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Mathematical Modeling of the Biological Pest Control of Rice Stem Borer using Logistic Model and its Equilibrium Points

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Abstract: ***Many agricultural problems have important ecological dimensions. Diseases, pests and epidemics are often treated more 'ecologically' it implies more 'holistically' and less mechanistically. The models are Ecological models often use more 'holistic' concepts. The differential equations and difference equations are considered. In this model can be age- Structured logistical model is presented. A simple mathematical model of interaction between rice stems borer (Scirpophaga Incertulas) and its egg parasitoid Trichogramma Japanicam. In this model the rice stem borer is represented by egg and larval stages. The parasitoid is considered in terms of parasitized eggs. Linear feedback control strategy is proposed to indicate how many natural enemies introduced into the environment. Using this control strategy directs the system to the stable equilibrium point.***

Keywords: Mathematical model, rice stems borer, *Trichogramma Japanicam,* equilibrium point

1. Introduction

The yellow stem borer (*Scirpophaga Incertulas)* is the most important rice pest in Tamil Nadu at the rainy season. The yellow stem borer builds internal galleries in the paddy field causing direct damages of weight loss. One of the challenges of the improvements in the farming and harvesting of rice is the biological pest control. A good strategy in biological pest control can increase the yield and also the benefits of environment. Biological control is the use of living organisms to control the pest population. Pest are spices that interfere with human activity it cause injury loss or irritation to a crop, stored product, animal or people. Our aim is to control the pest density in an equilibrium level below economic damage. The natural enemies play an important role in limiting of the potential pest population. The application of biological control method as the manipulation of natural enemies by man to control pests, natural biological control as control the occurs without human intervention.

2. Material and Methods

Mathematical modeling is an important tool used in agricultural problems. In this model applied to the problems of biological pest control of impact between the pest and its natural enemy. We propose a simple mathematical model of interaction between the rice stem borer *(Scirpophaga Incertulas)* and its egg and larval parasitoid Trichogramma Japanicum. This model the yellow stem borer is represented by the egg and the larval stages, and the parasitoid is considered in terms of the parasitized eggs and laval parasitoid Trichogramma Japanicum.

Mathematical models of interactions between the yellow stem borer and its egg and larval parasitoid (Trichogramma Japanicum)

Consider two main stages of development of the rice stem borer egg and larval stages. There exists only one egg parasitoid (Trichogramma Japanicum) in a common environment. The rate at which the infestation is acquired is proportional to the number of encounters between Trichogramma Japanicum and rice stem borers. That means the net rate of parasitism is γxz, (R.M.Aderson and R.M.May)

x is the number of uninfected individuals.

z the number of parasitoids egg.

γ the rate of parasitism.

Consider the assumption for the rice stem borer and parasitoid (Trichogramma Japanicum) interaction and also using the logistic growth for egg population based on this equation the rate of change of the egg population density N_1 as

$$\frac{dN_1}{dt} = \beta\left(1 - \frac{N_1}{k}\right)N_1 - a_1N_1 - b_1N_1 - \gamma N_1 z \text{-------}(1)$$

Here N_1 is egg density of rice borer

z the number of adult parasitoids

β the net reproductivity rate

K the carrying capacity

a_1 the fraction of eggs from which the larvae emerge at time t

γ rate of parasitism

b_1 natural mortality rate of the egg population

Next stage of rice stem borer

The rate of change of the parasitized egg N_2

$$\frac{dN_2}{dt} = \gamma N_1 z - a_2N_2 - b_2N_2 \text{-----------------}(2)$$

a_2 mortality rate of the parasitized egg

b_2 the fraction of the parasitized eggs from adult parasitoids emerge at time t.

Next stage of rice stem borer is emerge larvae population

The rate of charge of larvae population density N_3

$$\frac{dN_3}{dt} = b_1N_1 - a_3N_3 - b_3N_3 \text{ ------------(3)}$$

a_3 mortality rate of larvae population which moults into pupal stage at time t.

In the adult parasitoid infects eggs of rice stem borer at the beginning of second generation life is

$$z=\delta b_2N_2 \text{ --------------(4)}$$

δ the number of parasitoids emerged from one parasitized egg(δ=1 for *Trichogramma Japanicum*)

$$z=b_2N_2 \text{ --------------(5)}$$

Substitute the equation (5) in (1), (2) and (3)

$$\left.\begin{aligned} \frac{dN_1}{dt} &= \beta\left(1-\frac{N_1}{k}\right)N_1 - a_1N_1 - b_1N_1 - \gamma N_1b_2N_2 \\ \frac{dN_2}{dt} &= \gamma N_1b_2N_2 - a_2N_2 - b_2N_2 \\ \frac{dN_3}{dt} &= b_1N_1 - a_3N_3 - b_3N_3 \end{aligned}\right\} \text{----(6)}$$

The above equation to use the logistical model and interaction between the rice stem borer and its parasitoid *Trichogramma Japanicum*.

3. Equilibrium Points

The above set of modeling equation tends to equilibrium point is right hand side of the equation (6) tends to zero.

$$N_1{}^*\left(\beta\left(1-\frac{N_1{}^*}{k}\right) - a_1 - b_1 - \gamma b_2N_2{}^*\right) = 0 \text{-------(7)}$$

$$N_2{}^*(\gamma N_1{}^*b_2 - a_2 - b_2) = 0 \text{ -----------------------(8)}$$

$$b_1N_1{}^* - N_3{}^*(a_3 - b_3) = 0 \text{ ------------------------(9)}$$

Solve the equation (7), (8) and (9)

(7)⇒ $N_1{}^* = 0$

(8) ⇒ $N_2{}^* = 0$

$N_1{}^* = 0$ and $N_2{}^* = 0$ in (9) $\Rightarrow N_3{}^* = 0$

$N_1{}^* = 0, N_2{}^* = 0$ and $N_3{}^* = 0$ is a trivial solution

In equation (8) $\Rightarrow N_2{}^* = 0$

$N_2{}^* = 0$ in equation (7) $N_1{}^* = (-a_1 - b_1 + \beta)\frac{k}{\beta}$

$$= (\beta - a_1 - b_1)\frac{k}{\beta}$$

$N_3{}^* = -\frac{b_1k}{\beta}(\beta - a_1 - b_1)(\frac{-1}{a_3+b_3})$

$=\frac{b_1k}{\beta(a_3+b_3)}(\beta - a_1 - b_1)$

In equation (8) ⇒

$N_1{}^* = \frac{a_2+b_2}{\gamma b_2}$ $\quad N_2{}^* = \frac{1}{\gamma b_2}(\beta - \frac{\beta}{k}\left(\frac{a_2+b_2}{\gamma b_2}\right) - a_1 - b_1)$

$N_3{}^* = \frac{b_1(a_2+b_2)}{\gamma b_2(a_3+b_3)}$

The Equilibrium points are

EP_1=(0,0,0)

EP_2=($(\beta - a_1 - b_1)\frac{k}{\beta}, 0, \frac{b_1k}{\beta(a_3+b_3)}(\beta - a_1 - b_1)$)

Ep_3=($\frac{a_2+b_2}{\gamma b_2}, \frac{1}{\gamma b_2}\left(\beta - \frac{\beta}{k}\left(\frac{a_2+b_2}{\gamma b_2}\right) - a_1 - b_1\right), \frac{b_1(a_2+b_2)}{\gamma b_2(a_3+b_3)}$)

The equilibrium point EP_1 is a trivial point.

For EP_2 the condition $\beta > a_1 + b_1$ ensures the non-negativity of larvae population.

For EP_3 the condition for non-negativity of parasitized egg population is $\beta > a_1 + b_1 + \frac{\beta}{k}\left(\frac{a_2+b_2}{\gamma b_2}\right)$

4. Result and Discussions

The mathematical model for biological pest control of rice stem borer using logistical growth model there are three stages of stability analysis and its equilibrium points. The first equilibrium point is a trivial point. The second equilibrium point is non negativity of larvae population. The net reproductivity rate is more than the sum of the fraction of eggs from which the larvae emerge at time t and natural mortality rate of the egg population. The third equilibrium point is the condition for non negativity of parasitized egg population of the net reproductivity is more than the sum of the fraction of eggs from which the larvae emerge at time t and natural mortality rate of the egg population and a small positive quantity. In the linear feedback control strategy is proposed to indicate the equilibrium condition is not satisfied natural enemies introduced into the environment. Using this control strategy directs the system to the stable equilibrium point.

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