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Non Linear Programming Approach for Radiotherapy Treatment Planning with Partial Volume Constraints

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Abstract: ***Optimization has become an important tool in treatment planning for cancer radiation therapy .In radiation therapy the major test is to quantify optimization techniques. In this paper an attempt has been made to solve the 'radiotherapy treatment planning' by using non linear programming approach with partial volume constraints. The challenge in therapy preparation is to decide a treatment plan, to determine beam weights, beam directions, and appropriate use of beam modifiers such as wedges and blocks, with the aim of delivering a lethal dose to the tumor while sparing nearby organs at risk and normal tissue. The results are evaluated numerically and illustrated through dose volume histograms.***

Keywords: Non Linear Programming problem; Radiotherapy; Dose Volume Histograms; Intensity modulated radiotherapy (IMRT); Conventional 3D conformal radiotherapy (3DCRT)

1. Introduction

For many years, radiation has been used as a treatment for cancer. Bombarding a malignant tumor with high-energy particles can destroy the cancerous cells or at least slow down their growth. Since tumors grow in the presence of healthy tissue and even near critical organs, it is usually impossible to irradiate the tumor without allowing some damage to the nearby critical organs. In fact, complications can occur when neighboring critical organs receive too much of this collateral radiation. Moderate damage to critical organs may be acceptable, however, if the effect can be accurately predicted, since cancerous cells do not have the ability to repair themselves as efficiently as normal cells. Therefore, an important research activity in radiotherapy is to develop methods of escalating dosage delivered to the tumor while carefully controlling the dosage deposited in neighboring critical organs and healthy tissues.

Modern external radiotherapy seeks to conform the "shape" of the delivered radiation to specific three-dimensional structures (tumors, critical organs, etc.) within the patient using a linear accelerator that emits beams of high-energy photons from predetermined angles around the patient. Conventional 3D conformal radiotherapy (3DCRT) uses beams of uniform intensity. Parameters such as beam intensity and aperture are carefully adjusted to give the best conformation and least chance of complications. In recent years, intensity-modulated radiotherapy (IMRT) has been gaining support as a more advantageous method of delivering radiation treatments. The IMRT approach breaks each beam up into hundreds of tiny constituent beamlets, or pencil-beams, each of which is assigned its own intensity. The resulting IMRT beam has better resolution than a uniform 3DCRT beam for conforming to the 3D shape of the patient's tumor and avoiding the critical organs. For instance, in IMRT, intensities for certain beamlets which are collinear with critical organs might be set to low or zero values, while other beamlets collinear with the tumor can be set to high values. In standard 3DCRT, however, the entire beam must be one uniform intensity, so maintaining the same safety tolerances for the critical organ would require delivering far less dosage to the tumor. Although IMRT has many advantages over the conventional 3DCRT approach, it also complicates treatment planning. Whereas 3DCRT requires the assignment of only a few intensities (one for each beam), IMRT requires thousands (one for each beamlet). The considerable task of planning the IMRT beamlet intensities such that a desirable treatment is achieved necessitates extensive use of optimization formulations and techniques. The standard optimization formulation for the IMRT problem minimizes the average least squares deviation from the prescribed tumor dose, penalizing when critical organ dosages exceed some tolerance dose [1].

A concise mathematical description of the non-linear programming (NLP) problem is as follows:

$$\min \quad f(x) \tag{1.1}$$

$$\text{Subject to} \quad g(x) \le 0, \tag{1.2}$$

$$l \le x \le u. \tag{1.3}$$

Here, x is a vector of variables that are continuous real numbers, f(x) is the objective function, and g(x) represents the set of constraints. l and u are vectors of lower and upper bounds placed on the variables. With a non-linear formulation [2], there is an expanded range of possible objective functions and constraints as compared with linear programming.

The remainder of the paper is described as follows. Section 2 contains a non linear programming model of treatment panning problem that we tested. The model is described in section 2. We interpret and discuss the computational results in section 3 which are illustrated through graphically. Section 4 contains main conclusions.

2. Formulation of the Treatment Planning Problem

Implementation of Partial Volume Constraints Using Nonlinear Programming

Recall that the dose volume histogram displays the fraction of each region of the patient that receives at least a specified dose level. In some cases, the radiation oncologist is willing to sacrifice a portion of a region at risk in order to improve the probability of curing the disease. Oncologists often specify constraints of the form "No more than *x*% of this region at risk can exceed a dose of *y*." Thus, for a particular region at risk, the oncologist determines both a dose limit and a fraction of the structure that can exceed the dose limit. This type of requirement is called a partial volume constraint [3].

For our formulas, the dose limit will be denoted by Λ and the fraction of the volume allowed to exceed this limit will be denoted by Ω. We have taken up both nonlinear and mixed integer approach to the implementation of partial volume constraints.

The nonlinear formulation represents a new approach to partial volume constraints:

$$\min_{w} \sum_{(i,j)\in I} (D_{ij} - \delta_{ij})^2 \qquad (2.1)$$

$$\text{Subject to } D_{ij} = \sum_{p=1}^{n} w_p D_{ij}^p \quad \forall (i,j), \qquad (2.2)$$

$$\sum_{(k,l)\in R} erf(D_{kl} - \Lambda_{kl}) \le n_R \Omega_R, \qquad (2.3)$$

$$\sum_{(k,l)\in N} erf(D_{kl} - \Lambda_{kl}) \le n_N \Omega_N, \qquad (2.4)$$

$$w_p \ge 0. \qquad (2.5)$$

In these formulas, n_R is the number of pixels in the region at risk and n_N is the number of pixels in the normal tissue. The partial volume constraints were realized through the use of a ramp function. In this case the error function (erf) was used. For each partial volume constraint, the error function was shifted so that the center of the ramp matched the dose limit. With other shifts and scalings it may be possible to improve the solution quality and numerical performance. The goal of the optimizer was to minimize the sum of the squared difference between the prescribed and the actual doses over all of the pixels in the tumor subject to two partial volume constraints.

For this simulated treatment, the prescription dose level in the tumor was set at 70 Gy. The first partial volume constraint specified that 80% of the region at risk must be kept below 30 Gy. The second partial volume constraint specified that 95% of the normal tissue region must be kept below 50 Gy.

3. Numerical Evaluation

In the simulated treatments, the dose distribution for each beamlet is stored in a relatively dense 100x100 matrix. Up to 7000 beamlets are used in the optimization. We calculated by minimizing the squared differences between the prescribed and the actual doses summed over all of the pixels , given by the equation (2.2) numerically by considering a partial volume constraints specified that 80% of the region at risk must be maintained below the dose limit of 30Gy and the second partial volume constraint specified that 95% of the normal region must be kept below 50Gy. Form the above parameters, when the dose volume histogram is constructed the following conclusions are made. From the fig.1, it is observed that when 20% of the region at risk was allowed to exceed 30Gy and 5% of the region at risk was allowed to exceed 50%. The non-target required a relative dose of approximately 73%. Where as critical structure showed a response at a relative dose of 60Gy. The target was not subjected to any dose volume histogram constraint that was applied to non-target and critical structure, it is observed that the response of target was relatively similar that of the non-target i.e. the relative dose was 73Gy.

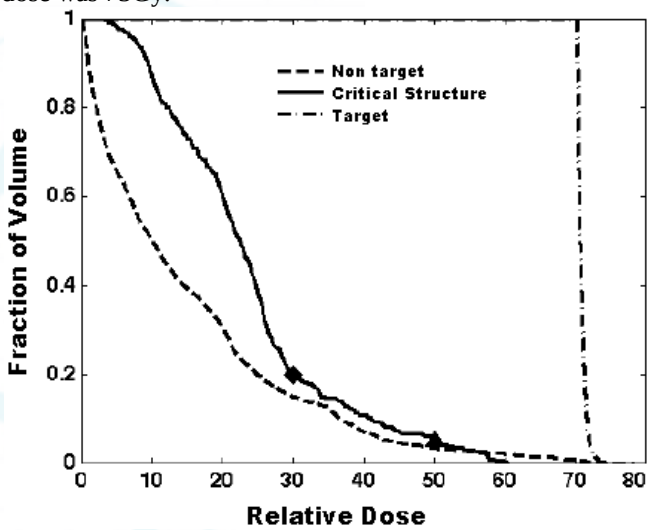

Figure 1: Cumulative dose volume histogram

4. Conclusion

In this paper an effort has been made to study the radiation treatment planning by using the non linear programming with partial volume constraints in which the profiles of critical structure, target and non target are illustrated using dose volume histograms. It can be concluded that when the weight of critical structure is increased, the relative dose of response appears to decrease whereas for non-target structures, an increase in weight dose does not show any difference in the relative dose of response. It can further be concluded that in the target structure a decrease in weight showed an increase in the relative dose.

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