

Scour Prediction around Bridge Pier without and with Collar using ANFIS

Virender Kumar Sarda

Professor, Civil Engineering, Department, National Institute of Technology, Hamirpur (HP), India

Abstract: A modeling tool based on Adaptive Network-based Fuzzy Inference System (ANFIS) to predict the scour depth around bridge piers with and without collars is presented in this paper. The input parameters that affect the scour phenomena are average flow velocity, critical threshold velocity of sediment movement, flow depth, median particle diameter, geometric standard deviation, uncontracted and contracted channel widths. Experimental data collected during the present study and available in the literature are used in the training and testing stages of the models. A sensitivity analysis determined the most important parameters in predicting the scour depth around bridge piers with collars. ANFIS's results were found to be accurate, precise and satisfactory and thus can be used as analysis and design tool.

Keywords: Clear water scour, bridge pier, collar, ANFIS, scour protection devices

1. Introduction

The local scour around pier has the potential to threaten the structural integrity of bridge piers, ultimately causing failure when the foundation of the pier is undermined. In addition to the human loss, bridge failures result into heavy financial costs due to direct expenditure for replacement and restoration as well as the indirect expenditure related to the disruption of transportation facilities. Therefore, it is important to predict the integrity of bridge piers subject to scour at regular intervals. In this regard, experimental methods for monitoring scour and tools for predicting and estimating maximum scour depth in different environments are essential. Many models to predict scour depth around bridge piers based on experimental data are often not very accurate due to specific experimental conditions and experimental uncertainties and complexity of the scouring phenomena.

The problem of scour around an isolated pier has been extensively studied and documented by many researchers [1-3]. The design guides, such as, HEC-18 [4] and the Indian Road Congress Code IRC-78 ("standard" 2014) [5] usually require deep and expensive pier embedment in rivers. To reduce this depth of embedment, efforts have been made to reduce the depth of scour around bridge piers by using scour countermeasures which has been reviewed in [1,2,6] and many others. A series of recent bridge failures due to pier scour [7-9] have led to researchers to develop improved solutions for protecting bridges against the damages due to scour.

Recently, different artificial intelligence approaches such as Artificial Neural Networks (ANNs), Genetic Programming (GP), Gene-Expression Programming (GEP), Model Tree (MT), and Group Method of Data Handling (GMDH) have become popular for solving scour problem around hydraulic structures [10-13]. However, ANFIS based methods have not been used for predicting the local scour depth around bridge piers with or without countermeasure such as collars. This paper develops an ANFIS model to predict the scour depth around bridge piers with collar in bed of rectangular

channels. The performance of the proposed model is compared with empirical methods.

2. Model Descriptions

The acronym ANFIS derives its name from adaptive neuro-fuzzy inference system. It is an adaptive network, a network of nodes and directional links. Associated with the network is a learning rule - for example back propagation algorithm. It is called adaptive because some, or all, of the nodes have parameters, which affect the output of the node. These networks are learning a relationship between inputs and outputs. This adjustment allows the fuzzy systems to learn from the data they are modeling. The performance of this method is like both adaptive neural network ANN and FL (fuzzy-logic).

In both ANN and FL case, the input passes through the input layer (by input membership function) and the output could be seen in output layer (by output membership functions). In this type of advanced fuzzy logic, neural network uses a learning algorithm. The parameters are changed until they reach the optimal solution. Actually, in this type the FL tries (by using the neural network advantages) to adjust its parameters. The difference between real output and the network output in ANN is one of the common method to evaluate its performance. Therefore, ANFIS uses either back propagation or a combination of least squares estimation and back propagation for membership function-parameter estimation [10]. A fuzzy inference system consists of three components:

- **Rules;** The if-then rules have are determined by 'knowledge acquisition' from an expert. It is usually a time consuming process that is fraught with problems.
- **Membership functions;** A fuzzy set is fully determined by its membership function. This has to be determined.
- **The reasoning mechanism;** to carry out inference procedure on the rules and given fact.

Figure 1 shows the block diagram of basic fuzzy inference process from input to output.

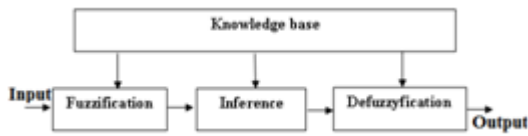


Figure 1: Block diagram of fuzzy inference process flow.

2.1 ANFIS Structure

The ANFIS model is based on a fusion of ideas from fuzzy control and neural networks and possesses the advantages of both neural networks and fuzzy control systems. In this way, one can bring the low-level learning and computational power of neural networks to fuzzy control systems and also provides the high-level, IF-THEN rule reasoning of fuzzy control systems to neural networks. Fundamentally, ANFIS is a graphical network representation of a Sugeno-type fuzzy system, endowed by neural learning capabilities. The network is composed of nodes with specific functions, or duties, collected in layers with specific functions [9]. In order to show the representative strength of ANFIS, the neural fuzzy control system considered in this paper is based on Tagaki-Sugeno-Kang (TSK) fuzzy rules [9] whose consequent parts are linear combinations of their preconditions. TSK fuzzy rules are in the forms Eq. 1.

$$R^j : IF x_1 \text{ is } A_1^j \text{ AND } x_2 \text{ is } A_2^j \text{ AND} \dots \text{ AND } x_n \text{ is } A_n^j \tag{1}$$

where $x_i (i = 1, 2, \dots, n)$ are input variables, y is the output variable, A_i^j are linguistic terms of the precondition part with membership functions $\mu_{A_i^j}(x_i)$, $a_j^i \in R$ are

coefficients of linear equations $f_i(x_1, x_2, \dots, x_n)$ and $j = 1, 2, \dots, m$. Assume that the fuzzy control system under consideration has two inputs x_1 and x_2 and one output y , then the rule base contains two TSK fuzzy rules as shown in Eqs. 2 and 3.

$$R^1 : IF x_1 \text{ is } A_1^1 \text{ AND } x_2 \text{ is } A_2^1 \text{ THEN } y = f_1 = a_0^1 + a_1^1 x_1 + a_2^1 x_2 \tag{2}$$

$$R^2 : IF x_1 \text{ is } A_1^2 \text{ AND } x_2 \text{ is } A_2^2 \text{ THEN } y = f_2 = a_0^2 + a_1^2 x_1 + a_2^2 x_2 \tag{3}$$

In fuzzy logic approaches, for given input values of x_1 and x_2 , the inferred output y^* is calculated by Eq. 4:

$$y^* = \mu_1 f_1 + \mu_2 f_2 / (\mu_1 + \mu_2) \tag{4}$$

Where μ_j are firing strengths of $R_j (j = 1, 2)$ and are given by Eq. 5:

$$\mu_j = \mu_{A_1^j} x_1 \times \mu_{A_2^j} x_2 \quad (j = 1, 2) \tag{5}$$

If product inference is used, the corresponding ANFIS architecture is shown in Fig. 2 [9], where node functions in the same layers are of the type described below.

Layer 1 Every node in this layer implies an input and it just passes external signals to the next layer.

Layer 2 Every node in this layer acts as a membership function (MF) $\mu_{A_1^j} x_1$ and its output specifies the degree to

which the given x_i satisfies the quantifier A_i^j . A sample (MF) is shown in Fig. 3.

Generally $\mu_{A_1^j} x_1$ is selected as bell shaped such as Eq. 6 and 7 with a maximum equal to 1 and minimum equal to 0:

$$\mu_{A_1^j} x_1 = \frac{1}{1 + \left[\frac{(x_i - m_i^j)}{\sigma_i^j} \right]^2} b_i^j \tag{6}$$

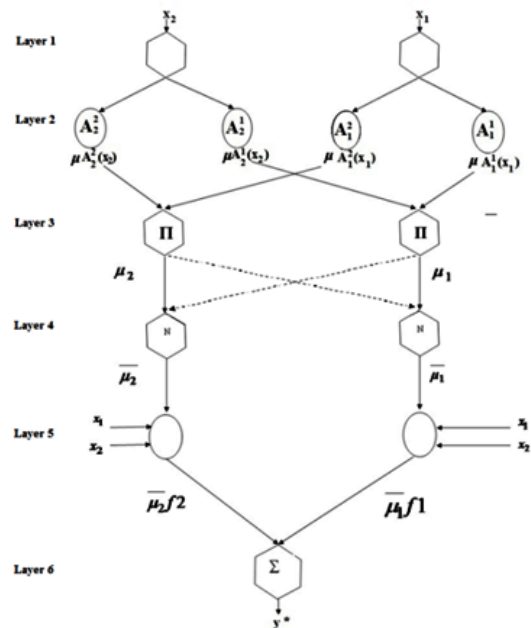


Figure 2: Structure of ANFIS

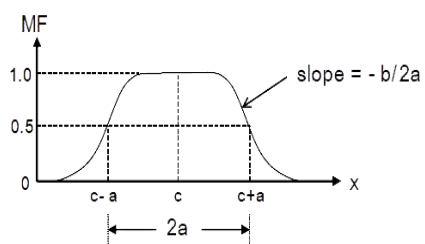


Figure 3: Sample membership function

$$\text{Or, } \mu_{A_1^j}(x_i) = \exp \left\{ - \left[\frac{(x_i - m_i^j)}{\sigma_i^j} \right]^2 b_i^j \right\} \tag{7}$$

Where $\{m_i^j, \sigma_i^j, b_i^j\}$ is the parameter set to be adapted. In fact, continuous and piecewise differentiable functions, such as commonly used trapezoidal or triangular membership functions, are also qualified candidates for node functions in this layer. Parameters in this layer are referred to as precondition parameters.

Layer 3 Every node in this layer is labeled $\bar{\mu}_j$ and multiplies the incoming signals $\mu_j = \mu_{A_1^j} x_1 \times \mu_{A_2^j} x_2$ and sends the product out. Each node output represents the firing strength of a rule.

Layer 4 Every node in this layer is labeled by N and calculates the normalized firing strength of a rule. That is the j^{th} node calculates the ratio of the firing strength of the j^{th} rule to that of all the rules by Eq. 8:

$$\bar{\mu}_j = \mu_j / \sum \mu_i \tag{8}$$

Layer 5 Every node j in this layer calculates the weighted consequent value $\bar{\mu}_j(a_0^j + a_1^j x_1 + a_2^j x_2)$, where $\bar{\mu}_j$ is the output of Layer 4 and $\{a_0^j, a_1^j, a_2^j\}$ is the set to be tuned. Parameters in this layer are referred to as consequent parameters.

Layer 6 The only node in this layer is labeled as Σ and it sums all incoming signals to obtain the final inferred result for the whole system [9].

2.2. Model Learning and Inference through ANFIS

Presented model consists of a number of membership functions and rules with adjustable parameters similarly to that of neural networks. The parameters associated with a given membership function may be chosen arbitrarily. However, in order to account for variations in the data values, the parameters were chosen to modify the membership functions to the input/output data. The neuro-adaptive learning techniques provide a method for the fuzzy modeling procedure to learn information about a data set, in order to compute the membership function parameters that best allow the associated fuzzy inference system to track the given input/output data. Using a given input/output data set, the toolbox function ANFIS constructs a fuzzy inference system (FIS) whose membership function parameters are adjusted using either a back-propagation algorithm alone, or in combination with a least squares (Hybrid) type of method. This allows your fuzzy systems to learn from the data they are modeling.

2.3 Graphical User Interface (GUI) Tool

Fuzzy Logic Toolbox Graphical User Interface Tool was used to build fuzzy inference System. Although, Fuzzy Logic Toolbox software does not limit the number of inputs, the number of inputs may be limited by the available memory of machine. If the number of inputs is too large, or the number of membership functions is too big, then it may also be difficult to analyze the FIS using the other GUI tools. ANFIS editor GUI in MATLAB ® V8.1.0.604 (R2013a) is opened by entering “anfisedit” in the command window. ANFIS editor GUI consists of (Fig. 4):

Membership Function Editor to define the shapes of all the membership functions associated with each variable.

Rule Editor to edit the list of rules that defines the behavior of the system.

Rule Viewer to view the fuzzy inference diagram. Rule viewer has been used here as diagnostic to see which rules are active, or how individual membership function shapes influence the results.

Surface Viewer to view the dependency of one of the outputs on any one or two of the inputs that is, it generates and plots an output surface map for the system.

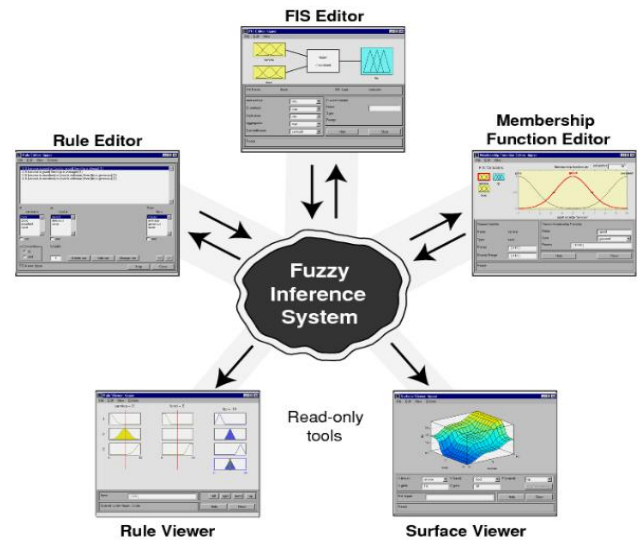


Figure 4: Fuzzy Inference Toolbox showing Graphical User Interface Tools.

2.4 Model Validation Using Checking and Testing Data Sets

To start training in ANFIS Editor GUI, first it needs to have a training data set that contains desired input/output data pairs of the target system to be modeled. Sometimes it also requires having the optional testing data set that can check the generalization capability of the resulting fuzzy inference system, or a checking data set that helps with model over-fitting during the training. Over-fitting is accounted for by testing the FIS trained on the training data against the checking data, and choosing the membership function parameters to be those associated with the minimum checking error if these errors indicate model over fitting. Training error plots should be examined closely in order to determine these errors. Usually these training and checking data sets are collected based on observations of the target system and are then stored in separate files.

3. Analysis of effective parameters on the scour depth around bridge pier without and with collar

Equilibrium scour depth around a circular pier in a steady flow of water over a bed of uniform, spherical and cohesion less sediment depends on numerous groups of variables such as; flow, sediment characters, and pier geometry. Thus, the following functional relationship can describe scour depth.

$$d_{sp} = f_1(h, U, d_{50}, b) \tag{9}$$

Where u = average velocity of approach flow; h = depth of flow; d_{50} = particle mean diameter; b = diameter of the pier and d_{sp} = equilibrium scour depth.

In case of pier fitted with collar, the equilibrium scour depth (d_{sc}) is related to the six input parameters: equilibrium scour depth in case of pier without appurtenances (d_{sp}), relative collar diameter ($B - b$), depth of collar below free water surface (h_c), pier diameter (b), depth of flow (h), mean sediment size (d_{50}). The functional relationship between input and output parameters is shown in Eq. 10.

$$d_{sc} = f_2(d_{sp}, B - b, h, b, h_c, d_{50}) \quad (10)$$

Equation (10) can be written in non-dimensional form as:

$$\frac{d_{sp} - d_{sc}}{d_{sp}} = f_3\left(\frac{d_{sp}}{b}, \frac{d_{sp}}{h}, \frac{B - b}{d_{sp}}, \frac{h - h_c}{h}, \frac{b}{d_{50}}\right) \quad (11)$$

Equation 11 gives reduction in scour depth due to placement of collar around bridge pier.

4. Implementations And Results

4.1 Pier without collar

To assess the performance of the models, the equilibrium scour data without collar consisting of 242 data points, collected from literature [20-31] were divided into two parts randomly - a training or calibration set consisting 170 data points and a validation or testing set consisting of 72 data points. Table 1 shows the range of different parameters present in data set.

Table 1: Range of different parameters for pier without collar

Parameter	Range
Flow depth, (h)	0.030 - 0.6 (m)
Average velocity of approach flow, u	0.172- 1.095 (m/s)
Mean sediment size, d_{50}	0.24 - 14.25 (mm)
Diameter of pier, b	0.028- 0.915 (m)
Equilibrium scour depth, d_{sp}	0.032 - 0.7 (m)

In order to check the effect of membership functions, ANFIS architecture utilizing Sugeno- type fuzzy-inference systems and two models were prepared using generalized bell shaped and triangular membership function to emulate a given training data set of scour without collar using Eq. 9. Three Generalized bell (gbell) type membership function and three triangular membership functions were applied to the fuzzy inference system (FIS) with a linear type membership function for the output in ANFIS models.

It was found that generalized bell shaped membership function model (Fig. 5 and 6) provides better predictions of

scour depth than the triangular membership function ANFIS model both in training and validation stage with the correlation coefficients 0.8936 and 0.9434 and RMSE of 16.132 and 5.949 respectively.

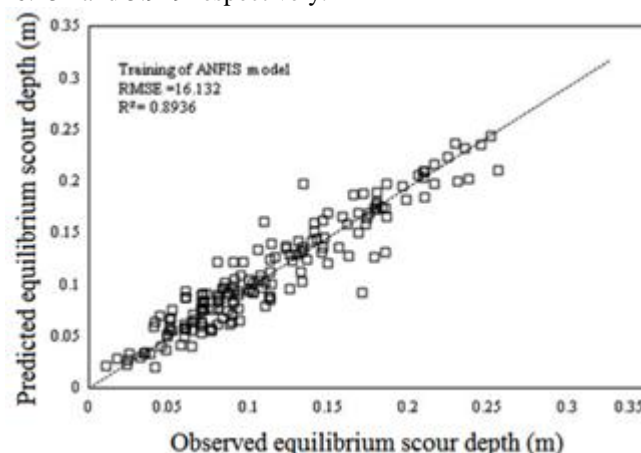


Figure 5: Scatter plot of observed and predicted equilibrium scour depth for training (gbell type MF)

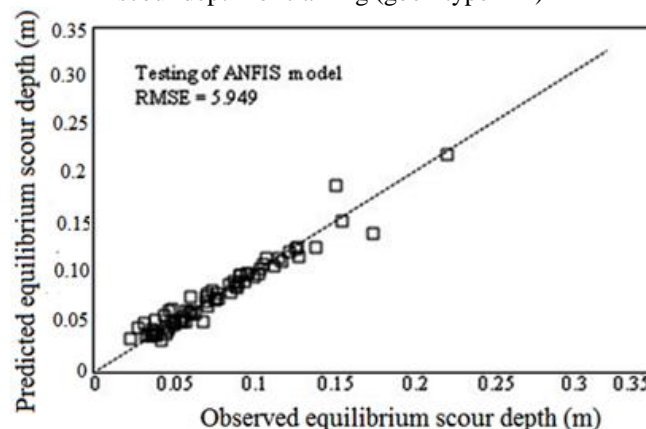


Figure 6: Scatter plot of observed and predicted equilibrium scour depth for validation (gbell type MF)

3.2 Pier with collar

In this study two ANFIS models using generalized bell shaped membership function with dimensional and non-dimensional parameters were developed. The data on equilibrium scour depth around bridge pier provided with collar was collected from literature [8, 24, 28, 32-37]. The range of different parameters present in data set is tabulated in Table 2.

Table 2: Range of different parameters present in data set for pier with collar

Parameter	Range
Flow Depth, h	0.046 - 0.2 (m)
Diameter of pier, b	0.030 - 0.244 (m)
Depth of collar below free water surface, h_c	0.046 - 0.251 (m)
Mean sediment size, d	0.280 - 1.9 (m)
Diameter of collar, B	0.060 - 0.488 (m)
Scour depth without appurtenances, d_{sp}	0.049 - 0.1675 (m)
Scour depth to collar, d_{sc}	0.014 - 0.143 (m)

The whole data set consisting of 130 data points was divided into two parts - 60% for training and 40% for testing or validation.

- a) First, ANFIS model was established using original data pertaining to parameters in dimensional form set to estimate scour depth in case of pier fitted with collar, using Eq. 10.
- b) Second, model was developed using parameters in non-dimensional form given in Eq. 11 to estimate the reduction in scour depth due to collar.

Data sets containing input combination and output are implemented in the ANFIS GUI for both cases and Sugeno type system of first order was created. For both of the models, two gbell type membership function were applied to the fuzzy inference system (FIS) with a linear type membership function for the output and sets of 64 and 32 rules were created based on two membership function for these models. For training purpose hybrid learning algorithms having combination of least-squares and back propagation gradient descent method was applied to the FIS. The epoch size was set as 3.

Figures 7 and 8 compare the observed and predicted values for the training and testing data when the ANFIS model trained by the dimensional data set i.e. Eq. 10 and non-dimensional data set i.e. Eq. 11. The statistical error parameter obtained from training stage for Eq. 10 were RMSE= 6.223 and $R^2 = 0.9043$ while for Eq. 11 these were RMSE= 0.0438 and $R^2 = 0.9283$ which clearly shows that data when expressed in nondimensional form gives better results. Also, during validation stage, these statistical parameters were found to be RMSE= 0.773 with $R^2 = 0.983$ and RMSE = 0.0157 with $R^2 = 0.993$ respectively. It is clear again that the RMSE value of Eq. 11 is minimum i.e. 0.01674 with a R^2 value of 0.993. Hence, ANFIS model with parameters expressed in non-dimensional form performs most satisfactory for prediction of reduction in scour depth due to collar.

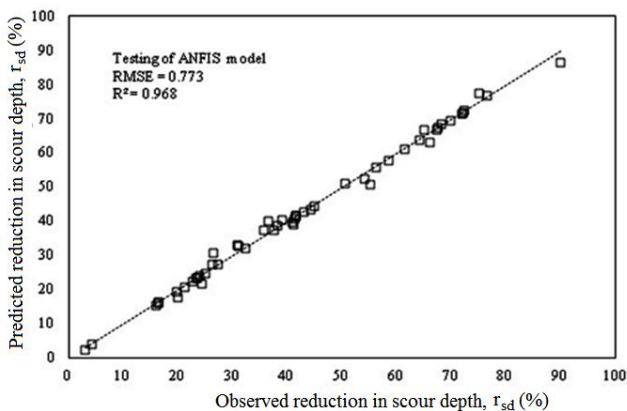


Figure 7: Scatter plot of observed and predicted scour depth due to collar with Eq. 10 for validation

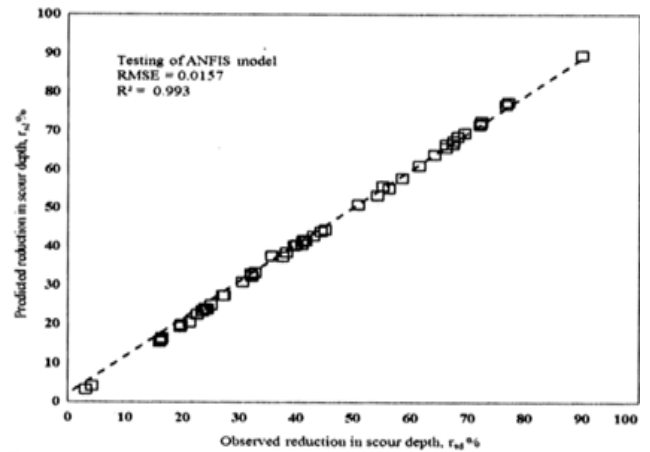


Figure 8: Scatter plot of observed and predicted reduction in scour depth due to collar with Eq. 11 for validation

Sensitivity analysis

Through the data analysis, process of sensitivity analysis was to quantify how much model output values were affected by variations in the input values. To determine the importance of each input variable on the scour depth, the ANFIS was applied to perform a sensitivity analysis. The analysis was conducted such that, one parameter of Eq. 11 was eliminated each time to evaluate the effect of that particular input on output. Results of the analysis are given in Table 3.

Table 3: Sensitivity analysis results for the parameters in Eq. 11

Method	RMSE	R^2	RANK
ANFIS Eq. 11	0.0157	0.993	-
ANFIS no. $(h - h_c) / h$	0.1719	0.226	1
ANFIS no. $(B - b) / b$	0.1050	0.714	2
ANFIS no. d_{sp} / h	0.0230	0.986	3
ANFIS no. d_{sp} / b	0.0666	0.989	4
ANFIS no. b / d_{50}	0.0011	0.990	5

It is apparent from Table 3 that $(h - h_c) / h$ and b / d_{50} have the highest and the least effect on reduction in scour depth $(d_{sp} - d_{sc}) / d_{sp}$ respectively. These findings are in consistent with existing understanding of the relative importance of the various parameters on scour depth. The effects of the non-dimensional variables on reduction in scour depth using collar $(d_{sp} - d_{sc}) / d_{sp}$ have also been ranked in Table 3.

5. Summary and Conclusions

In this paper, adaptive neuro-fuzzy inference system vector machines were developed to predict the scour depth around bridge pier without and with collar. Data sets for performing the training and testing stages were collected from literatures. Four inputs and one output parameter [Eq. 9] were defined for scour depth around pier without collar while for pier with collar five inputs and one output parameter [Eq. 11] were defined. Performance of ANFIS as soft computing

tool for the determination of both scour around bridge pier without collar with gbell type MF (i.e. RMSE= 5.949 with $R^2 = 0.9434$) and with collar (i.e. RMSE= 0.0157 with $R^2 = 0.993$) was found to be satisfactory.

Nomenclature

u = average velocity of approach flow

h = depth of flow

d_{50} = particle mean diameter

b = diameter of the pier

d_{sp} = equilibrium scour depth around bridge pier

d_{sc} = equilibrium scour depth around bridge pier with collar

$(B - b)$ = relative collar diameter

h_c = depth of collar below free water surface

References

- [1] Virender Kumar, Reduction of scour around bridge pier using protective devices. Ph. D. thesis, University of Roorkee, Roorkee, India, (1996)
- [2] Lu Lu Deng and C. S. Cai, Bridge Scour: Prediction, Modeling, Monitoring, and Countermeasures—Review. Practice Periodical on Structural Design and Construction. 15(2), May, (2010)
- [3] Ayush Vashisth and Bhawana Arora, Local scour: A Review for Hydraulic Engineers. International e-Journal of Mathematics and Engineering. 5(1), 2354 - 2359 (2014)
- [4] E. V. Richardson and S.R. Davis, Evaluating scour of bridges. Rep 000FHWA/NHI01-001, HEC-18, Federal Highway Administration of Transportation. (2001)
- [5] IRC:78, Standard Specifications Codes of Practice for Road Bridges. Sec: VII, Indian Road Congress, (2014)
- [6] Mubeen Beg and Salman Beg, Scour Reduction around Bridge Piers: A Review, International Journal of Engineering Inventions. 2(7), May, 7-15, (2013)
- [7] Kumalasari Wardhana and C. Hadipriono Fabian, Analysis of Recent Bridge Failures in the United States. Journal of Performance of Constructed Facilities. 17(3), 144-150 (2003)
- [8] A. Tafarajnoruz, R. Gaudio and F. Calomino, Evaluation of Flow-Altering Countermeasures against Bridge Pier Scour. J. Hydraul. Eng., 138(3), 297–305 (2012)
- [9] S. M. Bateni, D. S. Jeng and B.W. Melville, Bayesian neural networks for prediction of equilibrium and time-dependent scour depth around bridge piers. Adv. Eng. Software. 38 (2), 102–111 (2007)
- [10] T. L. Lee, D.S. Jeng, G. H. Zhang and J. H. Hong, Neural network modeling for estimation of scour depth around bridge piers. J. Hydrodynam. Ser. B 19 (3), 378–386 (2007)
- [11] A. Guven and M. Gunal, Genetic programming approach for prediction of local scour downstream hydraulic structures. J. Irrigation Drainage Eng. 134 (2), 241–249 (2008)
- [12] M.K. Ayoubloo, A. Etemad-Shahidi, and J. Mahjoobi, Evaluation of regular wave scour around a circular pile using data mining approaches. Appl. Ocean Res. 32 (1), 34–39 (2010)
- [13] H. Md. Azamathulla, A. AbGhani, N. A. Zakaria and A. Guven, Genetic Programming to Predict Bridge Pier Scour. J. Hydraul. Eng. ASCE. 136 (3), 165–169 (2010)
- [14] A. Etemad-Shahidi and N. Ghaemi, Model tree approach for prediction of pile groups scour due to waves. J. Ocean Eng. 38, 1522–1527 (2011)
- [15] A. Guven, H. MdAzamathulla and N. A. Zakaria, Linear genetic programming for prediction of circular pile scour. J. Ocean Eng. 36 (12–13), 985–991 (2009)
- [16] M. Najafzadeh and G.A. Barani, Comparison of group method of data handling based genetic programming and back propagation systems to predict scour depth around bridge piers. Sci. Iran 18 (6), 1207–1213 (2011)
- [17] M. Pal, N. K. Singh and N. K. Tiwari, M5 model tree for pier scour prediction using field dataset. KSCE J. Civil Eng. 16 (6), 1079–1084 (2012)
- [18] M. Najafzadeh, G.A. Barani and M. R. Hessami Kermani, GMDH network based back propagation algorithm to predict abutment scour in cohesive soils. J. Ocean Eng. 59, 100–106 (2013)
- [19] S. Akib, M. Mohammadhassani and A. Jahangirzadeh, Application of ANFIS and LR in prediction of scour depth in bridges. Computers and Fluids. 91, 77–86 (2014)
- [20] J. Chabert, and P. Engeldinger, Etude des affouillements autour des piles des ponts. Report, Laboratoire National d'Hydraulique, Chatou, France (1956)
- [21] H. W. Shen, V. R. Schneider and S. S. Karaki, Mechanics of local scour. Report CER66HWS22, US Department of Commerce, National Bureau of Standards. Institute for Applied Technology, Washington, DC (1969)
- [22] S. Hancu, Sur le calcul des affouillements locaux dans la zone des piles des ponts. In: 14th International Association of Hydraulic Research Congress. Paris, France, 299–313 (1971)
- [23] S. C. Jain, and E. E. Fischer, Scour around bridge piers at high Froude numbers. FHWA-RD-79-104, Federal Highway Administration, U.S. Department of Transportation, Washington, D. C (1980)
- [24] Y. M. Chiew, Local scour at bridge piers. Report No 355, Department of Civil Engineering, University of Auckland, New Zealand (1984)
- [25] R. Ettema, Scour at Bridge Piers. Report No. 216, School of Engg., University of Auckland, Auckland, New Zealand (1980)
- [26] U. C. Kothyari, Scour around bridge piers. Ph.D. thesis, Univ. of Roorkee, Roorkee, India (1989)
- [27] A. M. Yanmaz, and D. Altinbilek, Study of time-dependent local scour around bridge piers. J. Hydr. Eng., ASCE, 117(10), 1247-1268 (1991)
- [28] Virender Kumar, Protection of Bridge Piers from Scour. Indian Highways, 11-22, Oct (1998)
- [29] O. Oliveto and W. Hager, Temporal evolution of clear-water pier and abutment scour. J. Hydr. Eng., ASCE. 128(9), 811-820 (2002)
- [30] M. F. Mia and H. Nago, Design method of time-dependent local scour at circular bridge pier. J. Hydr. Eng., 129(6), 420–427 (2003)

- [31] S. Dey and R. V. Raikar, Characteristics of horseshoe vortex in developing scour holes at piers. *J. Hydr. Eng.*, 1334, 399–413 (2007)
- [32] D. E. Schneible, An investigation of the effect of bridge pier shape on the relative depth of scour. M.Sc. Thesis, Graduate College of the State, University of Iowa, Iowa City, Iowa (1951)
- [33] S. Tanaka and M. Yano, Local scour around circular cylinder. *Proc. 12th IAHR Congress*, Fort Collins, CO, 3, 125–134 (1967)
- [34] A. Etemad-Shahidi, R. Yasa and M. H. Kazeminezhad, Prediction of wave-induced scour depth under submarine pipelines using machine learning approach. *Appl. Ocean Res.* 33, 54–59 (2011)
- [35] V. Garg, B. Setia and D. V. S. Verma, Prevention of scour by combination of scour protection devices around bridge piers. *Proc. River Flow 2008*, Izmir, TR, 1637–1643 (2008)
- [36] A. T. Moncada-M., J. Aguirre-Pe, J. C. Bolívar and E. J. Flores, Scour protection of circular bridge piers with collars and slots. *J. Hydraulic Res.* 47(1), 119–126 (2009)
- [37] A. M. Negm, G. M. Abdelaal, Y. A. Mohamed and A. A. Fathy, Control Of Local Scour Around Bridge Piers Using Current Deflector. *13th International Water Technology Conference*, Iwtc 13, Hurghada, Egypt (2009)