International Journal of Scientific Engineering and Research (IJSER)
ISSN (Online): 2347-3878
Index Copernicus Value (2015): 62.86 | Impact Factor (2015): 3.791

On k-Super Mean Labeling

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Abstract: ***Let G be a (p, q) graph and*** $f: V(G) \to \{k, k+1, k+2, \dots, p+q+k-1\}$ ***be an injection. For each edge e = uv, let*** $f^*(e) = \frac{f(u)+f(v)}{2}$ ***if*** $f(u)+f(v)$ ***is even and*** $f^*(e) = \frac{f(u)+f(v)+1}{2}$ ***if*** $f(u)+f(v)$ ***is odd, then f is called k-Super mean labeling if*** $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$. ***A graph that admits k- Super mean labeling is called k-Super mean graph. In this paper, we investigate k-super mean labeling of*** $C_n + v_1v_3$, SL_n, $C_n \odot K_1$, A_n^m, $(P_m \, A \, K_{1,2}) \cup P_n$.

Keywords: $C_n + v_1v_3$, , SL_n, $C_n \odot K_1$, A_n^m, $(P_m \, A \, K_{1,2}) \cup P_n$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G. In this paper, we investigate k-super mean labeling of $C_n + v_1v_3$, SL_n, $C_n \odot K_1$, A_n^m, $(P_m \, A \, K_{1,2}) \cup P_n$.

Abbreviation: **SML** - super mean labeling.

Definition 1.1

Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called k-Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \dots, p+q+k-1$. A graph that admits k- Super mean labeling is called k-Super mean graph.

Definition 1.2

The graph $C_n+v_1v_3$ is obtained from the cycle $C_n : v_1v_2 \dots v_nv_1$ by adding an edge between the vertices v_1 and v_3.

Definition 1.3

A Slanting ladder $S(L_n)$ is a graph obtained from L_n by adding the edges u_iv_{i+1} ; $1 \le i \le n$ where $1 \le i \le n$ are the vertices of L_n such that $u_1u_2u_3\dots u_n$ and $v_1v_2v_3\dots v_n$ are two parts of length n in the graph L_n.

Definition 1.4

A **corona of a cycle** C_n is a cycle with the vertices $u_1, u_2, u_3, \dots, u_n$ and the edges $e_1, e_2, e_3, \dots, e_n$ and $v_1, v_2, v_3, \dots, v_n$ are the corresponding new vertices in $C_n \odot K_1$ and a_i be the edges joining $u_iv_i = 1$ to n.

Definition 1.5

The graph $P_m \, A \, K_{1,2}$ is obtained by attaching $K_{1,2}$ to each vertex of P_n.

2. Main Results

Theorem 2.1: The graph $C_n+v_1v_3$ is a k-Super mean graph for $n \ge 5$.

Proof:
Let $V(C_n+v_1v_3) = \{ v_i \ ; 1 \le i \le n \}$ and $E(C_n+v_1v_3) = \{ e' = v_1v_3 \} \cup \{e_i = v_i \, v_{i+1} \ ; 1 \le i \le n\}$ be the vertices and edges of $(C_n+v_1v_3)$ respectively.

Define $f : V(C_n+v_1v_3) \to \{1, 2, 3, \dots, 2n+1\}$ as follows:

Case 1: n is odd.

$$f(v_i) = \begin{cases} k+5; \ i = 1, \\ k; \ i = 2, \\ k+2; \ i = 3, \\ k+9; \ i = 4, \\ k+4i-6; \ 5 \le i \le \frac{n+3}{2}, \\ k+4(n-i)+7; \ \frac{n+3}{2}+1 \le i \le n-1, \\ k+8; \ i = n. \end{cases}$$

Case 2: n is even.

$$f(v_i) = \begin{cases} k+5; \ i = 1, \\ k; \ i = 2, \\ k+2; \ i = 3, \\ k+4i-7; \ 4 \le i \le \frac{n+2}{2}, \\ k+4(n-i)+8; \ \frac{n+4}{2} \le i \le n. \end{cases}$$

It can be verified that f is a super mean labeling of $C_n+v_1v_3$. Hence $C_n+v_1v_3$ is a super mean graph.

Example 2.1:

14-super mean labeling of $C_6+v_1v_3$ is given in figure 2.1:

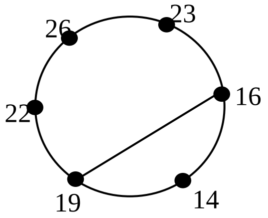

Fig 2.1: 14-SML of $C_6+v_1v_3$

Theorem 2.2: The slanting ladder SL_n is a k- super mean graph, for $n \geq 2$ and $n \neq 3t+1, t \geq 1$.

Proof:

Let $V(S(L_n)) = \{ u_i, v_i ; 1 \leq i \leq n\}$ and
$E(S(L_n)) = \{e_i = (v_i, v_{i+1}) ; 1 \leq i \leq n-1\} \cup$
$\{e_i' = (u_i, v_{i+1}) ; 1 \leq i \leq n-1\} \cup$
$\{e_i'' = (u_i, u_{i+1}); 1 \leq i \leq n-1 \}$
be the vertices and edges of $S(L_n)$ respectively.
Define $f : V(SL_n) \rightarrow \{1, 2, 3, \ldots, 5n-3\}$ as follows:
$f(u_{3i-2}) = 15i + k - 11 ;\ 1 \leq i \leq n-2,$
$f(u_{3i-4}) = 15i + k - 24 ;\ 2 \leq i \leq n-1,$
$f(u_{3i-6}) = 15i + k - 35 ;\ 3 \leq i \leq n$
$f(v_1) = k,$
$f(v_{3i-4}) = 15i + k - 28 ;\ 2 \leq i \leq n-1,$
$f(v_{3i-6}) = 15i + k - 34 ;\ 3 \leq i \leq n-1,$
$f(v_{3i-8}) = 15i + k - 46 ;\ 4 \leq i \leq n-2.$
Now, the induced edge labels are as follows:
$f^*(v_1v_2) = k+1,$
$f^*(v_{3i-4}v_{3i-3}) = 15i + k - 23 ;\ 2 \leq i \leq n-1,$
$f^*(v_{3i-6}v_{3i-5}) = 15i + k - 32 ;\ 3 \leq i \leq n-1,$
$f^*(v_{3i-8}v_{3i-7}) = 15i + k - 49 ;\ 4 \leq i \leq n-1,$
$f^*(u_{3i-2}u_{3i-1}) = 15i + k - 10 ; 1 \leq i \leq n-2,$
$f^*(u_{3i-4}u_{3i-3}) = 15i + k - 22 ; 2 \leq i \leq n-1,$
$f^*(u_{3i-6}u_{3i-5}) = 15i + k - 30 ; 3 \leq i \leq n-3,$
$f^*(u_1v_2) = k+3,$
$f^*(u_{3i-4}v_{3i-3}) = 15i + k - 21 ; 2 \leq i \leq n-1,$
$f^*(u_{3i-6}v_{3i-5}) = 15i + k - 33 ;\ 3 \leq i \leq n-3,$
$f^*(u_{3i-8}v_{3i-7}) = 15i + k - 42 ;\ 4 \leq i \leq n-2.$
Here p = 2n and q = 3(n - 1).
Clearly, $f(V) \cup \{f^*(e) : e \in E(S(L_n))\} =$
$\{k, k+1, \ldots, 5n+k-4\}.$
So, f is a k - super mean labeling.
Hence $S(L_n)$ is a k - super mean graph.

Example 2.2:

20- super mean labeling of SL_5 is given in figure 2.2:

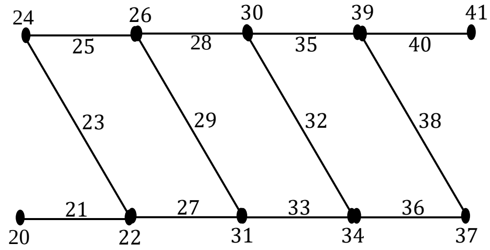

Figure 2.2: 220-SML of SL_5

Theorem 2.3:

Corona of a cycle C_n is a k-super mean graph for $n \geq 3$.

Proof:
Let $V(C_n \odot K_1) = \{ u_i, v_i ; 1 \leq i \leq n\}$ and
$E(C_n \odot K_1) = \{e_i = (u_i, u_{i+1}) ; 1 \leq i \leq n\} \cup$
$\{a_i = (u_i, v_i) ; 1 \leq i \leq n\}$
be the vertices and edges of $C_n \odot K_1$ respectively.

Define $f : V(C_n \odot K_1) \rightarrow \{1, 2, \ldots, 4n\}$ as follows:
Case 1: $\boldsymbol{n}$ ***is odd.*** $\boldsymbol{n = 2m+1, m = 1, 2, 3, \ldots}$
$f(u_1) = k+2,$
$f(u_i) = \begin{cases} 8(i-2) + k + 4 ;\ 2 \leq i \leq m+1, \\ 8(2m+1-i) + k + 11 ;\ m+2 \leq i \leq 2m+1, \end{cases}$
$f(v_1) = k,$
$f(v_i) = \begin{cases} 8(i-2) + k + 6 ; 2 \leq i \leq m+1, \\ 8(2m+1-i) + k + 9 ; m+2 \leq i \leq 2m+1, \end{cases}$

Now, the induced edge labels are as follows:
$f^*(e_1) = k+3,$
$f^*(e_i) = \begin{cases} 8(i-2) + k + 8 ; 2 \leq i \leq m+1, \\ 8(2m+1-i) + k + 7 ;\ m+2 \leq i \leq 2m+1, \end{cases}$
$f^*(a_1) = k+1,$
$f^*(a_i) = \begin{cases} 8(i-2) + k + 5 ;\ 2 \leq i \leq m+1, \\ 8(2m+1-i) + k + 10 ;\ m+2 \leq i \leq 2m+1. \end{cases}$
Case 2 : $\boldsymbol{n}$ ***is even.*** $\boldsymbol{n = 2m, m = 2, 3, \ldots}$
$f(u_1) = k+2,$
$f(u_i) = 8(i-2) + k + 4 ;\ 2 \leq i \leq m,$
$f(u_{m+1}) = 8m + k - 3,$
$f(u_i) = 8(2m-i) + k + 11 ;\ m+2 \leq i \leq 2m,$
$f(v_1) = k,$
$f(v_i) = 8(i-2) + k + 6 ; 2 \leq i \leq m,$
$f(v_{m+1}) = 8m + k - 1,$
$f(v_{m+2}) = 8m + k - 8,$
$f(v_i) = 8(2m-i) + k + 9 ;\ m+3 \leq i \leq 2m.$
Now, the induced edge labels are as follows:
$f^*(e_1) = k+3,$
$f^*(e_i) = 8(i-2) + k + 8 ;\ 2 \leq i \leq m-1,$
$f^*(e_m) = 8m + k - 7,$
$f^*(e_{m+1}) = 8m + k - 4,$
$f^*(e_i) = 8(2m-i) + k + 7;\ m+2 \leq i \leq 2m,$
$f^*(a_1) = k+1,$
$f^*(a_i) = 8(i-2) + k + 5 ;\ 2 \leq i \leq m,$
$f^*(a_{m+1}) = 8m + k - 2,$
$f^*(a_i) = 8(2m-i) + k + 10 ;\ m+2 \leq i \leq 2m.$
Here p = 2n and q = 2n
Clearly, $f(V) \cup \{f^*(e) : e \in E(C_n \odot K_1)\} =$
$\{k, k+1, \ldots, 4n+k-1\}.$
So, f is a k - super mean labeling.
Hence $C_n \odot K_1$ is a k - super mean graph.

Example 2.3: 30- mean labeling of $C_8 \odot K_1$ is shown in figure 2.3:

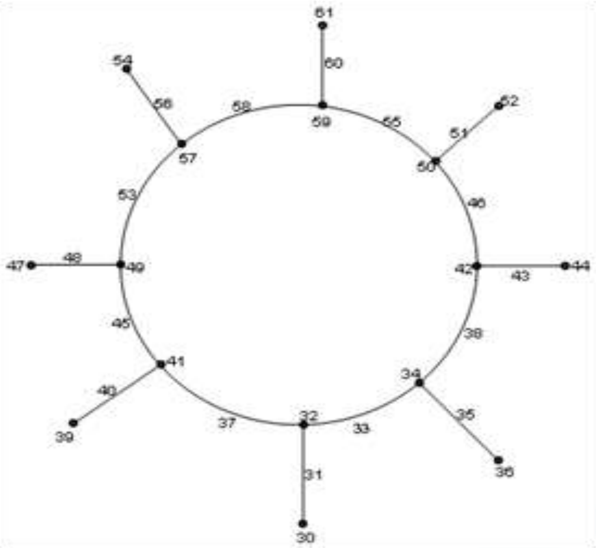

Figure 2.3: 30 - SML of $C_8 \odot K_1$

Theorem 2.4:

The generalized Antiprism A_n^m is a k-super mean graph for all m ≥ 2, n is even except for n = 4.

Proof:

Let V(A_n^m) = { v_i^j ; $1 \le i \le n, 1 \le i \le m$ } and
E(A_n^m) ={$e_i^j = (v_i^j v_{i+1}^j, v_n^j v_1^j)$;$1 \le i \le n-1, 1 \le j \le m$} ∪
{$a_i^j = (v_i^j v_i^{j+1}$; $1 \le i \le n, 1 \le j \le m-1$} ∪
{$b_i^j = (v_i^j v_{i-1}^{j+1}, v_1^j v_n^{j+1})$; $2 \le i \le n, 1 \le j \le m-1$}
be the vertices and edges of A_n^m respectively.
Define f : V(A_n^m) → { $1, 2, 3, \ldots . 4mn - 2n$} as follows:

f(v_1^j) = $4(j-1)n + k$; $1 \le j \le m$,
f(v_2^j) = $4(j-1)n + k + 2$; $1 \le j \le m$,
f(v_3^j) = $4(j-1)n + k + 6$; $1 \le j \le m$,
f(v_4^j) = $4(j-1)n + k + 11$; $1 \le j \le m$,
f(v_i^j) = $4(j-1)n + 4i + 2n + k - 6$; $5 \le i \le \frac{n+2}{2}$, $1 \le j \le m$,
f($v_{\frac{n+2+2i}{2}}^j$) = $4(j-1)n + 2n - 4i + k$; $1 \le i \le \frac{n-6}{2}$, $1 \le j \le m$,
f(v_{n-1}^j) = $4(j-1)n + k + 8$; $1 \le j \le m$,
f(v_n^j) = $4(j-1)n + k + 5$; $1 \le j \le m$.

It can be verified that f is a super mean labeling of A_n^m . Hence A_n^m is a super mean graph.

Example 2.4:
74 – super mean labeling of A_6^2 is shown in figure 2.4:

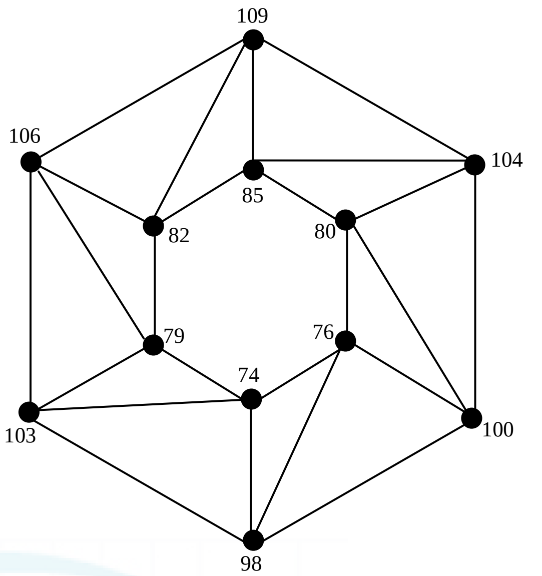

Figure 2.4: 74 - SML of A_6^2

Theorem 2.5:

The generalized Antiprism A_n^m is a k-super mean graph for all m ≥ 2,
n is odd.

Proof:

Let V(A_n^m) = { v_i^j ; $1 \le i \le n$, $1 \le i \le m$} and
E(A_n^m) = {$e_i^j = (v_i^j v_{i+1}^j, v_n^j v_1^j)$;$1 \le i \le n-1, 1 \le j \le m$} ∪
{ $a_i^j = (v_i^j v_i^{j+1}$; $1 \le i \le n, 1 \le j \le m-1$} ∪
{$b_i^j = (v_i^j v_{i-1}^{j+1}, v_1^j v_n^{j+1})$; $2 \le i \le n, 1 \le j \le m-1$}
be the vertices and edges of A_n^m respectively.
Define f : V(A_n^m) → { $1, 2, 3, \ldots . 4mn - 2n$} as follows:

$f(v_1^j) = 4(j-1)n + 2i + k - 2$; $1 \le i \le \frac{n+1}{2}, 1 \le j \le m$,
$f\,(v_{\frac{n+3}{2}}^j) = 4(j-1)n + n + k + 2$; $1 \le j \le m$,

$f\,(v_{\frac{n+3+2i}{2}}^j) = 4(j-1)n + n + k + 2i + 2$;
$$1 \le i \le \frac{n-3}{2}, \ 1 \le j \le m.$$

Now, the induced edge labels are as follows:

$f^*\left(e_i^j\right) = 4(j-1)n + 2i + k - 1$;
$$1 \le i \le \frac{n-1}{2}, 1 \le j \le m,$$
$f^*\,(e_{\frac{n-1+2i}{2}}^j) = 4(j-1)n + n + k + 2i - 1$;
$$1 \le i \le \frac{n-1}{2}, 1 \le j \le m,$$
$f^*\left(e_n^j\right) = 4(j-1)n + n + k$; $1 \le j \le m$,
$f^*\left(a_i^j\right) = 4(j-1)n + 2n + k + 2i - 2$;
$$1 \le i \le \frac{n+1}{2}, 1 \le j \le m,$$
$f^*\,(a_{\frac{n+1+2i}{2}}^j) = 4(j-1)n + 3n + k + 2i$;
$$1 \le i \le \frac{n+1}{2}, 1 \le j \le m,$$
$f^*\left(b_i^j\right) = 4(j-1)n + 2n + 2i + k - 1$;
$$1 \le i \le \frac{n-1}{2}, 1 \le j \le m,$$

$f^*\left(b^j_{\frac{n-1+2i}{2}}\right) = 4(j-1)n + 3n + k + 2i - 1\ ;$

$$1 \le i \le \frac{n-1}{2}, 1 \le j \le m,$$

$f^*\left(b^j_n\right) = 4(j-1)n + 3n + k\ ;\ \ 1 \le j \le m.$

Clearly, f(V) ∪ { f *(e): e ∈ E(A^m_n) =

$$\{k, k+1, \dots, 4mn - 2n + k - 1\}.$$

So, f is a k - super mean labeling.

Hence A^m_n is a k - super mean graph.

Example 2.5:

100 – super mean labeling of A^3_5 is shown in figure 2.5:

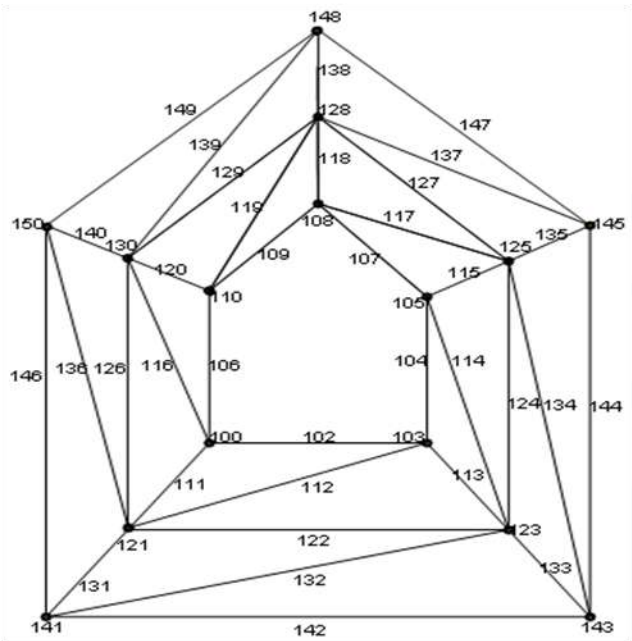

Figure 2.4: 74 - SML of A^2_6

Theorem 2.6

The graph $(P_m\ A\ K_{1,2}) \cup P_n$ is a k- super mean graph for every m, and n ≥ 2.

Proof:

Let $V\left((P_m\ A\ K_{1,2}) \cup P_n\right) = \{u_i\ ; 1 \le i \le m\} \cup$

$$\{z_i\ ; 1 \le i \le n\} \cup$$
$$\{v_i, w_i\ ; 1 \le i \le m\}$$

$E\left((P_m\ A\ K_{1,2}) \cup P_n\right) = \{e_i = (u_i, u_{i+1})\ ; 1 \le i \le m-1\} \cup$

$$\{a_i = (u_i, v_i)\ ; 1 \le i \le m\} \cup$$
$$\{b_i = (u_i, w_i)\ ; 1 \le i \le m\} \cup$$
$$\{c_i = (z_i, z_{i+1})\ ; 1 \le i \le n-1\}$$

be the vertices and edges of $(P_m\ A\ K_{1,2}) \cup P_n$ respectively.

Define f : $V\left((P_m\ A\ K_{1,2}) \cup P_n\right) \to \{1, 2, \dots, 6m + 2n - 2\}$ as follows:

$f(u_i) = 6i + k - 4\ ; 1 \le i \le m,$
$f(v_i) = 6i + k - 6\ ; 1 \le i \le m,$
$f(w_i) = 6i + k - 2\ ; 1 \le i \le m,$
$f(z_i) = 6m + 2i + k - 3\ ; 1 \le i \le n.$

It can be verified that f is a k-super mean labeling. Hence $(P_m\ A\ K_{1,2}) \cup P_n$ is a k-super mean graph.

Example 2.6:

126 – super mean labeling of $(P_4\ A\ K_{1,2}) \cup P_5$ is shown in figure 2.6:

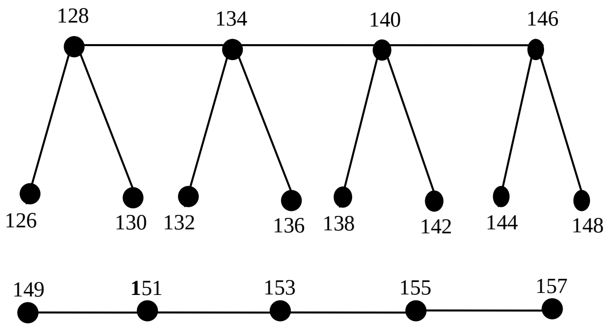

Figure 2.6: 126 – SML of $(P_4\ A\ K_{1,2}) \cup P_5$

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