(σ, τ) –Lie Ideals with Some New Circulars

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Abstract: Let R be a ring optional, U be an additive subgroup of R and $\sigma,\tau:\mathrm{R}\to\mathrm{R}$ be twomapping $[x,y]_{\sigma,\tau}=x\sigma(x)-\tau(\mathrm{y})x$ in most of our study we will consider R is prime ring with characteristic not equal 2, and (σ, τ) functions equivalent automorphsem. The basic aim of this study Is the study of circulating Lieidealto $(\sigma.\tau)$ - Lie ideal mainstreaming some results at $(\sigma.\tau)$ - Lie ideal using theorems non-generalized for the purpose of circulating provable and the use of these proofs generalized to help prove theorems other non-generalized.We can prove a lot of non-generalized theorems on the subject Lie ideal by these theorems that have been circulated in this research. We have used in this research

$$\{R\ni x\forall R;c(\mathrm{y})+\tau(x)\mathrm{c}\}=C_{\sigma,\tau}$$

$$d(\mathrm{xy})=\mathrm{d(x)}\sigma(\mathrm{y})+\tau(\mathrm{x})\mathrm{d(y)},\forall x,y\in R$$

Results are as follows

1) **Theorem** (2.1).Let $d_1:R\to R$ be a $(\sigma.\tau)$ – derivation and $d_2:R\to R$ be an (α_j,α_j) - derivation and $d_3:R\to R$ be an (β,β)-derivation. Such that $d_3\beta=\beta d_3, d_2\beta=\beta d_2, d_1\beta=\beta d_1$, where β is automorphisn of R. If $U\neq(0)$ is an ideal of R such that $d_3(\mathrm{U})\subset\mathrm{U}$ and $d_1d_2d_3(U)=0$, Then either $d_1=0$ or $d_2=0$ or $d_3=0$.

2) **Theorem** (2.2)(In general). Let $d_1:R\to R$ be $(\sigma.\tau)$ – derivation and $d_i:R\to R$ be an (α_j,α_j) − derivation such that $d_1\alpha_1=\alpha_1 d_1, d_i\alpha_j=\alpha_j d_i, i=2.......n,\ j=n-1$, Where is α_j an outomorphism of R if $U\neq(0)$ s an ideal of R such that $d_1(\mathrm{U})\subset\mathrm{U}$ A $d_1d_2.......d_n(\mathrm{U})=(0),\mathrm{n}\in\mathrm{N}$,

Then either $d_1=0$ or $d_2=0$ ......or $d_n=0$

3) **Theorem** (2.3).Let U be nonzero ideal of R and $a,b,c\in U$,
if $[c,[\mathrm{a},[\mathrm{b},\mathrm{x}]]]_{\sigma,\tau}=0,\forall\mathrm{x}\in\mathrm{U}$ for all x∈ U, then either
1. $c,a\in C_{\sigma,\tau}$ or $b\in Z(\mathrm{R})$.
2. $c\in C_{\sigma,\tau}$ or $a,b\in Z(\mathrm{R})$.

4) **Theorem** (2.4) (In general)**.** Let U be nonzero ideal of R and,
if $a_1,a_2,.........,a_n\in U$, $\forall x\in U$ then either
1. $a_n,a_{n-1},..........,a_2\in C_{\sigma,\tau}$ or $a_1\in Z(\mathrm{R})$
2. $a_n\in C_{\sigma,\tau}$ or $a_{n-1},..........,a_1\in Z(\mathrm{R})$

Keywords: Lie ideals, ring, semi-ring, characteristic ring, derivation, homomorphic.

1.Introduction

This work is a continuation of a series results that have been obtained by some researchers(K. A. Jassim) and Dr. (A.A. Hameed) see [25-26].

Let R be a ring, U be an additive subgroup of R, U is called a lie ideal of R if $[U,R]\subset U$.

We generalized this definition that : If we have $\sigma,\tau:\mathrm{R}\to\mathrm{R}$ be two mapping then

1) U is called a $(\sigma.\tau)$ right Lie ideal of R if $[U,R]_{\sigma,\tau}\subset U$.

2) U is called a $(\sigma.\tau)$ lift Lie ideal of R if $[R,U]_{\sigma,\tau}\subset U$.

3) U is called a $(\sigma.\tau)$ - Lie ideal of R if U is both $(\sigma.\tau)$ - right Lie ideal and $(\sigma.\tau)$ - lift Lie ideal of R.

This chapter consists of two sections, in section one we give the basic definitions and study the relations among them, we illustrate some of these results by some examples.In most of our study we will consider R is prime ring with characteristic not equal 2, and σ, τ functions equivalent automorphsem.In section two, we gave important results when U is a (σ,τ) right Lie ideal of R, such that if $[U,U]_{\sigma,\tau} \subset C_{\sigma,\tau}$ then either $U \subset Z(\mathrm{R})$ or $U \subset C_{\sigma,\tau}$. Also if U is a subring of Rwith new theorems which represents the generalization to lemma (3.1) when n=3 and when n in general, as well as we concluded a new theorem (3.3) we got some relations between the center of the R, denoted by Z(R) and between the centralizer of x in R, denoted by $C_R(x)$.

The aim of the study

1) Study is a generalization of the concept is perfect for Lie ideal to (σ,τ) - Lie ideal and a generalization some of the results of the ideal for ideal to (σ, τ)- Lie ideal as well as provable.

2) To study the relationship between the derivative and the (σ,τ) - Lie ideal and give some important results, we have used in this study

$\{R \ni x \forall R; c\sigma(\mathrm{y})+\tau(\mathrm{x})\mathrm{c}; \mathrm{R} \in \mathrm{c}\} = C_{\sigma,\tau}$ as well as we used the following formula to complete derivation process

$d(xy)=d(x)\sigma(y)+\tau(x)d(y), \forall x,y \in R$.

2. Basic Concepts

Definition (2.1).A ring R is called a prime ring if $aRb=(0), \mathrm{a},\mathrm{b} \in \mathrm{R}$, implies that $a=0$ or $b=0$.

Definition(2.2).A ring R is called a semi prime ring if $aRb=(0), \mathrm{a},\mathrm{b} \in \mathrm{R}$, implies that $a=0$.

Definition (3.3).Let R be an arbitrary ring. If there exists a positive integer n such that $na=0$ for all a∊R, then the smallest positive integer with this property is called the characteristic of the ring, by symbols we write $chR=n$ .If no such positive integer exists (that is, n=0 is the only integer for which $na=0$, for all a in R), then R is said to be of characteristic zero.

Remark (2.1).We can show easily that if R is a prime ring with characteristic not equal n is equivalent to n-torsion free.

Definition (2.4).Let R be a ring. Define a Lie product $[,]$ on R as follows

$[x,y]=xy-yx \forall x,y \in R.$.

Remark (2.2).Let R be a ring, then $\forall x,y \in R$ we have: -

$[x,\mathrm{y}\,z]=y[x,\mathrm{z}]+[x,y]z$

$[x+y,z]=[\mathrm{x},z]+[\mathrm{y},z]$

$[xy,z]=x[y,z]+[x,z]y$

Definition (2. 5).Let A be a Lie subring of a ring R. An additive subgroup $U \subset A$ is said to be a Lie ideal of A, if whenever $u \in U$ and $a \in A$, then $[u,a] \in U$.

Definition (2. 6).Let R be a ring Define the product $[x,y]_{\sigma,\tau} = x\sigma(\mathrm{y})-\tau(\mathrm{y})\mathrm{x}$, $\forall x,y \in R$.

Remark(2.3). Let R be a ring and let a, $\sigma,\tau : R \to R$ be two mappings.The $\forall x,y,z \in R$, we have:

1) $[x+y,z]_{\sigma,\tau} = [x,z]_{\sigma,\tau} + [y,z]_{\sigma,\tau}$.

2) $[xy,z]_{\sigma,\tau} = x[y,z]_{\sigma,\tau} + [x,(\mathrm{z})]_{\sigma,\tau}\, y = x[y,\sigma(\mathrm{z})]+[x,z]_{\sigma,\tau}\, y$.

Remark (2. 4).Let R be a ring and let, $\sigma,\tau : R \to R$ be two homomorphism's. Then $\forall x,y \in R$, we have:

$[x,yz]_{\sigma,\tau} = \tau(\mathrm{y})[x,z]_{\sigma,\tau} + [x,y]_{\sigma,\tau}\, \sigma(\mathrm{z})$

Definition (2. 7).Let R be a ring, U be an additive subgroup of R and, $\sigma,\tau : R \to R$ be twomappings.Then

1) U is called a (σ,τ)- right Lie ideal of R if $[U,R]_{\sigma,\tau} \subset U$.

2) U is called a (σ,τ)- left Lie ideal of R if $[R,U]_{\sigma,\tau} \subset U$.

3) U is called a (σ,τ)- Lie ideal of R if U is both (σ,τ)-right Lie ideal and (σ,τ) left Lie ideal of R.

Definition (2. 8).Let R be a ring, the center of R, denoted by $Z(R)$, is the-set $\{a \in R; ar = ra \forall r \in R\}$.

Definition (2. 9).Let Xbe a nonempty subset of R, the centralizer of X in. R, denoted, by $C_R(X)$, is the set. $\{a \in R; [x,a] = 0 \forall r \in R\}$.

Definition (2.10).Let R be a ring and let, $\sigma,\tau : R \to R$ be two mappings. (σ,τ)- centralizer of R, denoted by $C_{\sigma,\tau}$ is the set $\{c \in R; c\sigma(\mathrm{x}) = \tau(\mathrm{x})\mathrm{c} \,\forall\, \mathrm{x} \in \mathrm{R}\}$.

Definition (2.11).Let R be a ring. An additive mapping d:R →R is called a derivation on R if $d(xy) = d(x)y + xd(y) \forall \mathrm{x,y} \in R$. We say that d is an inner derivation if there exists an element a∊ R. such that $d(x) = [a,x] \forall x \in R$.

Definition (2.12).Let R be a ring. An additive mapping $d : R \to R$ is called a (σ,τ)-derivation where, $\sigma,\tau : R \to R$ be two mappings, if $d(xy) = d(x)\sigma(y) + \tau(x)d(y) \forall \mathrm{x,y} \in \mathrm{R}$ It is clear that every derivation is a (σ,τ)-derivation.

Definition (2.13). Let $d : R \to R$ be an additive mapping then we say that d is a (σ,τ)-inner derivation if there exists an element a∊R such that $d(x) = [a,x]_{\sigma,\tau}, \forall x \in R$

3.(σ,τ)-Right Lie ideals

The following lemmas help us to prove the main theorems

Lemma(3.1).Let dbe a (σ,τ)-derivation of R and $a \in R$, U be a nonzero ideal of R. If $(\mathrm{U}) = (0)$ or $(d(\mathrm{U})\mathrm{a} = (0))$, then either $a = 0$ or $d = 0$.

Lemma (3.2).Let $d_1 = R \to R$ be a (σ,τ)-derivation and $d_2 = R \to R$ be an (α,α) derivation such that $d_2\alpha = \alpha d_2, d_1\alpha = \alpha d_1$, where α is an automorphism ofR. If $U \neq (0)$ is an ideal of R such that $d_2(\mathrm{U}) \subset \mathrm{U}$ and $d_1d_2(\mathrm{U}) = (0)$, then either

$d_1 = 0$ or $d_2 = 0$.

Proof.For any $u,v \in U, uv \in U$. By hypothesis $d_1d_2(\mathrm{U}) = (0)$, So,

$$0 = d_1d_2(\mathrm{uv}) = d_1(d_2(u)\alpha(\mathrm{v}) + \alpha(u)d_2(\mathrm{v})) = d_1(d_2(u)\,\alpha(\mathrm{v})) + d_1(\,\alpha(u)d_2(\mathrm{v})) =$$
$$= d_1(d_2(u))\sigma(\alpha(\mathrm{v})) + \tau(d_2(u))d_1(\,\alpha(\mathrm{v})) + d_1(\,\alpha(u))\,\sigma(\mathrm{d}_2(\mathrm{v})) + \tau(\,\alpha(u))\,d_1(\mathrm{d}_2(\mathrm{v})).$$

Since $d_1d_2(\mathrm{U}) = (0)$, also $-\ \tau(d_2(u))d_1(\alpha(\mathrm{v})) + d_1(\alpha(u)\sigma(d_2(\mathrm{v})) = 0$, that is,

$$d_1(\alpha(u)\sigma(d_2(\mathrm{v})) + \tau(d_2(u))d_1(\,\alpha(\mathrm{v})) = 0 \;,\; \forall \boldsymbol{u}, \boldsymbol{v} \in \boldsymbol{U} \,..........\mathbf{(1)}$$

Replacing u by $d_2(u)$ in (1). We get

$d_1(\alpha(d_2(u)))\sigma(d_2(\mathrm{v})) + \tau(d_2(d_2(u)))d_1(\alpha(\mathrm{v})) = 0$ Using $\alpha d_1 = d_1\alpha$ we have $\boldsymbol{\tau(d_2^2(u))d_1(\alpha(\mathrm{v})) = 0, \forall\, \mathrm{u,v} \in \mathrm{U}\,........(2)}$, also, we have $\tau(d_2^2(u)\alpha(d_1(\mathrm{v})) = 0$.

Since α is an automorphism, hence α^{-1} exists such that by Lemma (2.1)we get $\boldsymbol{d_2^2(u) = 0 \forall\, \mathrm{u} \in \mathrm{U}}$ or $d_1 = 0$. That is $d_2^2(U) = (0)$ or $d_1 = 0$.

Suppose $d_1 \neq 0$, then $d_2^2(U) = (0)$. For any u, v∊U, so uv ∊ U.Hence,

$$0 = d_2^2(uv) = d_2(d_2(uv)) = d_2(d_2(u)\alpha(\mathrm{v}) + \alpha(u)d_2(\mathrm{v})) = d_2(d_2(u)\,\alpha(\mathrm{v})) + d_2(\,\alpha(u)d_2(\mathrm{v})) =$$
$$= d_2(d_2(u))\alpha(\alpha(\mathrm{v})) + \alpha(d_2(u))d_2(\alpha(\mathrm{v})) + d_2(\,\alpha(u))\,\alpha(\,d_2(\mathrm{v})) + \alpha(\,\alpha(u))\,d_2(\,d_2(\mathrm{v}))\;.$$

But $d_2^2(U) = 0$, we get $\boldsymbol{\alpha(d_2(u))d_2(\alpha(\mathrm{v})) + d_2(\alpha(u))\alpha(d_2(\mathrm{v})) = 0}$.

Also, $\alpha d_2 = d_2\alpha$ we have $d_2(\alpha(u))d_2(\alpha(v)) + d_2(\alpha(u))d_2(\alpha(v)) = 0$. So $2d_2(\alpha(u))d_2(\alpha(v)) = 0$. Since R is a prime ring with $ChR \neq 2$, then $d_2(\alpha(u))d_2(\alpha(v)) = 0$.So, $\alpha(d_2(u)d_2(v)) = 0$ and $d_2(u)d_2(v) = 0, \forall u, v \in U$. Therefore $d_2(U)d_2(U) = (0)$. By Lemma (2.1), we get either $d_2(U) = (0)$ or $d_2 = 0$.

If $d_2(U) = (0)$, then $d_2(ru) = 0, u \in U, r \in R$. This implies $0 = d_2(r)\alpha(ru) = \alpha(r)d_2(u) = d_2(r)\alpha(u)$. That is, $d_2(r)\alpha(u) = 0 \forall u \in U, r \in R$. Now $0 = d_2(r)\alpha(ru) = d_2(r)\alpha(r)\alpha(u) \forall u \in U, r \in R$.

Since R is a prime ring, then $d_2(r) = 0 \forall r \in R$. That is, $d_2 = 0$.

Corollary (3.1). Let U be a nonzero ideal of R and $a, b \in U$. If $\left[a,[b,x]\right]_{\sigma,\tau} = 0, \forall x \in U$, then either $a \in C_{\sigma,\tau}$ or $b \in Z(R)$.

Proof. The map $d_1 : R \to R$, defined by $d_1(x) = [a, x]$, is a (σ,τ)-derivation and the map $d_2 : R \to R$, defined by $d_2(x) = [b, x]$, is a derivation.

Moreover $d_2(U) = [b,U] \subset U$, that is $d_2(U) \subset U$ and $d_1d_2(U) = d_2(d_2(U)) = \left[a,[b,U]\right]_{\sigma,\tau} = (0)$ by assumption.Hence, in view of Lemma (3.2), we obtain $d_1 = 0$ or $d_2 = 0$.This implies that $a \in C_{\sigma,\tau}$ or $b \in Z(R)$.

Theorem (3:1). Let $d_1 : R \to R$ be a (α,τ) – derivation and $d_2 : R \to R$ be an (α,α)- derivation and $d_3 : R \to R$ be an (β,β)-derivation. Such that $d_3\beta = \beta d_3, d_2\beta = \beta d_2, d_1\beta = \beta d_1$ where β is automorphisn of R. If $U \neq (0)$ is an ideal of Rsuch that $d_3(U) \subset U$ and $d_1d_2d_3(U) = 0$, then either $d_1 = 0$ or $d_2 = 0$ or $d_3 = 0$.

Proof: Let $u, v \in U, uv \in U$. By hypothesis $d_2d_3(U) = 0$, so

$$0 = d_1d_2d_3(uv) = d_1d_2(d_3(u)\beta(v) + \beta(u)d_3(v))d_1(d_2(d_3(u)(\beta(v)) + d_2(\beta(u)d_3(v)) =$$

$$= d_1(d_2(d_3(u)\beta(v) + d_1(d_3(u)d_2(\beta(v)) + d_2(\beta(v))\alpha d_3(v) + \alpha\beta(u))d_2(d_3(v)) =$$

$$= d_1(d_2(d_3(u)\alpha\beta(v) + d_1(\alpha(d_3(u))d_2(\beta(v)) + d_1(d_2(\beta(u)\alpha d_3(v)) + d_1(\alpha(\beta(u)d_2(d_3(v)) =$$

$$d_1(d_2(d_3(u)\sigma(\alpha(\beta(v) + \tau(d_2(d_3(u)d_1(\alpha(\beta(v) + d_1(\alpha(d_3(u)\sigma d_2((v) + \tau d_3(u)d_1(d_2(\beta(v) +$$

$$+ d_1(\alpha(\beta(u)\sigma d_2(d_3(v)) + \tau(\alpha(\beta(u)d_1(d_2(d_3(v)) = 0 \text{ since } (d_1d_2d_3 = 0)$$

$$\tau(d_2(d_3(d_3(u)d_1(\alpha(\beta(v) + \alpha d_1(d_3(d_3(u))\sigma(d_2(\beta(v)) + \tau d_3(d_3(u) + d_1(d_2(\beta(v) + d_1(d_2(\beta(u)\sigma(\alpha(d_3(v) +$$

$$+ \tau d_3(d_3(u) + d_1(d_2(\beta(v) + d_1(d_2(\beta(u)\sigma(\alpha(d_3(v) \text{ So,}$$

$$\sigma(d_2(d_3(d_3(u)d_1(\alpha(\beta(u) + \alpha d_1(d_3(d_3(u))\sigma(d_2(\beta(v)d_3(d_3(u) = 0$$

$$\tau(d_2(d_3^2(u)d_1(\alpha(\beta(v)) + \alpha d_1(d_3^2(u)\sigma(d_2\beta(v))\sigma + \tau d_3^2(u)d_1(d_2(\beta)(v) = 0$$

Whereas $\tau d_3^2(u)d_1(d_2(\beta(v)) = 0$

$\tau d_3^2(u)d_1(\beta(d_2(v)) = 0,$ (using $d_2\beta = \beta d_2$)

$\tau d_3^2(u)\beta(d_1(d_2(v)) = 0,$ (using $d_1\beta = \beta d_1$)

sinceβ is outomophism, hence β^{-1} exists $\tau d_3^2(u)(d_1(d_2(v)) = 0$ By Lemma (2.1) we get $d_3^2(u) = 0 \forall u \in U$ or $d_1(d_2(v)) = 0 \forall v \in V$ This $d_3^2(U) \neq 0$ or $d_1(d_2(v)) = 0$

- Suppose $d_3^2(U) \neq 0$, then $d_1(d_2(v)) = 0$ by Lemma(2.2) we get $d_1 = 0$ or $d_2 = 0$.

- Suppose if $d_1(d_2(v)) \neq 0$, then $d_3^2(U) = 0$

For any u, v∈U, so $uv \in U$, Hence

$0 = d_3^2(U) = d_3^2(uv) = d_3(d_3(uv) = d_3(d_3(u)\beta(v) + \beta(u)d_3(v) =$

$= d_3(d_3(u)\beta(\beta(v)) + \beta(d_3(u)d_3(\beta(v)) + d_3(\beta(u))\beta(d_3(v)) + \beta(\beta(v))d_3(d_3(v))$

but $d_3^2(U) = (0)$ we get $\beta(d_3(u))d_3(\beta(v)) + d_3(\beta(u))\beta(d_3(v)) = 0.$ Also $\beta d_3 = d_3\beta$, we have $d_3(\beta(u))d_3(\beta(v)) + d_3(\beta(u))d_3(\beta(v)) = 0$, so,

$2d_3(\beta(u)d_3(\beta(v)) = 0$, since R is aprime with ch R≠2 then

$d_3(\beta(u)d_3(\beta(v)) = 0$, so, $\beta(d_3(u)d_3(v)) = 0$ and $(d_3(u)d_3(v)) = 0 \forall u, v \in U$

There fore $d_3(U)d_3(U) = 0$, by Lemma (3.1) we get lither $d_3(U) = (0)$ or $d_3 = 0$

If $d_3(U) = (0)$, then $d_3(ru) = 0, u \in U, r \in R$ this is implies

$0 = d_3(3)\beta(u) + \beta(r)d_3(u) = d_3(r)\beta(u)$ that is

$d_3(r)\beta(u) = 0, \forall u \in U, r \in R$ .Now $0 = d_3(v)(ru) = d_3(r)\beta(r)\beta(u) \forall u \in U, r \in R$ (since R is aprime ring) then $d_3(r) = 0 \forall r \in R$ that is $d_3 = 0$

Theorem (3.2) (In general). Let $d_1 : R \to R$ be (σ, τ) – derivation and $d_i : R \to R$ be an (α_j, α_j) – derivation such that $d_1\alpha_1 = \alpha_1 d_1, d_i\alpha_j = \alpha_j d_i, i = 2, ..., \mathrm{n}, \mathrm{j} = \mathrm{n}-1, \mathrm{n} \geq 2,$, Where α_j is an outomorphism of R if $U \neq (0)$ is an ideal of R such that $d_1(U) \subset \mathrm{U}$

and $d_1 d_2 ....d_n(U) = (0), n \in N$ Then either $d_1 = 0$ or $d_2 = 0$ ......or $d_n = 0$

Proof. Let $u, v \in U, uv \in U$, $d_1 d_2 ......d_n(U) = (0)$, so

$0 = d_1 d_2 ....d_n(uv) = d_1 d_2 ....d_{n-1}(d_n(u)\alpha_{n-1}(v) +$

$+\alpha_{n-1}(u)\alpha_n(v))d_1(d_2....(...d_{n-2}(d_{n-1}(u)\alpha_{n-1}(v)) + d_2......d_{n-2}(\alpha_{n-1}(u)d_n(v)) =$

$= d_1(d_2(.....(....d_{n-2}(d_{n-1}(u) + \alpha_{n-1}(u)) + d_1(d_{n-1}(u)d_2.....d_{n-2}(\alpha_{n-1}(u)) +$

$+d_2(\alpha_{n-1}(v)\alpha_{n-1}d_n(v) + \alpha_{n-1} + \alpha_{n-2}\alpha_{n-1}(u)d_2.....d_{n-1}(d_n(v)) =$

$= d_1(d_2....d_{n-1}(d_n(u)\alpha_{n-2}\alpha_{n-1}(v) + d_1(\alpha_{n-2}(d_n(u)d_2....d_{n-2}(\alpha_{n-1}((v)) +$

$+d_1(d_2.....d_{n-1}(\alpha_{n-1}(u))\alpha_{n-2}d_n(v) + d_1(\alpha_{n-1}(u))d_2....d_{n-1}(d_{n-1}(v)) =$

$= d_1(d_2....d_{n-1}(d_n(u)\sigma(\alpha_{n-2}(\alpha_{n-1}(v) + \tau(d_2....d_{n-1}(d_n(u))d_1(\alpha_{n-2}(\alpha_{n-1}(v)) +$

$+d_1(\alpha_{n-2}(d_{n-1}(u))\sigma(d_2....d_{n-1}(\alpha_{n-1}(v)) + \tau d_n(u)d_1(d_2.... + d_1 d_2....d_{n-1}(\alpha_{n-1}(u)\sigma(\alpha_{n-2}(d_n(v)) +$

$+\tau d_2....d_{n-1}(\alpha_{n-1}(u)d_1(\alpha_{n-2}(d_n(u)) + d_1(\alpha_{n-2}(\alpha_{n-1}(u))\sigma d_2....d_{n-2}(d_n(v) +$

$+\tau(\alpha_{n-2}(\alpha_{n-1}(u))d_1(d_2....d_{n-1}d_3(v)) = 0$ since $d_1 d_2 ....d_n = 0$

$\tau(d_2....d_{n-1}(d_n(d_n(u)d_1(\alpha_{n-2}(\alpha_{n-1}(v)) + \alpha_{n-2}d_1(d_n(d_n(u))\sigma(d_2....d_{n-1}(\alpha_{n-1}(v)) +$

$+\tau d_n(d_n(u)) + d_1 d_2....d_{n-1}(\alpha_{n-1}(v) + d_1(d_2....d_{n-1}(\alpha_{n-1}(u)\sigma(\alpha_{n-2}(d_n(v) +$

$+\tau d_2....d_{n-1}(\alpha_{n-1}(u))d_1(\alpha_{n-2}(d_{n-1}(v) + d_1(\alpha_{n-1}(\alpha_{n-1}(u)\sigma d_2....d_{n-1}(d_n(v)) = 0$ So,

$\sigma(d_2....d_{n-1}(d_n(d_n(u))d_1(\alpha_{n-2}(\alpha_{n-1}(u)) + \alpha_{n-2}d_1(d_n(d_n(u)\sigma(d_2....d_{n-1}(\alpha_{n-1}(u)d_n(d_n(u)) = 0$

$(d_2....d_{n-1}(d_n^2(u)d_1(\alpha_{n-2}(\alpha_{n-1}(u)) + \alpha_{n-2}d_1(d_n^2(u)\sigma(d_2....d_{n-1}\alpha_{n-1}(v))\sigma + \tau d_{n-1}^2(u)d_1(d_2....d_{n-1}(\alpha_{n-1}(v)) = 0$

where as

$\tau d_n^2(u)d_1(d_2....d_{n-1}\alpha_{n-1}(v)) = 0 \ (d_1\alpha_1 = \alpha_1 d_1, d_i\alpha_j = \alpha_j d_i, i = 2........n, \ j = n-1, n \geq 2)$

$\tau d_n^2(u)d_1(\alpha_{n-1}(d_2....d_{n-1}(v)) = 0 \ (d_1\alpha_1 = \alpha_1 d_1, d_i\alpha_j = \alpha_j d_i, i = 2........n, \ j = n-1, n \geq 2)$

$\tau d_n^2(u)\alpha_{n-1}(d_1(d_2....d_n(v)) = 0 \ (d_1\alpha_1 = \alpha_1 d_1, d_i\alpha_j = \alpha_j d_i, i = 2........n, \ j = n-1, n \geq 2)$

since α_{n-1}^{-1} is automorphismhence α_{n-1}^{-1} exist

$\tau d_n^2(u)(d_1(d_2....d_{n-1}(v))=0$, by Lemma (3.1) we get $d_n^2(\mathrm{u})=0$ for all u ∈U or $(d_1(d_2....d_{n-1}(v))=0$, that is $d_n^2(U)=0$ or $(d_1(d_2....d_{n-1}(v))=0(n\in N)$

1. Suppose $d_n^2(U)\neq 0$, $\Rightarrow$ then $(d_1(d_2....d_{n-1}(v))=0(n\in N)$

by Lemma (3.2) we get $d_1=0$ or $(d_2....d_{n-1}(v))=0$

- Suppose $d_1\neq 0$, $\Rightarrow$ then $(d_2....d_{n-1}(v))=0$, by Lemma (2.2)

$d_2=0$ or $(d_3....d_{n-1}(v))=0$

Thus, is the same way, we get

$d_1=0$ or $d_2=0$ … or $d_{n-1}=0$

2. Suppose if $(d_1(d_2....d_{n-1}(v))\neq 0$, then $d_n^2(U)=0$ for any u, v∈U, so u∈U, hence

$$0=d_n^2(U)=d_n^2(uv)=d_n(d_n(uv))=d_n(d_n(u)\alpha_{n-1}(v)+\alpha_{n-1}(u)(d_n(v))=$$
$$=d_n(d_n(u)\alpha_{n-1}(v)+d_n(\alpha_{n-1}(u)d_n(v)=d_n(d_n(u)\alpha_{n-1}(\alpha_{n-1}(v)+\alpha_{n-1}(d_n(u)d_n(\alpha_{n-1}(v))+$$
$$+d_n(\alpha_{n-1}(u)\alpha_{n-1}(\alpha_{n-1}(v)+(\alpha_{n-1}(d_n(u)+d_n(\alpha_{n-1}(v)+\alpha_{n-1}(\alpha_{n-1}(v))d_n(d_n(v)).$$

But $d_n^2(U)=0$ we get $\alpha_{n-1}(d_n(u)d_n(\alpha_{n-1}(v)+d_n(\alpha_{n-1}(d_n(v))=0$

Also $\alpha_{n-1}d_n=d_n\alpha_{n-1}$, we have $d_n(\alpha_{n-1}(u))d_n(\alpha_{n-1}(v))+d_n(\alpha_{n-1}(u))d_n(\alpha_{n-1}(v))=0$,

so $2d_n(\alpha_{n-1}(u))d_n(\alpha_{n-1}(v))=0$, since R is aprim with $chR\neq 2$ then

$d_n(\alpha_{n-1}(u)d_n(\alpha_{n-1}(v))=0$, so $\alpha_n(d_n(u)d_n(v))=0$ and $(d_n(u)d_n(v))=0\forall u,v\in U$

There fore $d_n(U)d_n(U)=(0)$ by Lemma(3.1), we get either $d_n(U)=(0)$ or $d_n=0$

if $d_n(U)=(0)$, then $d_n(\mathrm{ru})=0, u\in U, r\in R$

This is implies $0=d_n(r)\alpha_{n-1}(u)+\alpha_{n-1}(r)d_n(u)=d_n(r)\alpha_{n-1}(u)$ that is $d_n(r)\alpha_{n-1}(u)=0\forall u\in U, r\in R$, now.

$0=d_n(v)\alpha_{n-1}(ru)=d_n(r)\alpha_{n-1}(r)\alpha_{n-1}(u)\forall u\in U, r\in R$ (Since R is prime ring), then $d_n(r)=0\forall r\in R$, that is $d_n=0$.

Theorem(3.3)**.**Let U be nonzero ideal of R and $a,b,c\in U$, if $[c,[\mathrm{a},[\mathrm{b},\mathrm{x}]]]_{\sigma,\tau}=0, \forall\, \mathrm{x}\in \mathrm{U}$, then either

1. $c,a\in C_{\sigma,\tau}$ or $b\in Z(\mathrm{R})$.
2. $c\in C_{\sigma,\tau}$ or $a,b\in Z(R)$.

Proof. The map $d_1:R\to R$ define by $d_1(x)=[c,x]_{\sigma,\tau}$ is a $(\sigma,\tau)-$ derivation and the map $d_2:R\to R$ defined by $d_2(x)=[a,x]$, is derivation and the map $d_3:R\to R$ defined by $d_3(x)=[b,x]$ is a derivation moreover $d_3(u)=[b,U]\subset U$ this is $d_3(u)\subset \mathrm{U}$ and $d_1d_2d_3(u)=d_1(d_2(d_3(u)))=[c,[a,[b,u]]]_{\sigma,\tau}=(0)$, by assumption.

Hence, in view of Lemma (3.2) use obtain

1.If $d_1=0$ or $d_2d_3=0$ this is implies that if $d_1=0$ this implies $c\in C_{\sigma,\tau}$,

if $d_2d_3=0\Rightarrow$ by Lemma (2.2) either $d_2=0$ or $d_3=0$

This implies $a\in C_{\sigma,\tau}$ or $b\in Z(R)\Rightarrow c,a\in C_{\sigma,\tau}$ or $b\in Z(R)$

2.If $d_1d_2d_3=0$, then $d_1d_2=0$ or $d_3=0$ If $d_1d_2=0$ (by Lemma (2.2))either

$d_1=0$ or $d_2=0$ Hence $d_1=0$ this implies $c\in C_{\sigma,\tau}$ or $a\in Z(a)$

If $d_3=0$ this implies $b\in Z(R)$ So $c\in C_{\sigma,\tau}$ or $a,b\in Z(R)$

Theorem(3.4) (In general)**.**Let U be anonzero ideal of R and $a_1,a_2,.....a_n\in U$, If $\left[a_n\left[a_{n-1},\left[.......,\left[a_1,x\right]\right]\right].......\right]_{\sigma,\tau}=0\forall x\in U$, then either

1. $a_n, a_{n-1}, \ldots.. a_2 \in C_{\sigma,\tau}$ or $a_1 \in Z(R)$.
2. $a_n \in C_{\sigma,\tau}$ or $a_{n-1}, \ldots.. a_1 \in Z(R)$.

Proof. The map $d_1 : R \to R$ define by $d_1(x) = [a_n, x]_{\sigma,\tau}$ is a (σ,τ) – derivation and the map $d_2 : R \to R$ definedby $d_2(x) = [a_{n-1}, x]$, is derivation and the map $d_n : R \to R$ defined by $d_n(x) = [a_1, x]$ is a derivation moreoven $d_n(U) = [a_1, U] \subset U$ this is $d_n(U) \subset U$ and $d_1 d_2 \ldots\ldots.. d_n(U) = d_1(d_2(\ldots\ldots d_n(U)) = \left[a_n \left[a_{n-1}, \left[\ldots\ldots., [a_1, x]\right]\right] \ldots\ldots.\right]_{\sigma,\tau} = 0 \forall x \in U$ by assumption.Hence, in view of Lemma (3.2)use obtain

1.If $d_1 d_2 \ldots\ldots d_n = 0$, then $d_1(d_2 \ldots\ldots d_n) = 0$ this is implies that if $d_1 = 0$ this implies $a_n \in C_{\sigma,\tau}$ (1)

or $d_2 \ldots\ldots.. d_n = 0$ (by Theorem (3.2))either $d_2 = o$ or $d_3 \ldots\ldots d_n = 0$

if $d_2 = o$ This implies $a_{n-1} \in C_{\sigma,\tau}$ (2), Similarly $d_{n-1} d_n = 0$, by corollary (3.1)

Then either $a_2 \in C_{\sigma,\tau}$ or $a_1 \in Z(R)$ (3), From (1), (2) and (3) We get $a_n, a_{n-1}, \ldots.. a_2 \in C_{\sigma,\tau}$ or $a_1 \in Z(R)$.

2.If $d_1 d_2 \ldots\ldots d_n = 0$, then $(d_1 d_2 \ldots\ldots d_{n-1}) d_n = 0$ (by Theorem (3.2))
then either $(d_1 d_2 \ldots\ldots.. d_{n-1}) = 0$ or $d_n = 0$ implies that if $d_n = 0$ this implies $a_1 \in Z(R)$ (1)
or $(d_1 d_2 \ldots\ldots.. d_{n-1}) = 0$, We can write them $(d_1 d_2 \ldots\ldots. d_{n-2}) d_{n-1} = 0$ (by Theorem (3.2)) theneither $d_1 d_2 \ldots\ldots d_{n-2} = 0$ or $d_{n-1} = 0$ implies that if $d_{n-1} = 0$, then this implies $a_2 \in Z(R)$ (2), Similarly $d_1 d_2 = 0$ (by lemma (3.2))
then either $d_1 = 0$ or $d_2 = 0$, by corollary (3.1), then either
$a_n \in C_{\sigma,\tau}$ or $a_{n-1} \in Z(R)$ (3) from (1), (2) and (3) we get $a_n \in C_{\sigma,\tau}$ or $a_{n-1}, \ldots\ldots a_1 Z(R)$.

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