

On k-Super Mean Labeling of Some Graphs

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Abstract: Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. A graph that admits a super mean labeling is called super mean graph. Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called k -super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$. A graph that admits a k -super mean labeling is called k -super mean graph. In this paper, we investigate k -super mean labeling of $L_n \odot K_1, S(T_n), S(T_n \odot k_1), (P_n : C_4), [P_n : C_6^2]$.

Keyword: k -super mean labeling, k -super mean graph, $L_n \odot K_1, S(T_n), S(T_n \odot k_1), (P_n : C_4), [P_n : C_6^2]$

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [1]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G . In this paper, we investigate k -super mean graphs of

$L_n \odot K_1, S(T_n), S(T_n \odot k_1), (P_n : C_4), [P_n : C_6^2]$.

Definition 1.1:

Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **super mean labeling** if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. A graph that admits a super mean labeling is called **super mean graph**.

Definition 1.2:

Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called **k -super mean labeling** if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, \dots, p+q+k-1\}$. A graph that admits a k -super mean labeling is called **k -super mean graph**.

Definition 1.3:

A **ladder graph** is a product of $P_2 \times P_n$.

Definition 1.4:

A **triangular snake** (T_n) is obtained from a path by identifying each edge of the path with an edge of the cycle C_3 .

Definition 1.5

If G is a graph, then $S(G)$ is a graph by subdividing each edge of G by a vertex.

Definition 1.6

The graph $G = (P_n : C_4)$ is obtained from a path P_n by fusing one edge of one cycle C_4 at each vertices of the P_n denoted by $(P_n : C_4)$.

Definition 1.7:

The graph $G = [P_n : C_6^2]$ is obtained from a path P_n by fusing one vertex of two cycle C_6 at each vertices of the P_n denoted by $[P_n : C_6^2]$.

2. Main Results

Theorem 2.1:

$L_n \odot K_1$ is a k -Super mean labeling graph for all $n \geq 2$.

Proof:

Let $V(L_n \odot K_1) = \{u_i, v_i : 1 \leq i \leq n\} \cup \{w_i, x_i : 1 \leq i \leq n\}$ and $E(L_n \odot K_1) = \{a_i : (w_i, v_i) : 1 \leq i \leq n\} \cup \{b_i : (v_i, u_i) : 1 \leq i \leq n\} \cup \{c_i : (u_i, x_i) : 1 \leq i \leq n\} \cup \{d_i : (u_i, u_{i+1}) : 1 \leq i \leq n-1\} \cup \{e_i : (v_i, v_{i+1}) : 1 \leq i \leq n-1\}$ be the vertices and edges of $L_n \odot K_1$ respectively. First we label the vertices of $L_n \odot K_1$ as follows.

$$f(w_i) = \begin{cases} k; & i = 1 \\ k + 9i - 10; & 2 \leq i \leq n \end{cases}$$

$$f(v_i) = k + 9i - 7; 1 \leq i \leq n$$

$$f(u_i) = k + 9i - 5; 1 \leq i \leq n$$

$$f(x_i) = k + 9i - 3; 1 \leq i \leq n.$$

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, \dots, k+9n-3\}.$$

It can be verified that f is a k -Super mean labeling.

Hence, f is a k -Super mean labeling and hence $L_n \odot K_1$ is a k -Super mean graph.

Example 2.1:

20 – super mean labeling of $L_5 \odot k_1$ is shown in figure 2.1

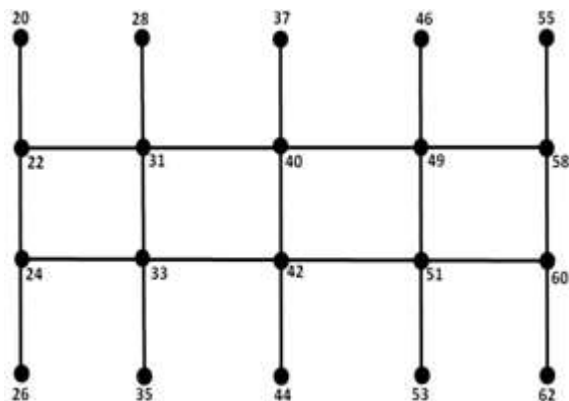


Figure 2.1: 20 – SML of $L_5 \odot k_1$

$$f^*(e_i) = \begin{cases} k + 7, i = 1 \\ k + 11i - 8; 2 \leq i \leq n - 1 \end{cases}$$

$$f^*(e'_i) = \begin{cases} k + 10, i = 1 \\ k + 11i - 2; 2 \leq i \leq n - 1 \end{cases}$$

Here $p = 5n - 4$, $q = 6n - 6$, $p + q = 11n - 10$.

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k + 1, \dots, k + 11n - 11\}.$$

Hence, f is a k – Super mean labeling and hence $S(T_n)$ is a k – Super mean graph.

Example 2.2:

346 - Super mean labeling of $S(T_4)$ is shown in figure 2.2

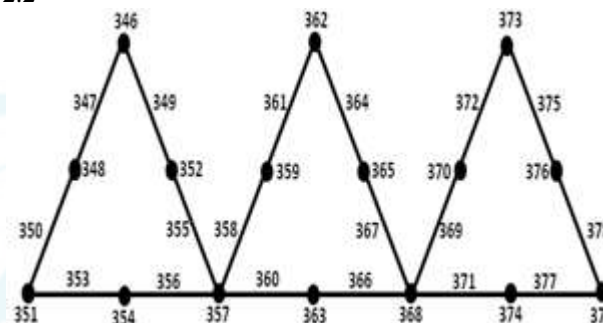


Figure 2.2: 346 – SML of T_4

Theorem 2.2:

$S(T_n)$ is a k – Super mean labeling graph for all $n \geq 2$.

Proof:

Let $S(T_n)$ be the graph obtained by subdividing all the edges.

$$V(S(T_n)) = \{w_i : 1 \leq i \leq n\} \cup \{u_i, v_i, x_i, y_i : 1 \leq i \leq n - 1\}$$

$$E(S(T_n)) = \{a_i : (u_i, v_i) : 1 \leq i \leq n - 1\} \cup \{b_i : (u_i, x_i) : 1 \leq i \leq n - 1\} \cup \{c_i : (v_i, w_i) : 1 \leq i \leq n - 1\} \cup \{d_i : (x_i, w_{i+1}) : 1 \leq i \leq n - 1\} \cup \{e_i : (w_i, y_i) : 1 \leq i \leq n - 1\} \cup \{e'_i : (y_i, w_{i+1}) : 1 \leq i \leq n - 1\}$$

be the vertices and edges of $S(T_n)$ respectively.

First we label the vertices of T_n as follows.

$$f(u_i) = \begin{cases} k, i = 1 \\ k + 11i - 6; 2 \leq i \leq n - 1 \end{cases}$$

$$f(v_i) = \begin{cases} k + 2, i = 1 \\ k + 11i - 9; 2 \leq i \leq n - 1 \end{cases}$$

$$f(w_i) = \begin{cases} k + 5, i = 1 \\ k + 11i - 11; 2 \leq i \leq n \end{cases}$$

$$f(x_i) = \begin{cases} k + 6, i = 1 \\ k + 11i - 3; 2 \leq i \leq n - 1 \end{cases}$$

$$f(y_i) = \begin{cases} k + 8, i = 1 \\ k + 11i - 5; 2 \leq i \leq n - 1. \end{cases}$$

Now the induced edge labels are as follows:

$$f^*(a_i) = \begin{cases} k + 1, i = 1 \\ k + 11i - 7; 2 \leq i \leq n - 1 \end{cases}$$

$$f^*(b_i) = \begin{cases} k + 3, i = 1 \\ k + 11i - 4; 2 \leq i \leq n - 1 \end{cases}$$

$$f^*(c_i) = \begin{cases} k + 4, i = 1 \\ k + 11i - 10; 2 \leq i \leq n - 1 \end{cases}$$

$$f^*(d_i) = \begin{cases} k + 9, i = 1 \\ k + 11i - 1; 2 \leq i \leq n - 1 \end{cases}$$

Theorem 2.3:

The graph $S(T_n \odot k_1)$ is a k – Super mean labeling graph for all $n \geq 2$.

Proof:

$$V(S(T_n \odot k_1)) = \{u_i, u'_i, v_i : 1 \leq i \leq n + 1\} \cup \{v'_i, w_i, w'_i, x_i, y_i, z_i : 1 \leq i \leq n\}$$

$$E(S(T_n \odot k_1)) = \{a_i : (v'_i, w_i) : 1 \leq i \leq n\} \cup \{a'_i : (w_i, z_i) : 1 \leq i \leq n\} \cup \{b_i : (v_i, v'_i) : 1 \leq i \leq n\} \cup \{b'_i : (z_i, v_{i+1}) : 1 \leq i \leq n\} \cup \{c_i : (v_i, y_i) : 1 \leq i \leq n\} \cup \{c'_i : (y_i, v_{i+1}) : 1 \leq i \leq n\} \cup \{d_i : (v_i, u'_i) : 1 \leq i \leq n + 1\} \cup \{d'_i : (u'_i, u_i) : 1 \leq i \leq n + 1\} \cup \{e_i : (x_i, w'_i) : 1 \leq i \leq n\} \cup \{e'_i : (w'_i, w_i) : 1 \leq i \leq n\}.$$

be the vertices and edges of $S(T_n \odot k_1)$ respectively.

First we label the vertices of $S(T_n \odot k_1)$ as follows.

$$f(u_i) = \begin{cases} k; i = 1 \\ k + 19i - 21; 2 \leq i \leq n + 1 \end{cases}$$

$$f(u'_i) = \begin{cases} k + 2; i = 1 \\ k + 19i - 19; 2 \leq i \leq n + 1 \end{cases}$$

$$f(v_i) = k + 19i - 15; 1 \leq i \leq n + 1$$

$$f(v'_i) = k + 19i - 13; 1 \leq i \leq n$$

$$f(w_i) = k + 19i - 9; 1 \leq i \leq n$$

$$f(w'_i) = k + 19i - 7; 1 \leq i \leq n$$

$$f(x_i) = k + 19i - 5; 1 \leq i \leq n$$

$$f(y_i) = k + 19i - 10; 1 \leq i \leq n$$

$$f(z_i) = k + 19i + 1; 1 \leq i \leq n.$$

Now the induced edge labels are as follows:

$$f^*(a_i) = k + 19i - 11; 1 \leq i \leq n$$

$$\begin{aligned}
 f^*(a'_i) &= k + 19i - 4; 1 \leq i \leq n \\
 f^*(b_i) &= k + 19i - 14; 1 \leq i \leq n \\
 f^*(b'_i) &= k + 19i + 3; 1 \leq i \leq n \\
 f^*(c_i) &= k + 19i - 12; 1 \leq i \leq n \\
 f^*(c'_i) &= k + 19i - 3; 1 \leq i \leq n
 \end{aligned}$$

$$f^*(d_i) = \begin{cases} k + 2; i = 1 \\ k + 19i - 17; 2 \leq i \leq n + 1 \end{cases}$$

$$f^*(d'_i) = \begin{cases} k + 1; i = 1 \\ k + 19i - 20; 2 \leq i \leq n + 1 \end{cases}$$

$$\begin{aligned}
 f^*(e_i) &= k + 19i - 6; 1 \leq i \leq n \\
 f^*(e'_i) &= k + 19i - 8; 1 \leq i \leq n.
 \end{aligned}$$

Here $p = 9n + 3, q = 10n + 2, p + q = 19n + 5$
Clearly,

$$\begin{aligned}
 f(v) \cup \{f^*(e) : e \in E(G)\} \\
 = \{k, k + 1, \dots, k + 19n + 4\}
 \end{aligned}$$

So, $S(T_n \odot K_1)$ is a k -Super mean graph.

Example 2.3:

1073 – Super mean labeling of $S(T_3 \odot K_1)$ is shown in figure 2.3

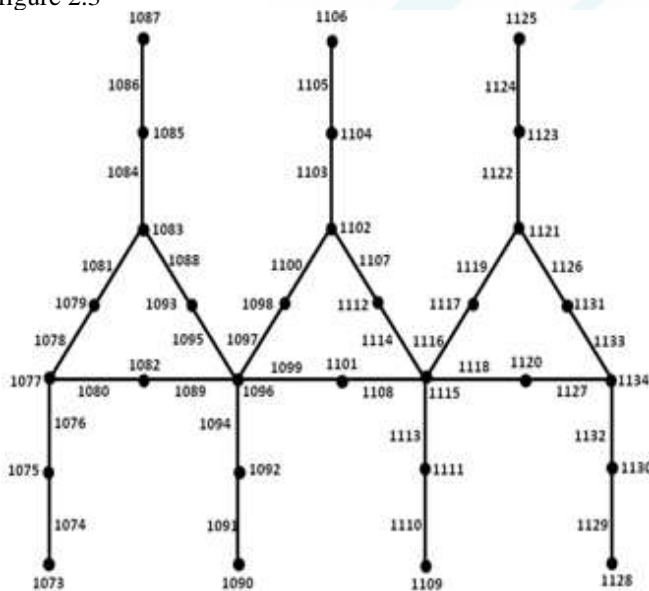


Figure 2.3: 1073 – SML of $S(T_3 \odot K_1)$

Theorem 2.4:

The graph $(P_n : C_4)$ is a k -Super mean labeling graph for all $n \geq 2$.

Proof:

Let $V((P_n : C_4)) = \{u_i, v_i, w_i, x_i, y_i; 1 \leq i \leq n\}$ and
 $E((P_n : C_4)) = \{a_i = (u_i, v_i); 1 \leq i \leq n\} \cup$
 $\{b_i = (u_i, w_i); 1 \leq i \leq n\} \cup$
 $\{c_i = (v_i, x_i); 1 \leq i \leq n\} \cup$
 $\{d_i = (w_i, x_i); 1 \leq i \leq n\} \cup$
 $\{e_i = (x_i, y_i); 1 \leq i \leq n\} \cup$
 $\{e'_i = (y_i, y_{i+1}); 1 \leq i \leq n - 1\}$
 be the vertices and edges of $(P_n : C_4)$ respectively.

First we label the vertices of $(P_n : C_4)$ as follows.

$$f(u_i) = \begin{cases} k; i = 1 \\ k + 11i - 10; 2 \leq i \leq n \end{cases}$$

$$f(v_i) = k + 11i - 2, 1 \leq i \leq n$$

$$f(w_i) = \begin{cases} k + 2; i = 1 \\ k + 11i - 12; 2 \leq i \leq n \end{cases}$$

$$f(x_i) = k + 11i - 7, 1 \leq i \leq n$$

$$f(y_i) = k + 11i - 3, 1 \leq i \leq n.$$

Now the induced edge labels are as follows:

$$f^*(a_i) = k + 11i - 6, 1 \leq i \leq n$$

$$f^*(b_i) = \begin{cases} k + 1; i = 1 \\ k + 11i - 11; 2 \leq i \leq n \end{cases}$$

$$f^*(c_i) = k + 11i - 4, 1 \leq i \leq n$$

$$f^*(d_i) = \begin{cases} k + 2; i = 1 \\ k + 11i - 9; 2 \leq i \leq n \end{cases}$$

$$f^*(e_i) = k + 11i - 5, 1 \leq i \leq n$$

$$f^*(e'_i) = k + 11i + 3, 1 \leq i \leq n - 1.$$

Here $p = 5n, q = 6n - 1, p + q = 11n - 1$

Clearly,

$$\begin{aligned}
 f(V) \cup \{f^*(e) : e \in E(G)\} \\
 = \{k, k + 1, \dots, k + 11n - 2\}.
 \end{aligned}$$

Hence, f is a k -Super mean labeling and hence $(P_n : C_4)$ is a k -Super mean graph.

Example 2.4:

567 – Super mean labeling of $(P_4 : C_4)$ is shown in figure 2.4

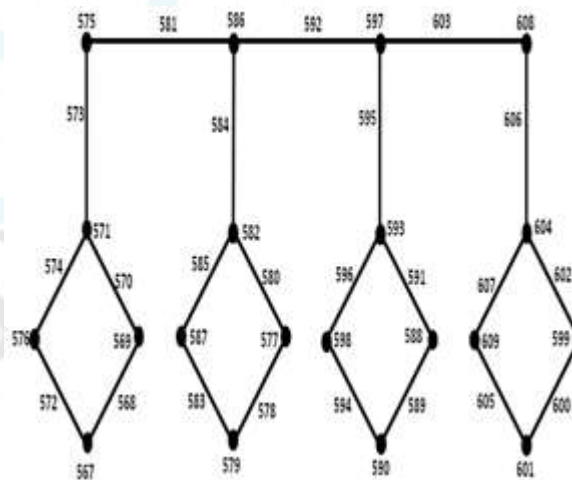


Figure 2.4: 567 – SML of $(P_4 : C_4)$

Theorem 2.5:

The graph $[P_n : C_6^2]$ is a k -Super mean labeling graph for all $n \geq 2$.

Proof:

Let $V([P_n : C_6^2]) = \{u_i, v_i, w_i, x_i, y_i, z_i; 1 \leq i \leq n\} \cup$
 $\{u'_i, v'_i, w'_i, y'_i, z'_i; 1 \leq i \leq n\}$ and
 $E([P_n : C_6^2]) = \{a_i = (u_i, v_i); 1 \leq i \leq n\} \cup$
 $\{a'_i = (u'_i, v'_i); 1 \leq i \leq n\} \cup$
 $\{b_i = (v_i, w_i); 1 \leq i \leq n\} \cup$
 $\{b'_i = (v'_i, w'_i); 1 \leq i \leq n\} \cup$
 $\{c_i = (w_i, x_i); 1 \leq i \leq n\} \cup$

$$\begin{aligned} & \{c_i' = (w_i', x_i) ; 1 \leq i \leq n\} \cup \\ & \{d_i = (x_i, y_i) ; 1 \leq i \leq n\} \cup \\ & \{d_i' = (x_i, y_i') ; 1 \leq i \leq n\} \cup \\ & \{e_i = (y_i, z_i) ; 1 \leq i \leq n\} \cup \\ & \{e_i' = (y_i', z_i') ; 1 \leq i \leq n\} \cup \\ & \{e_i'' = (z_i, u_i') ; 1 \leq i \leq n\} \cup \\ & \{e_i''' = (u_i', z_i') ; 1 \leq i \leq n\} \cup \\ & \{e_i^{iv} = (x_i, x_{i+1}) ; 1 \leq i \leq n - 1\} \end{aligned}$$

be the vertices and edges of $[P_n : C_6^2]$ respectively.

First we label the vertices of $[P_n : C_6^2]$ as follows.

$$f(u_i) = k + 24i - 22; 1 \leq i \leq n$$

$$f(v_i) = k + 24i - 24; 1 \leq i \leq n$$

$$f(v_i') = k + 24i - 19; 1 \leq i \leq n$$

$$f(w_i) = k + 24i - 18; 1 \leq i \leq n$$

$$f(w_i') = k + 24i - 16; 1 \leq i \leq n$$

$$f(x_i) = k + 24i - 13; 1 \leq i \leq n$$

$$f(y_i) = k + 24i - 11; 1 \leq i \leq n$$

$$f(y_i') = k + 24i - 7; 1 \leq i \leq n$$

$$f(z_i) = k + 24i - 8; 1 \leq i \leq n$$

$$f(z_i') = k + 24i - 2; 1 \leq i \leq n$$

$$f(u_i') = k + 24i - 5; 1 \leq i \leq n.$$

Now the induced edge labels are as follows:

$$f^*(a_i) = k + 24i - 23; 1 \leq i \leq n$$

$$f^*(a_i') = k + 24i - 20; 1 \leq i \leq n$$

$$f^*(b_i) = k + 24i - 21; 1 \leq i \leq n$$

$$f^*(b_i') = k + 24i - 17; 1 \leq i \leq n$$

$$f^*(c_i) = k + 24i - 15; 1 \leq i \leq n$$

$$f^*(c_i') = k + 24i - 14; 1 \leq i \leq n$$

$$f^*(d_i) = k + 24i - 12; 1 \leq i \leq n$$

$$f^*(d_i') = k + 24i - 10; 1 \leq i \leq n$$

$$f^*(e_i) = k + 24i - 9; 1 \leq i \leq n$$

$$f^*(e_i') = k + 24i - 4; 1 \leq i \leq n$$

$$f^*(e_i'') = k + 24i - 6; 1 \leq i \leq n$$

$$f^*(e_i''') = k + 24i - 3; 1 \leq i \leq n$$

$$f^*(e_i^{iv}) = k + 24i - 1; 1 \leq i \leq n - 1$$

$$\text{Here } p = 11n, q = 13n - 1, p + q = 24n - 1.$$

Clearly,

$$\begin{aligned} f(V) \cup \{f^*(e) : e \in E(G)\} \\ = \{k, k + 1, \dots, k + 24n - 2\}. \end{aligned}$$

Hence, f is a k – Super mean labeling and hence $[P_n : C_6^2]$ is a k – Super mean graph.

Example 2.5:

1 – Super mean labeling of $[P_3 : C_6^2]$ is shown in figure 2.5

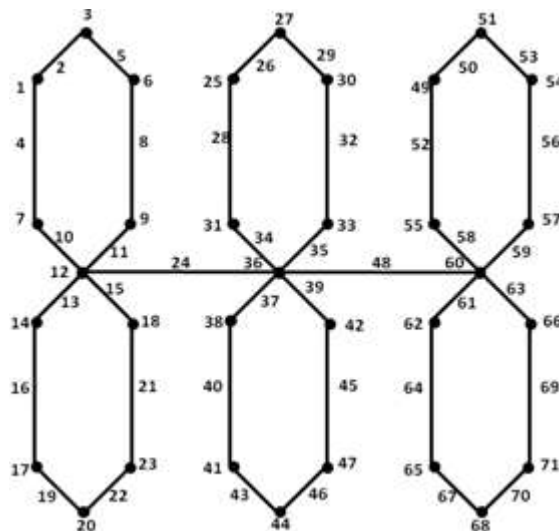


Figure 2.5: 1 – SML of $[P_3 : C_6^2]$

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