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Stabilization of a Vertical Inverted Pendulum Using Conventional PID Controllers

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Abstract: ***This paper presents conventional PID controllers for stabilizing a vertical inverted pendulum, which can move in the XZ horizontal and vertical plane with the help of horizontal and vertical forces. Since the vertical inverted pendulum is a two input three-output system hence their PID controllers are used, two for position control while the remaining one for angle control. Simulation results carried out using Matlab Simulink show the effectiveness of the proposed controllers.***

Keywords: Vertical Inverted Pendulum, PID controller, Stabilization

1. Introduction

Inverted pendulums are under-actuated, nonlinear and non-minimum phase systems with lesser control inputs than the degrees of freedom (Astrom et al., 1999). Designing a controller for such a system becomes very difficult which results in the inverted pendulum system a classical benchmark problem for designing controllers in control theory (Boubakar, 2012). The control of many real time systems such as segways, rocket launchers, crane lifting containers and self-balancing robots resembles the control of an inverted pendulum. Various types of controllers for stabilization of traditional inverted pendulum have been discussed in the literature. Besides this some researchers worked on the other kinds of inverted pendulums, such as spherical or $X-Y$ inverted pendulum (Wai and Chang, 2006, Chang and Lee, 2007) and $X-Z$ inverted pendulum (Maravall, 2004, Maravall et al., 2005). In an $X-Z$ inverted pendulum also known as a vertical inverted pendulum the control objective is to keep the pendulum in the upright position while following a desired reference trajectory by the base. To attain the control objective three controllers are required, two for the position control of the pivot (x and z positions) while the other for angle control of the pendulum. A proportional derivative controller for controlling an $X-Z$ inverted pendulum has been reported by (Maravall, 2004). The proposed controller guarantees the global stability by applying Lyapunov's direct method. Later a hybrid controller which comprises PD control into a Takagi–Sugeno fuzzy control structure for stabilizing an $X-Z$ inverted pendulum has been presented (Maravall et al., 2005). Stabilization and tracking control of $X-Z$ inverted pendulum using PID controllers and sliding mode control strategy has been considered (Wang, 2011 and Wang, 2012). A survey on inverted pendulum as a benchmark in nonlinear control theory has been presented in the literature, (Boubakar, 2012). In this survey various control techniques for stabilization of an inverted pendulum has been discussed (Furuta et al., 1992, Chung and Hauser, 1995, Spong, 1995, Fantoni et al., 2000, Zhao et al., 2001, Muskinja et al., 2006 and Tsai, 2007). In this paper PID controllers are designed to stabilize a vertical inverted pendulum system.

This paper is organized as follows: Section II gives the state equations of the vertical inverted pendulum. The designing of PID controller has been explained in Appendix A. The simulation results have been presented in section III followed by the conclusion of the paper in section IV.

2. $X-Z$ Inverted Pendulum

A vertical inverted pendulum mounted on a base has been shown in Figure 1.

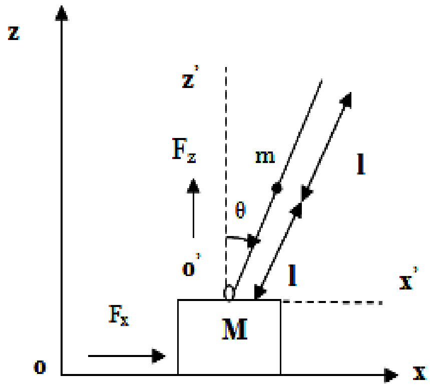

Figure 1: Vertical inverted pendulum

The stabilization of the inverted pendulum in the upright position depends upon the horizontal and vertical displacements of the pivot, which in turn depends upon the applied forces F_x and F_Z in the XZ plane. As shown in Figure 1, l is the distance from the pivot to the mass center of the pendulum. M and m is the mass of the base and the pendulum respectively. The parameters of the inverted pendulum system are given in Table 1 (Wang, 2011).

Table 1: Inverted pendulum parameters (Wang, 2011)

M (kg)	m (kg)	l (m)	g (m/s^2)
1	0.1	0.3	9.8

The state space model of the inverted pendulum system is presented as follows:

Defining $x_1 = x, x_3 = z$ and $x_5 = \theta$

$\dot{x}_1 = x_2$ (1)

$$-m\sin\theta\cos\theta(F_z)+(F_x)$$

$$\dot{x}_2=\frac{(M+m-m\sin^2\theta)+Mml\sin\theta\dot{\theta}^2}{M(M+m)} \quad (2)$$

$$\dot{x}_3=x_4 \quad (3)$$

$$\dot{x}_4=\frac{m(F_z)(1-\cos^2\theta)-m\sin\theta\cos\theta(F_x)+M(F_z-Mg)+Mml\cos\theta\dot{\theta}^2}{M(M+m)} \quad (4)$$

$$\dot{x}_5=x_6 \quad (5)$$

$$\dot{x}_6=\frac{-\cos\theta(F_x+F_{xfric})+\sin\theta(F_z+F_{zfric})}{Ml} \quad (6)$$

The simulink model of the vertical inverted pendulum with angle and position PID controllers is shown in Figure 2.

Three PID controllers are used; two for the position control while the other for angle control. The designing of PID controllers is explained in Appendix A. The reference inputs for x and z positions are the step inputs. Since the movement of the pivot in the z direction will not affect the angle of the inverted pendulum, hence the angle PID controller is only associated with x position control and not with z position control.

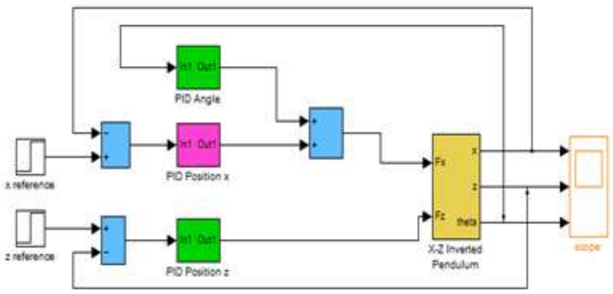

Figure 2: $x-z$ inverted pendulum system with adaptive gain scheduling PID controllers

3. Simulation Results

The simulation results for position and angle control are given from Figure 3 to Figure 5.

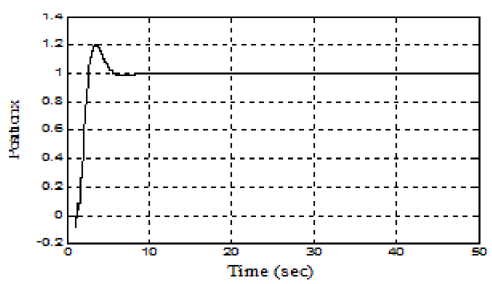

Figure 3: Position XVs time

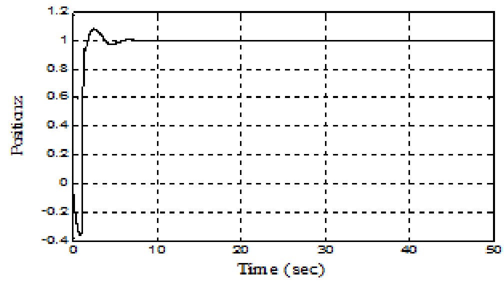

Figure 4: x Position Vs time

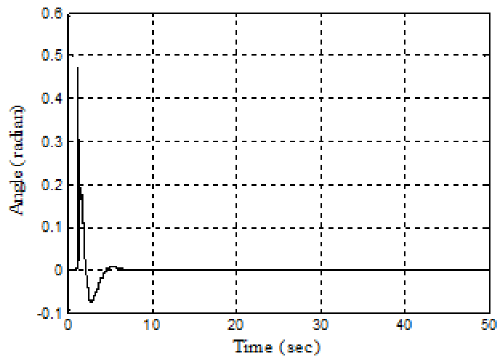

Figure 5: z Position Vs time

For angle control the two responses are almost same but still a slight improvement in the inverse response behavior of the system and settling time.

The above simulation result show that in case of x position control the system response becomes faster with proposed

4. Appendix A

Using Table 1 the state, input and output matrices of X inverted pendulum are given as:

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.0414 & 11.0126 & 0\\ 0 & 0 & 0 & 0\\ 0 & -0.121 & 171.48 & 0\end{bmatrix},\ B=\begin{bmatrix}0\\ 0.347\\ 0\\ 1.0142\end{bmatrix},\ C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

The positive semi definite symmetric matrix Q comes out to be [14]:

$$Q=\begin{bmatrix}65000 & 0 & 0 & 0\\ 0 & 2600 & 0 & 0\\ 0 & 0 & 2600 & 0\\ 0 & 0 & 0 & 130\end{bmatrix}$$

The optimal state feedback gains are:

$$K=\begin{bmatrix}-2.5495 & -4.7965 & 370.9584 & 28.6162\end{bmatrix}$$

The eigen values of (A–BK) are calculated as:

-13.1520, -13.0388, -0.6042 + 0.5945i, -0.6042 - 0.5945i

These eigen values act as the system closed loop poles.

Using the above four poles, and selecting the fifth pole to be six times the real part of the dominant pole among these four poles, the coefficients of the desired characteristic equation and finally finding PID gains can be calculated. These values come out to be:

K_{p1}=78.9243 K_{i1}=152.158 K_{d1}=10
K_{p2}= -21.16 K_{i2}=-13.0887 K_{d2}=-1.9413

5. Conclusions

In this paper conventional PID controllers are used for stabilizing a vertical inverted pendulum. Since the vertical inverted pendulum is a two input three output system hence there PID controllers are used, two for position control while the remaining one for angle control. Simulation results carried out using Matlab Simulink show the effectiveness of the proposed controllers.

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