Properties of Operations on Total Regular Intuitionistic Fuzzy Graphs

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Abstract: In this paper, some properties of union and join on total regular intuitionistic fuzzy graphs are derived, and also theorems related to these concepts are stated and proved.

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1. Introduction

In 1975, Rosenfeld [15], Yeh and Banh [17] have carried out experiments on the applications of fuzzy sets to develop the structure of Fuzzy Graphs (FGs). Intuitionistic fuzzy graph theory was introduced by K.T. Atanassov [3] in 1999. Research on the theory of Intuitionistic fuzzy sets [IFS] has been witnessing an exponential growth in mathematics and its application. M. G. Karunambigai and R. Parvathi [7] introduced intuitionistic fuzzy graph as a special case of Atanassov’s IFS. A. Nagoo Gani and S. Sajitha Begum [12] defined degree, order and size in intuitionistic fuzzy graphs and extends the properties. This leads to the consideration of the operations on IFGs. In this paper, properties of union and join on total regular IFGs are presented.

2. Basic Definitions

Definition 2.1: An intuitionistic fuzzy graph (IFG) is of the form G: (V, E) where
(i) The function \( \mu_i: V \to [0,1] \) & \( \gamma_i: V \to [0,1] \) denote the degree of membership and non-membership of the element \( v \in V \) respectively. Such that \( 0 \leq \mu_i(v) + \gamma_i(v) \leq 1 \) for every \( v \in V \), \( i = 1, 2, \ldots, n \).
(ii) The function \( \mu_2: V \times V \to [0,1] \) & \( \gamma_2: V \times V \to [0,1] \) are defined by
\[
\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)) \\
\gamma_2(v_i, v_j) = \max(\gamma_1(v_i), \gamma_1(v_j))
\]

Definition 2.2: An IFG \( G = (V, E) \) is said to be regular IFG, if there is a vertex which is adjacent to vertices with same degrees.

Definition 2.3: An intuitionistic fuzzy graph \( G \) is said to be strong IFG, if \( \mu_{2ij} = \min(\mu_1(v_i), \mu_1(v_j)) \) and \( \gamma_{2ij} = \max(\gamma_1(v_i), \gamma_1(v_j)) \), \( \forall (v_i, v_j) \in E \).

Definition 2.4: An IFG \( G = (V, E) \) is said to be complete IFG if \( \mu_{2ij} = \min(\mu_1(v_i), \mu_1(v_j)) \) and \( \gamma_{2ij} = \max(\gamma_1(v_i), \gamma_1(v_j)) \) for every \( (v_i, v_j) \in E \).

Definition 2.5: Let \( G = ((\mu_1, \gamma_1), (\mu_2, \gamma_2)) \) be an IFG. Then the degree of a vertex \( v_i \) is defined as
\[
d(v_i) = \left( \sum_{(v_i, v_j) \in E} \mu_2(v_i, v_j) \right) + \left( \sum_{(v_i, v_j) \in E} \gamma_2(v_i, v_j) \right)
\]
Where \( \mu_2(v_i, v_j) = \gamma_2(v_i, v_j) = 0 \) Whenever there exists no edge between \( v_i, v_j \).

Definition 2.6: Let \( G = (V, E) \) be an IFG. If \( (d_\mu(v), d_\gamma(v)) = (k_1, k_2) \) for all \( v \in V \) that is if each vertex has same membership degree \( k_1 \) and same nonmembership degree \( k_2 \) then \( G \) is said to be a regular intuitionistic fuzzy graph.

Definition 2.7: Let \( G = (V, E) \) be an IFG. Then the total degree of a vertex \( u \in V \) is defined by,
\[
\begin{align*}
td(\mu(u)) & = (d_\mu(u) + \sum_{v \in V} \mu_1(v), \sum_{v \in V} \gamma_1(v)) \\
\end{align*}
\]
If each vertex of \( G \) has same membership total degree \( k_1 \) and same nonmembership total degree \( k_2 \), then \( G \) is said to be a total regular IFG.

Definition 2.8: Union on Intuitionistic Fuzzy Graph: Let \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \) be two intuitionistic fuzzy graphs with \( V_1 \cap V_2 \neq \emptyset \) and \( G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2) \). Then the union of IFGs \( G_1 \) and \( G_2 \) is an IFG with the condition that \( \gamma_2 \leq \min(\gamma_1) \) and \( \gamma_2 \leq \min(\gamma_1) \) is defined by,
\[
(\mu_1 \cup \mu_2)(v) = \begin{cases} 
\mu_1(v), & \text{if } v \in V_1 \\
\mu_2(v), & \text{if } v \in V_2 \\
\mu_1(v) \vee \mu_2(v), & \text{if } v \in V_1 \cap V_2
\end{cases}
\]
\[
(\gamma_1 \cup \gamma_2)(v) = \begin{cases} 
\gamma_1(v), & \text{if } v \in V_1 \\
\gamma_2(v), & \text{if } v \in V_2 \\
\gamma_1(v) \wedge \gamma_2(v), & \text{if } v \in V_1 \cap V_2
\end{cases}
\]

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\[
\begin{align*}
\mu_{2i} \cup \mu_{2j} (v, v_i) &= \mu_{2i} (v, v_i) \cup \mu_{2j} (v, v_i) \\
&= \max \{ \mu_{2i} (v, v_i), \mu_{2j} (v, v_i) \} \\
&= \mu_{2i} (v, v_i) \lor \mu_{2j} (v, v_i)
\end{align*}
\]

Proof:

Let \( G_1 \) and \( G_2 \) be two intuitionistic fuzzy graphs such that \( V_1 \cap V_2 = \emptyset \). Then \( G_1 \cup G_2 \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph if and only if \( G_1 \) and \( G_2 \) are \((k_1, k_2)\) total regular intuitionistic fuzzy graphs.

Theorem 3.1:

Let \( G_1 \) and \( G_2 \) be two intuitionistic fuzzy graphs such that \( V_1 \cap V_2 = \emptyset \). Then \( G_1 \cup G_2 \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph if and only if \( G_1 \) and \( G_2 \) are \((k_1, k_2)\) total regular intuitionistic fuzzy graphs.

Proof:

Since \( V_1 \cap V_2 = \emptyset \),

\[
td_{G_1 \cup G_2} (u) = \begin{cases} td_{G_1} (u) , & \text{if } u \in V_1 \\ td_{G_2} (u) , & \text{if } u \in V_2 \end{cases}
\]

Hence \( G_1 \cup G_2 \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph if and only if \( G_1 \) and \( G_2 \) are \((k_1, k_2)\) total regular intuitionistic fuzzy graphs.

Theorem 3.2:

Let \( G_1 \) be an intuitionistic fuzzy subgraph of \( G_2 \). Then \( G_1 \cup G_2 \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph if and only if \( G_2 \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph.

Proof:

Let \( G_1 = (\mu_1, \gamma_1) \) be an intuitionistic fuzzy subgraph of \( G_2 = (\mu_2, \gamma_2) \). Then \( \mu_1 \leq \mu_2 \) and \( \gamma_1 \leq \gamma_2 \).

Therefore \( \mu_1 \cup \mu_1' = \mu_1 \) and \( \gamma_1 \cup \gamma_1' = \gamma_1 \).

Hence \( G_1 \cup G_2 = (\mu_1 \cup \mu_2, \gamma_1 \cup \gamma_2) \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph.

Theorem 3.3:

Let \( G_1 \) and \( G_2 \) be two intuitionistic fuzzy graphs such that \( V_1 \cap V_2 = \emptyset \). Then \( G_1 \cup G_2 \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph if and only if \( G_2 \) is a \((k_1, k_2)\) total regular intuitionistic fuzzy graph.
We know that, from theorem 3.

Let \( G_1 \) and \( G_2 \) be two intuitionistic fuzzy graphs such that \( V_1 \cap V_2 = \emptyset \). If \( G_1 \) and \( G_2 \) are total regular intuitionistic fuzzy graphs then \( G_1 \cup G_2 \) is not total regular intuitionistic fuzzy graph.

Proof:
Since \( V_1 \cap V_2 = \emptyset \).
\[
\text{td}_{G_1 \cup G_2}(u) = \begin{cases} 
\text{td}_{G_1}(u), & \text{if } u \in V_1 - V_2 \\
\text{td}_{G_2}(u), & \text{if } u \in V_2 - V_1 
\end{cases}
\]
\[
\text{td}_{G_1 \cup G_2}(u) = (k_1, k_2), \text{ if } u \in V_1 - V_2 \\
\text{td}_{G_1 \cup G_2}(u) = (k_3, k_4), \text{ if } u \in V_2 - V_1
\]
That is, \( \text{td}_{G_1}(u) = (k_1, k_2), \forall u \in V_1 \) and \( \text{td}_{G_2}(u) = (k_3, k_4), \forall u \in V_2 \).

Hence \( G_1 \cup G_2 \) is not total regular intuitionistic fuzzy graph.

Theorem 3.5:
Let \( G_1 \) and \( G_2 \) be two intuitionistic fuzzy graphs such that \( V_1 \cap V_2 \neq \emptyset \). If \( G_1 \) and \( G_2 \) are total regular intuitionistic fuzzy graphs then \( G_1 \cup G_2 \) is not total regular intuitionistic fuzzy graph.

Proof:
Since \( V_1 \cap V_2 \neq \emptyset \).
Given \( G_1 \) and \( G_2 \) are total regular intuitionistic fuzzy graphs,

That is, \( \text{td}_{G_1}(u) = (k_1, k_2), \forall u \in V_1 \) and \( \text{td}_{G_2}(u) = (k_3, k_4), \forall u \in V_2 \).

We know that,

\[
\text{td}_{G_1 \cup G_2}(u) = \text{td}_{G_1}(u) \cup \text{td}_{G_2}(u)
\]

Similarly \( G_2 \) is strong intuitionistic fuzzy graph, hence

(i) \( \mu_2(u, u') = \min \{ \mu_1(u, u'), \mu_1(u', u) \} \)

(ii) \( \gamma_2(u, u') = \max \{ \gamma_1(u, u'), \gamma_1(u', u) \} \), for every \( u, u' \in E \)

\( V_1 \cap V_2 = \emptyset \) implies there is no edge in common.

So \( G_1 \cup G_2(e) = G_1(e) \) and \( G_2(e) \), where \( e \) be edges.

Hence \( G_1 \cup G_2(u, u') = G_1(u, u') = (\mu_2, \gamma_2)(u, u') \) and \( G_2(u, u') = (\mu_2, \gamma_2)(u, u') \).

Therefore \( G_1 \cup G_2 \) is strong intuitionistic fuzzy graph.

Hence if \( G_1 \) and \( G_2 \) are \( (k_1, k_2) \) total strong regular intuitionistic fuzzy graph, then \( G_1 \cup G_2 \) is \( (k_1, k_2) \) total strong regular intuitionistic fuzzy graph such that \( V_1 \cap V_2 = \emptyset \).

Case 2: \( V_1 \cap V_2 \neq \emptyset \).

As from theorem 3.5, \( G_1 \cup G_2 \) is not \( (k_1, k_2) \) total regular intuitionistic fuzzy graph. We know that union of two strong intuitionistic fuzzy graphs need not be strong.
Hence we conclude that, for the case $V_1 \cap V_2 \neq \emptyset$, if $G_1$ and $G_2$ are $(k_1, k_2)$ total strong regular intuitionistic fuzzy graph, then $G_1 \cup G_2$ is need not be total strong regular intuitionistic fuzzy graph.

4. Properties of Join on Total Regular Intuitionistic Fuzzy Graphs

The join of two total regular intuitionistic fuzzy graphs need not be a total regular intuitionistic fuzzy graph.

**Theorem 4.1**

Let $G_1$ and $G_2$ be two $(k_1, k_2)$ total regular intuitionistic fuzzy graphs, with $V_1 \cap V_2 = \emptyset$. Then $G_1 + G_2$ is not total regular intuitionistic fuzzy graph.

**Proof:**

Given $G_1$ and $G_2$ are $(k_1, k_2)$ total regular intuitionistic fuzzy graphs.

$$td_{G_1}(u) = (k_1, k_2), \forall u \in V_1 \quad \text{and} \quad td_{G_2}(v) = (k_1, k_2), \forall v \in V_2$$

To prove $td_{G_1+G_2}(u) = (k_1, k_2), \forall u \in V_1 \cup V_2$

Since $V_1 \cap V_2 = \emptyset$,

$$td_{G_1+G_2}(u) = td_{G_1}(u) + \sum_{uv \in E} [\mu_1(u) \land \mu'_1(v), \gamma_1(u) \lor \gamma'_1(v)]$$

For every $u \in V_1$,

$$td_{G_1+G_2}(u_1) = td_{G_1}(u_1) + \sum_{uv \in E} [\mu_1(u_1) \land \mu'_1(v_2), \gamma_1(u_1) \lor \gamma'_1(v_2)]$$

Hence, $td_{G_1+G_2}(u_1) \neq td_{G_1+G_2}(u_2)$

So join of two $(k_1, k_2)$ total regular intuitionistic fuzzy graphs is not total regular intuitionistic fuzzy graph.

**Theorem 4.2:**

Let $G_1$ and $G_2$ be two intuitionistic fuzzy graphs, such that $V_1 \cap V_2 = \emptyset$. If $G_1 + G_2$ is $(k_1, k_2)$ total regular intuitionistic fuzzy graphs, then $G_1$ and $G_2$ are total regular intuitionistic fuzzy graphs.

**Proof:**

Since $V_1 \cap V_2 = \emptyset$.

For every $u \in V_1$,

$$td_{G_1+G_2}(u_1) = td_{G_1}(u_1) + \sum_{uv \in E} [\mu_1(u_1) \land \mu'_1(v_1), \gamma_1(u_1) \lor \gamma'_1(v_1)]$$

Therefore, $td_{G_1+G_2}(u_1) = (k_1, k_2)$.

In a similar way, we can prove for every vertices of $V_2$, $td_{G_2}(v) = (k_3, k_4)$.

Hence $G_2$ is total regular intuitionistic fuzzy graph.

5. Conclusion

In this paper, some new properties of union and join on total regular intuitionistic fuzzy graph are discussed. It will be more useful for doing further research in the field of regular IFG.

References


