On k-Super Mean Labeling of Some Graphs

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Abstract: ***Let G be a*** (p, q) ***graph and*** $f: V(G) \to \{1, 2, 3, \ldots p+q\}$ ***be an injection. For each edge*** $e = uv$, ***let*** $f^*(e) = \frac{f(u)+f(v)}{2}$ ***if*** $f(u)+f(v)$ ***is even and*** $f^*(e) = \frac{f(u)+f(v)+1}{2}$ ***if*** $f(u)+f(v)$ ***is odd, then f is called super mean labeling if*** $f(V) \cup \{f^*(e): e \in E(G) = \{1, 2, 3, \ldots p+q\}$. ***A graph that admits a super mean labeling is called super mean graph. Let G be a*** (p, q) ***graph and*** $f: V(G) \to \{k, k+1, k+2, \ldots p+q+k-1\}$ ***be an injection. For each edge*** $e = uv$, ***let*** $f^*(e) = \frac{f(u)+f(v)}{2}$ ***if*** $f(u)+f(v)$ ***is even and*** $f^*(e) = \frac{f(u)+f(v)+1}{2}$ ***if*** $f(u)+f(v)$ ***is odd, then f is called k - super mean labeling if*** $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \ldots p+q+k-1\}$. ***A graph that admits a k - super mean labeling is called k - super mean graph. In this paper, we investigate k-super mean labeling of*** $L_n \odot K_1$, $S(T_n)$, $S(T_n \odot k_1)$, $(P_n{:}C_4)$, $[P_n{:}C_6^2]$.

Keyword: k - super mean labeling, k - super mean graph, $L_n \odot K_1$, $S(T_n)$, $S(T_n \odot k_1)$, $(P_n{:}C_4)$, $[P_n{:}C_6^2]$

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [1]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G. In this paper, we investigate k-super mean graphs of

$L_n \odot K_1$, $S(T_n)$, $S(T_n \odot k_1)$, $(P_n{:}C_4)$, $[P_n{:}C_6^2]$.

Definition 1.1:

Let G be a (p, q) graph and $f: V(G) \to \{1, 2, 3, \ldots p+q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called **super mean labeling** if $f(V) \cup \{f^*(e): e \in E(G) = \{1, 2, 3, \ldots p+q\}$. A graph that admits a super mean labeling is called **super mean graph.**

Definition 1.2:
Let G be a (p, q) graph and
$f: V(G) \to \{k, k+1, k+2, \ldots p+q+k-1\}$ be an injection. For each edge $e = uv$, let
$f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd, then f is called **k - super mean labeling** if
$f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \ldots, p+q+k-1\}$. A graph that admits a k - super mean labeling is called **k - super mean graph.**

Definition 1.3:
A **ladder graph** is a product of $P_2 \times P_n$.

Definition 1.4:
A **triangular snake** $(\mathbf{T_n})$ is obtained from a path by identifying each edge of the path with an edge of the cycle C_3.

Definition 1.5
If G is a graph, then **S (G)** is a graph by subdividing each edge of G by a vertex.

Definition 1.6
The graph $\mathbf{G = (P_n{:}C_4)}$ is obtained from a path P_n by fusing one edge of one cycle C_4 at each vertices of the P_n denoted by $(\mathbf{P_n{:}C_4})$.

Definition 1.7:
The graph $\mathbf{G = [P_n{:}C_6^2]}$ is obtained from a path P_n by fusing one vertex of two cycle C_6 at each vertices of the P_n denoted by $[P_n{:}C_6^2]$.

2. Main Results

Theorem 2.1:
$\mathbf{L_n \odot k_1}$ is a k – Super mean labeling graph for all $n \geq 2$.

Proof:
Let $V(L_n \odot k_1) = \{u_i, v_i : 1 \leq i \leq n\} \cup \{w_i, x_i : 1 \leq i \leq n\}$ and
$E(L_n \odot k_1) = \{a_i : (w_i, v_i) : 1 \leq i \leq n\} \cup \{b_i : (v_i, u_i) : 1 \leq i \leq n\} \cup \{c_i : (u_i, x_i) : 1 \leq i \leq n\} \cup \{d_i : (u_i, u_{i+1}) : 1 \leq i \leq n-1\} \cup \{e_i : (v_i, v_{i+1}) : 1 \leq i \leq n-1\}$
be the vertices and edges of $L_n \odot k_1$ respectively.
First we label the vertices of $L_n \odot k_1$ as follows.

$$f(w_i) = \begin{cases} k\,; & i = 1 \\ k + 9i - 10\,; & 2 \leq i \leq n \end{cases}$$

$f(v_i) = k + 9i - 7\,;\ 1 \leq i \leq n$
$f(u_i) = k + 9i - 5\,;\ 1 \leq i \leq n$
$f(x_i) = k + 9i - 3\,;\ 1 \leq i \leq n.$
Clearly,
$f(V) \cup \{f^*(e): e \in E(G) = \{k, k+1, \ldots, k+9n-3\}$.

It can be verified that f is a k-Super mean labeling.

Hence, **f** is a **k** – Super mean labeling and hence $\mathbf{L_n \odot k_1}$ is a **k** – Super mean graph.

Example 2.1:
20 – super mean labeling of $L_5 \odot k_1$ is shown in figure 2.1

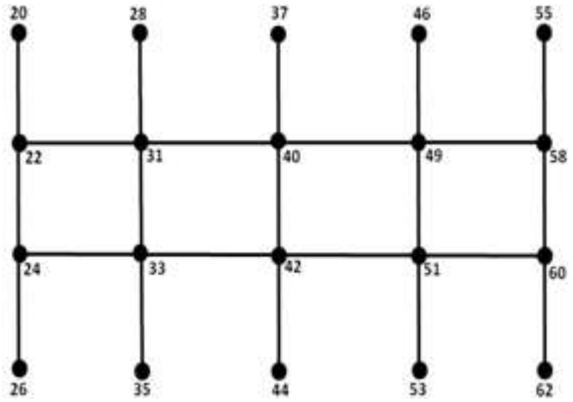

Figure 2.1: 20 – SML of $L_5 \odot k_1$

Theorem 2.2:
$S(T_n)$ is a **k** – Super mean labeling graph for all $n \geq 2$.

Proof:
Let $S(T_n)$ be the graph obtained by subdividing all the edges.
Let $V(S(T_n)) = \{w_i : 1 \leq i \leq n\}$ U
$\{u_i, v_i, x_i, y_i : 1 \leq i \leq n-1\}$ and
$E(S(T_n)) = \{a_i : (u_i, v_i) : 1 \leq i \leq n-1\}$ U
$\{b_i : (u_i, x_i) : 1 \leq i \leq n-1\}$ U
$\{c_i : (v_i, w_i) : 1 \leq i \leq n-1\}$ U
$\{d_i : (x_i, w_{i+1}) : 1 \leq i \leq n-1\}$ U
$\{e_i : (w_i, y_i) : 1 \leq i \leq n-1\}$U
$\{e_i' : (y_i, w_{i+1}) : 1 \leq i \leq n-1\}$
be the vertices and edges of $S(T_n)$ respectively.

First we label the vertices of T_n as follows.

$$f(u_i) = \begin{cases} k, i = 1 \\ k + 11i - 6;\ 2 \leq i \leq n-1 \end{cases}$$

$$f(v_i) = \begin{cases} k + 2, i = 1 \\ k + 11i - 9;\ 2 \leq i \leq n-1 \end{cases}$$

$$f(w_i) = \begin{cases} k + 5, i = 1 \\ k + 11i - 11; 2 \leq i \leq n \end{cases}$$

$$f(x_i) = \begin{cases} k + 6, i = 1 \\ k + 11i - 3;\ 2 \leq i \leq n-1 \end{cases}$$

$$f(y_i) = \begin{cases} k + 8, i = 1 \\ k + 11i - 5;\ 2 \leq i \leq n-1. \end{cases}$$

Now the induced edge labels are as follows:

$$f^*(a_i) = \begin{cases} k + 1, i = 1 \\ k + 11i - 7;\ 2 \leq i \leq n-1 \end{cases}$$

$$f^*(b_i) = \begin{cases} k + 3, i = 1 \\ k + 11i - 4;\ 2 \leq i \leq n-1 \end{cases}$$

$$f^*(c_i) = \begin{cases} k + 4, i = 1 \\ k + 11i - 10;\ 2 \leq i \leq n-1 \end{cases}$$

$$f^*(d_i) = \begin{cases} k + 9, i = 1 \\ k + 11i - 1;\ 2 \leq i \leq n-1 \end{cases}$$

$$f^*(e_i) = \begin{cases} k + 7, i = 1 \\ k + 11i - 8;\ 2 \leq i \leq n-1 \end{cases}$$

$$f^*(e_i') = \begin{cases} k + 10, i = 1 \\ k + 11i - 2; 2 \leq i \leq n-1 \end{cases}$$

Here p = 5n-4 , q = 6n-6 , p+q = 11n-10.
Clearly,

$$f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, \ldots, k + 11n - 11\}.$$

Hence , **f** is a **k** – Super mean labeling and hence **$S(T_n)$** is a **k** – Super mean graph.

Example 2.2:
346 - Super mean labeling of $S(\mathbf{T_4})$ is shown in figure 2.2

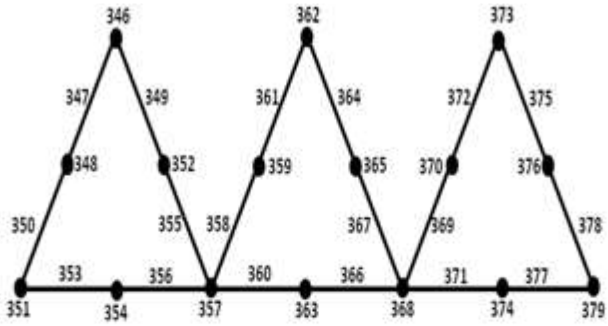

Figure 2.2: 346 – SML of T_4

Theorem 2.3:
The graph **$S(T_n \odot k_1)$** is a **k** – Super mean labeling graph for all $n \geq 2$.

Proof:
Let $V(S(T_n \odot k_1)) = \{u_i, u_i', v_i : 1 \leq i \leq n+1\}$ U
$\{v_i', w_i, w_i', x_i, y_i, z_i : 1 \leq i \leq n\}$ and
$E(S(T_n \odot k_1)) = \{a_i : (v_i', w_i) : 1 \leq i \leq n\}$ U
$\{a_i' : (w_i, z_i) : 1 \leq i \leq n\}$ U
$\{b_i : (v_i, v_i') : 1 \leq i \leq n\}$ U
$\{b_i' : (z_i, v_{i+1}) : 1 \leq i \leq n\}$ U
$\{c_i : (v_i, y_i) : 1 \leq i \leq n\}$ U
$\{c_i' : (y_i, v_{i+1}) : 1 \leq i \leq n\}$ U
$\{d_i : (v_i, u_i') : 1 \leq i \leq n+1\}$U
$\{d_i' : (u_i', u_i) : 1 \leq i \leq n+1\}$U
$\{e_i : (x_i, w_i') : 1 \leq i \leq n\}$ U
$\{e_i' : (w_i', w_i) : 1 \leq i \leq n\}$.
be the vertices and edges of $S(T_n \odot k_1)$ respectively.
First we label the vertices of $S(T_n \odot k_1)$ as follows.

$$f(u_i) = \begin{cases} k;\ i = 1 \\ k + 19i - 21; 2 \leq i \leq n+1 \end{cases}$$

$$f(u_i') = \begin{cases} k + 2;\ i = 1 \\ k + 19i - 19; 2 \leq i \leq n+1 \end{cases}$$

$f(v_i) = k + 19i - 15;\ 1 \leq i \leq n+1$
$f(v_i') = k + 19i - 13; 1 \leq i \leq n$
$f(w_i) = k + 19i - 9;\ 1 \leq i \leq n$
$f(w_i') = k + 19i - 7;\ 1 \leq i \leq n$
$f(x_i) = k + 19i - 5;\ 1 \leq i \leq n$
$f(y_i) = k + 19i - 10;\ 1 \leq i \leq n$
$f(z_i) = k + 19i + 1;\ 1 \leq i \leq n.$
Now the induced edge labels are as follows:
$f^*(a_i) = k + 19i - 11; 1 \leq i \leq n$

$f^*(a_i') = k + 19i - 4;\ 1 \le i \le n$
$f^*(b_i) = k + 19i - 14; 1 \le i \le n$
$f^*(b_i') = k + 19i + 3;\ 1 \le i \le n$
$f^*(c_i) = k + 19i - 12;\ 1 \le i \le n$
$f^*(c_i') = k + 19i - 3;\ 1 \le i \le n$

$$f^*(d_i) = \begin{cases} k+2; i = 1 \\ k + 19i - 17; 2 \le i \le n+1 \end{cases}$$

$$f^*(d_i') = \begin{cases} k+1; i = 1 \\ k + 19i - 20; 2 \le i \le n+1 \end{cases}$$

$f^*(e_i) = k + 19i - 6;\ 1 \le i \le n$
$f^*(e_i') = k + 19i - 8;\ 1 \le i \le n.$
Here p = 9n + 3, q = 10n +2, p+q = 19n + 5
Clearly,

$$f(v)\ U\ \{f * (e) :\ e \in E(G)\ = \{k, k+1, \ldots, k + 19n + 4\}$$

So, **S(T$_n$ ʘ k$_1$)** is a ***k*** – Super mean graph.

Example 2.3:
1073 – Super mean labeling of **S(T$_3$ ʘ k$_1$)** is shown in figure 2.3

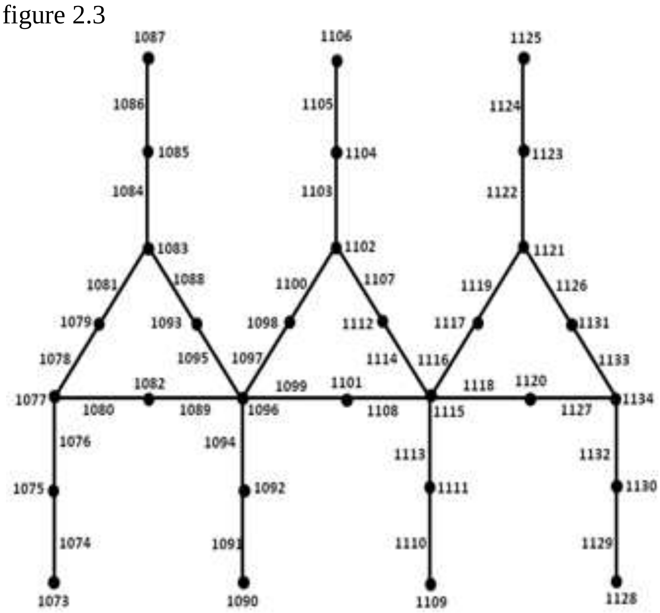

Figure 2.3: 1073 – SML of S(T$_3$ ʘ k$_1$)

Theorem 2.4:
The graph **(P$_n$: C$_4$)** is a **k** – Super mean labeling graph for all $n \ge 2$.

Proof:
Let V((P$_n$: C$_4$)) = {u$_i$, v$_i$, w$_i$, x$_i$, y$_i$; 1≤ i ≤ n} and
E((P$_n$: C$_4$)) = {a$_i$ = (u$_i$, v$_i$) ; 1≤ i ≤ n} U
{b$_i$ = (u$_i$, w$_i$) ; 1≤ i ≤ n} U
{c$_i$ = (v$_i$, x$_i$) ; 1≤ i ≤ n} U
{d$_i$ = (w$_i$, x$_i$) ; 1≤ i ≤ n} U
{e$_i$ = (x$_i$, y$_i$) ; 1≤ i ≤ n} U
{e$_i$′= (y$_i$, y$_{i+1}$) ; 1≤ i ≤ $n - 1$}
be the vertices and edges of (P$_n$: C$_4$) respectively.

First we label the vertices of (P$_n$: C$_4$) as follows.

$$f(u_i) = \begin{cases} k\ ;\ i = 1 \\ k + 11i - 10; 2 \le i \le n \end{cases}$$

$f(v_i) = k + 11i - 2, 1 \le i \le n$

$$f(w_i) = \begin{cases} k + 2\ ;\ i = 1 \\ k + 11i - 12; 2 \le i \le n \end{cases}$$

$f(x_i) = k + 11i - 7, 1 \le i \le n$
$f(y_i) = k + 11i - 3, 1 \le i \le n.$
Now the induced edge labels are as follows:
$f^*(a_i) = k + 11i - 6, 1 \le i \le n$

$$f^*(b_i) = \begin{cases} k + 1\ ;\ i = 1 \\ k + 11i - 11; 2 \le i \le n \end{cases}$$

$f^*(c_i) = k + 11i - 4, 1 \le i \le n$

$$f^*(d_i) = \begin{cases} k + 2\ ;\ i = 1 \\ k + 11i - 9; 2 \le i \le n \end{cases}$$

$f^*(e_i) = k + 11i - 5, 1 \le i \le n$
$f^*(e_i') = k + 11i + 3, 1 \le i \le n - 1.$
Here p = 5n , q = 6n-1 , p+q = 11n-1

Clearly,

$$f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, \ldots, k + 11n - 2\}.$$

Hence , **f** is a **k** – Super mean labeling and hence **(P$_n$: C$_4$)** is a **k** – Super mean graph.

Example 2.4:
567 – Super mean labeling of (P$_4$: C$_4$) is shown in figure 2.4

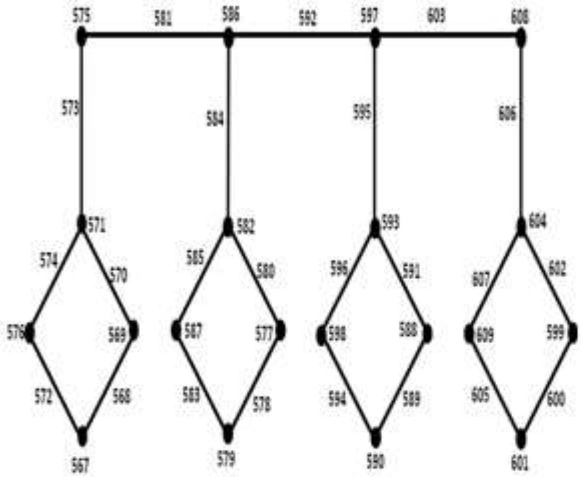

Figure 2.4: 567 – SML of (P$_4$: C$_4$)

Theorem 2.5:
The graph **[P_n:C_6^2]** is a **k** – Super mean labeling graph for all $n \ge 2$.

Proof:
Let V([P_n:C_6^2]) = {u$_i$, v$_i$, w$_i$, x$_i$, y$_i$,z$_i$; 1≤ i ≤ n} U
{u$_i$′, v$_i$′, w$_i$′, y$_i$′, z$_i$′ ; 1≤ i ≤ n} and
E([P_n:C_6^2]) = {a$_i$ = (u$_i$, v$_i$) ; 1≤ i ≤ n} U
{a$_i$′ = (u$_i$, v$_i$′) ; 1≤ i ≤ n} U
{b$_i$ = (v$_i$, w$_i$) ; 1≤ i ≤ n} U
{b$_i$′ = (v$_i$′, w$_i$′) ; 1≤ i ≤ n} U
{c$_i$ = (w$_i$, x$_i$) ; 1≤ i ≤ n} U

{$c_i' = (w_i', x_i)$; $1 \leq i \leq n$} U
{$d_i = (x_i, y_i)$; $1 \leq i \leq n$} U
{$d_i' = (x_i, y_i')$; $1 \leq i \leq n$} U
{$e_i = (y_i, z_i)$; $1 \leq i \leq n$} U
{$e_i' = (y_i', z_i')$; $1 \leq i \leq n$} U
{$e_i'' = (z_i, u_i')$; $1 \leq i \leq n$} U
{$e_i''' = (u_i', z_i')$; $1 \leq i \leq n$} U
{$e_i'^{v} = (x_i, x_{i+1})$; $1 \leq i \leq n-1$}

be the vertices and edges of $[P_n : C_6^2]$ respectively.
First we label the vertices of $[P_n : C_6^2]$ as follows.

$f(u_i) = k + 24i - 22;\ 1 \leq i \leq n$
$f(v_i) = k + 24i - 24;\ 1 \leq i \leq n$
$f(v_i') = k + 24i - 19;\ 1 \leq i \leq n$
$f(w_i) = k + 24i - 18;\ 1 \leq i \leq n$
$f(w_i') = k + 24i - 16;\ 1 \leq i \leq n$
$f(x_i) = k + 24i - 13;\ 1 \leq i \leq n$
$f(y_i) = k + 24i - 11;\ 1 \leq i \leq n$
$f(y_i') = k + 24i - 7;\ 1 \leq i \leq n$
$f(z_i) = k + 24i - 8;\ 1 \leq i \leq n$
$f(z_i') = k + 24i - 2;\ 1 \leq i \leq n$
$f(u_i') = k + 24i - 5;\ 1 \leq i \leq n.$

Now the induced edge labels are as follows:

$f^*(a_i) = k + 24i - 23;\ 1 \leq i \leq n$
$f^*(a_i') = k + 24i - 20;\ 1 \leq i \leq n$
$f^*(b_i) = k + 24i - 21;\ 1 \leq i \leq n$
$f^*(b_i') = k + 24i - 17;\ 1 \leq i \leq n$
$f^*(c_i) = k + 24i - 15;\ 1 \leq i \leq n$
$f^*(c_i') = k + 24i - 14;\ 1 \leq i \leq n$
$f^*(d_i) = k + 24i - 12;\ 1 \leq i \leq n$
$f^*(d_i') = k + 24i - 10;\ 1 \leq i \leq n$
$f^*(e_i) = k + 24i - 9, 1 \leq i \leq n$
$f^*(e_i') = k + 24i - 4;\ 1 \leq i \leq n$
$f^*(e_i'') = k + 24i - 6;\ 1 \leq i \leq n$
$f^*(e_i''') = k + 24i - 3;\ 1 \leq i \leq n$
$f^*(e_i'^{v}) = k + 24i - 1;\ 1 \leq i \leq n - 1$

Here p = 11n , q = 13n-1 , p+q = 24n-1.
Clearly ,

$$f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, \dots, k+24n-2\}.$$

Hence, **f** is a **k** – Super mean labeling and hence $[P_n : C_6^2]$ is a **k** – Super mean graph.

Example 2.5:
1 – Super mean labeling of $[P_3 : C_6^2]$ is shown in figure 2.5

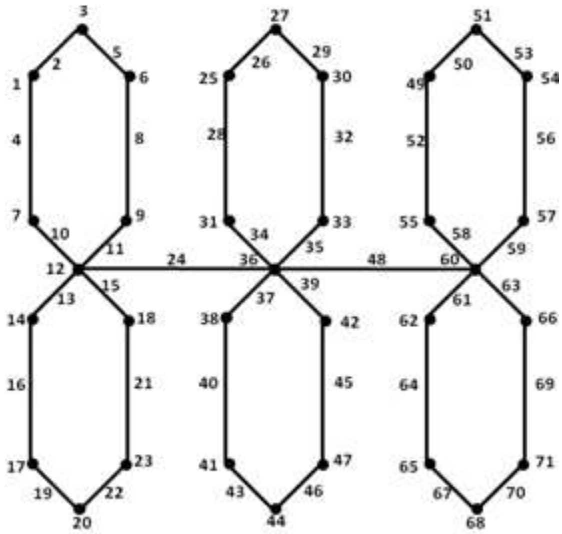

Figure 2.5: 1 – SML of $[P_3 : C_6^2]$

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