)ΓUΓa={0}, for all bϵN. (Since D(U)Γa={0})
Since D is a nonzero * derivation on N, we have that a=0 by the statement (i)
(iii) Suppose aΓD(U)={0} for aϵN. Then for all uϵU, bϵN and α, βϵΓ.
0=aαD(uβb)=aα{ uβD(b)+ D(u)βb*}
= aαuβD(b)+aα D(u)βb*
= aαuβD(b)
Hence aΓUΓD(b)={0} for all bϵN from the statement (i) and D is a nonzero * derivation on N, we have that a=0
We assume that N is a Γ-ring and U is a right N- subset. ◊

**Theorem 1:** Let N be a prime Γ-nearring and U be a right N-subset of N.If D is a nonzero * derivation on N such that $D^2(u)=0$ then $D^2=0$

**Proof:** For all u, vϵU and αϵΓ,
We have $D^2(u\alpha v)=0$.

Then $0= D^2(u\alpha v)= D(D(u\alpha v))=D\{D(u)\alpha v^*+ u\alpha D(v)\}$
$= D^2(u)\alpha(v^*)^*+ D(u)\alpha D(v^*)+ D(u)\alpha D(v^*)+ u\alpha D^2(v)$
Thus $D(U)\Gamma D(U)=\{0\}$ for all $u\epsilon U$.
From lemma 4(iii), we have $D(U)=0$.
Now, for all $b\epsilon N$, $u\epsilon U$ and $\alpha\epsilon\Gamma$
$D^2(u\alpha b)=D(D(a\alpha b))=D(D(u)\alpha b^*+u\alpha D(b))$
$=u\alpha D^2(b)+ D^2(u)\alpha b^*+ D(u)\alpha D(b)+ D(u)\alpha D(b^*)$
Hence $U\Gamma D^2(b)=\{0\}$ for all $b\epsilon N$.
By the lemma 4, we have $D^2(b)=0$ for all $b\epsilon N$. Consequently $D^2=0$. ◊

**Lemma 4:** Let D be a * derivation of a prime Γ-nearring N and 'a' be an element of N. If $a\alpha D(x)=0$( or $D(x)\alpha a=0$) for all $x\epsilon N$, $\alpha\epsilon\Gamma$, then either a=0 or D=0.

**Proof:** We assume that $a\alpha D(x)=0$ for all $x\epsilon N$, $\alpha\epsilon\Gamma$.
Change x by $x\beta y$ (for all $\beta\epsilon\Gamma$), we have that
$a\alpha D(x\beta y)=0= a\alpha D(x)\ \beta y^*+ a\alpha x\ \beta D(y)$.
Then by lemma 2,
Since $a\alpha D(x)=0$, $a\alpha x\ \beta D(y)=0$ for all x, $y\epsilon N$, α, $\beta\epsilon\Gamma$.
Since N is a Γ-prime nearring, either a=0 or D=0. ◊

**Theorem 2:** Let N be a Γ-prime nearring with nonzero * derivation and $D_1$ and $D_2$ such that for all x, $y\epsilon N$ and $\alpha\epsilon\Gamma$, $D_1(x)\alpha D_2(y)= - D_2(x)\alpha D_1(y)$. Then N is an abelian Γ-nearring

**Proof:** Let x, u, $v\epsilon N$, $\alpha\epsilon\Gamma$.
We have $D_1(x)\alpha D_2(y)= - D_2(x)\alpha D_1(y)$
Substitute y by u+v
$0= D_1(x)\alpha D_2(u+v)+ D_2(x)\alpha\ [D_1(u)+D_1(v)]$
$= D_1(x)\alpha\ [D_2(u)+D_2(v)]+ D_2(x)\alpha\ [D_1(u)+D_1(v)]$
$= D_1(x)\alpha D_2(u)+ D_1(x)\alpha D_2(v)- D_1(x)\alpha D_2(u)- D_1(x)\alpha D_2(v)$
$= D_1(x)\alpha\ [D_2(u)+ D_2(v)- D_2(u)- D_2(v)]$
$= D_1(x)\alpha D_2(u+v-u-v)$
Thus $D_1(N)\Gamma D_2(u+v-u-v)=\{0\}$ (1)
By the Lemma 4, we have
$D_2(u+v-u-v)=\{0\}$ (2)
Now, we substitute $x\beta u$ and $x\beta v$ ($\beta\epsilon\Gamma$) instead of u and v respectively in (2) .Then from Lemma 1, we deduce that for all x, u, $v\epsilon N$, $\beta\epsilon\Gamma$.
$0=D_2(x\beta u+ x\beta v- x\beta u- x\beta v)$
$=D_2\ [x\beta(u+v-u-v)]$
$=D_2(x)\beta(u+v-u-v)^*+ x\beta D_2(u+v-u-v)$ (by Lemma 4)
$=D_2(x)\beta(u+v-u-v)^*$
Substitute $(u+v-u-v)^*$ by u+v-u-v.
Again apply by Lemma 4, we see that for all u, $v\epsilon N$, u+v-u-v=0.
⇒u+v =v+u
Consequently, N is an abelian Γ-nearring. ◊

**Theorem 3:** Let N be a prime Γ-nearring of 2-torsion free and let $D_1$and $D_2$ be * derivations with the condition $D_1(a)\alpha D_2(b)= D_2(b)\alpha D_1(a)$ for all a, $b\epsilon N$, $\alpha\epsilon\Gamma$ on N .Then $D_1D_2$ is a * derivation on N if and only if either $D_1=0$, $D_2=0$.

**Proof:** We assume that $D_1D_2$ is a * derivation. Then we get for $\alpha\epsilon\Gamma$,
$D_1D_2(a\alpha b)=a\alpha D_1D_2(b)+D_1D_2(a)\alpha b^*$ (3)
Also, since $D_1$ and $D_2$ are * derivations, we obtain
$D_1D_2(a\alpha b)= D_1(D_2(a\alpha b))= D_1(a\alpha D_2(b)+D_2(a)\alpha b^*)$
$= D_1(a\alpha D_2(b))+ D_1(D_2(a)\alpha b^*)$
$= a\alpha D_1D_2(b)+D_1(a)\alpha D_2(b^*)+ D_2(a)\alpha D_1(b^*)^*+ D_1D_2(a)\alpha b^*$
$= a\alpha D_1D_2(b)+D_1(a)\alpha D_2(b^*)+ D_2(a)\alpha D_1(b)+ D_1D_2(a)\alpha b^*$ (4)
From (3) and (4) for $D_1D_2(a\alpha b)$ for all a, $b\epsilon N$, $\alpha\epsilon\Gamma$,
$D_1(a)\alpha D_2(b^*)+D_2(a)\alpha D_1(b)=0$ (5)
Hence from Theorem 2, we know that N is an abelian Γ-nearring.
Substitute a by $a\alpha D_2(c)$ in (5) and using Lemma 1 and * derivations in right distributive laws, we get that
$0= D_1(a\alpha D_2(c))\alpha D_2(b^*)+ D_2(a\alpha\ D_2(c))\alpha D_1(b)$
$=\{D_1(a)\alpha D_2(c^*)+a\alpha D_1D_2(c)\}\alpha D_2(b^*)+ \{a\alpha D^2_2(c^*)+ D_2(a)\alpha D_2(c)\}\alpha D_1(b)\}$
$=D_1(a)\alpha D_2(c^*)\ \alpha D_2(b^*)+a\alpha D_1D_2(c)\alpha D_2(b^*)+ a\alpha D^2_2(c^*)\ \alpha D_1(b)+ D_2(a)\alpha D_2(c)\alpha D_1(b\ =D_1(a)\alpha D_2(c^*)\ \alpha D_2(b^*)+ a\alpha\{D_1D_2(c)\alpha D_2(b^*)+ D^2_2(c^*)\ \alpha D_1(b)\}+D_2(a)\alpha D_2(c)\alpha D_1(b)$
Again change a by $D_2(c)$ in (5), we see that
$D_1(D_2(c)\alpha D_2(b^*))+ D_2(D_2(c))\alpha D_1(b)=0$
This implies that $a\alpha\{ D_1D_2(c)\alpha D_2(b^*)+ D^2_2(c))\alpha D_1(b)\}=0$
Hence, from the above long equality, we have the following equality,
$D_1(a)\alpha D_2(c^*)\ \alpha D_2(b^*)+ D_2(a)\alpha D_2(c)\ \alpha D_1(b)=0$, for all a, $b\epsilon N$, $\alpha\epsilon\Gamma$. (6)
Replace a and b by c in (5), we get
$D_2(c)\ \alpha D_1(b)= - D_1(c)\ \alpha D_2(b^*)$, $D_1(a)\alpha D_2(c^*)= -D_2(a)\ \alpha D_1(c)$
So that (6) becomes,
$0= \{-D_2(a)\ \alpha D_1(c^*)\}\alpha\ D_2(b^*)+ D_2(a)\alpha\{ -D_1(c)\ \alpha D_2(b^*)\}$
$=D_2(a)\ \alpha(-D_1(c^*))\alpha\ D_2(b^*)+ D_2(a)\alpha( -D_1(c))\alpha D_2(b^*)$
$=D_2(a)\beta\{-D_1(c^*))\alpha\ \ D_2(b^*)-D_1(c)\alpha D_2(b^*)\}$, for all a, $b\epsilon N$, $\alpha\epsilon\Gamma$.
If $D_2\neq 0$ then by Lemma 4,
We have the equality, $(-D_1(c^*))\alpha\ D_2(b^*)\ -D_1(c)\alpha D_2(b^*)=0$.
That is $D_1(c^*)\alpha D_2(b^*)=(-D_1(c))\alpha D_2(b^*)$, for all b, $c\epsilon N$, $\alpha\epsilon\Gamma$. (7)
Thus, using this condition of our theorem, we obtain
$(-D_1(c))\alpha D_2(b^*)=D_1(-c)\alpha D_2(b^*)=-D_2(b^*))\alpha D_1(-c)= D_2(b^*)\alpha(-D_1(c))= -D_2(b^*)\alpha D_1(c)$
$=-D_1(c))\alpha D_2(b^*)$ (8)
From (7) and (8) we have that, for all b, $c\epsilon N$, $\alpha\epsilon\Gamma$,
$2D_1(c)\alpha D_2(b^*)=0$.
Since N is 2-torsion free, $D_1(c)\alpha D_2(b^*)=0$
Also, since $D_2$ is not zero, by Lemma 4, we see that $D_1(c)=0$ for all $c\epsilon N$. Therefore $D_1=0$
Consequently, either $D_1=0$ or $D_2=0$.
The converse verification is obvious ◊.

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