Some Five Dimensional Cylindrically Symmetric Interior Solutions for Perfect Fluid in *f(R)* Theory of Gravity

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Abstract: ***In this paper, we obtained some five dimensional cylindrically symmetric interior solutions for perfect fluid in f(R) theory of gravity. Some solutions are obtained by considering three assumptions and discussed them by using isotropy of pressure and subluminal sound speed as physical acceptability conditions in f(R) theory of gravity. Here we obtained some physically acceptable solutions. We explore these five dimensional solutions in terms of functions of Ricci scalar, energy density and pressure.***

Keywords: Cylindrical Symmetry, Interior solutions, Perfect fluid, Physical acceptability, *f(R)* gravity

1. Introduction

General theory of relativity is the successful theory which explain gravitational phenomenon but modification of GR is necessary to explain the problems regarding dark matter, dark energy and accelerated expansion of the universe. The accelerated expansion of the universe is due to components called dark matter and dark energy. Recent data of CMBR and Supernova surveys confirmed that energy of the universe is a combination of dark energy (76%), dark matter (20%) and ordinary baryonic matter (4%). Recent observations [3-6] indicate that dark energy and dark matter are unknown form of energy which is responsible for cosmic accelerating expansion of the universe. However, study of exact nature of dark energy is still challenging in the field of cosmology. Einstein tried to resolve such problems by introducing cosmological constant Λ in his structure but later he considered as biggest mistake of his life. There are several difficulties to obtain solutions of such problems in GR. To explain the issue like accelerating expansion of the universe and dark energy, there is a need to modify the general theory of relativity.

It is proved that the modification of general relativity is a effective way to describe the nature of dark energy and expansion of the universe. Several modifications of GR have been made like Brans-Dicke theory, Scalar tensor theory of gravitation, Self-Creation theory, f(R, T) theory of gravity, f(T) theory, *f(R)* theory of gravity. Among various modified theory of gravity *f(R)* theory is the most appropriate and suitable due to its huge application in the field of cosmology. The problem of dark matter, dark energy and also unification of early time inflation and late-time acceleration are addressed in *f(R)* theory of gravity.

Nojiri et. al. [7] studied the problem of dark energy and dark matter in *f(R)* theory of gravity. Also *f(R)* theory clears the pictures of universe about accelerating expansion [8-9]. Singularity is the major issue in GR and by using the higher order curvature term, there does not exist singularity in *f(R)* theory of gravity [10]. *f(R)* actions were first introduced by Weyl and Edington[11, 12] In the context of non-singular oscillating cosmologies, Buchdahl[13]studied these actions rigorously. Using *f(R)* theory of gravity Multamaki and Vilja[14, 15] investigated the static spherically symmetric vacuum solutions and non-vacuum solutions by taking fluid respectively. Capozziello et al.[16]used Noether Symmetry approach to study spherically symmetric solutions in *f(R)* theory of gravity. Hollenstein and Lobo [17] studied the exact solutions of statics spherically symmetric space time in *f(R)* gravity coupled to non-linear hydrodynamics. Shojai Ali et. al. [18] discussed some static spherically symmetric interior solutions of *f(R)* gravity.

Some solutions of the field equations are obtained by using cylindrical symmetric metric in *f(R)* theory of gravity. In metric *f(R)* theory of gravity Azadi et. al. [19] analysed cylindrically symmetric vacuum solutions. Momeni D. and H. Gholizade [20] extended this work and studied static plane symmetric vacuum solutions using the assumption of constant scalar curvature which may be zero or non-zero. By using power law *f(R)* cosmological model, capozziello et. al. [21], discussed the dark energy and dust matter phases. Sharif M et. al. [22] generated energy distribution and non-vacuum cylindrically symmetric solutions by using assumption of constant curvature. M. Farasat Shamir [23] used the assumption of constant and non-constant curvature solutions and obtained dust static cylindrically symmetric solutions in *f(R)* gravity. Sharif and Arif [24] investigated the static cylindrically symmetric interior solutions in metric *f(R)* gravity. M.T. Rincon-Ramires et. al. [25] studied the cylindrically symmetric solutions in metric *f(R)* gravity with constant R. Sharif M and Z. Yousaf [26] formed cylindrically thin-shell wormholes in *f(R)* gravity.

Kaluza and Klein [27, 28] tried to unify gravity with electromagnetic interaction by introducing an extra dimension. The study of higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution. There is nothing in the field equations of relativity which restrict to four dimensions only. The higher dimension space-time have long been a subject of discussion due to fact that our senses perceive only three dimension of space and one dimension of time. The advances of super string theory in 10-D indicate that the higher dimensionality of space is required for interaction over the distance $r << 10^{-16}$ cm. The study of higher dimensional space-time is important because of the under lying idea that the cosmos at early stage of evolution might have had a higher dimensional era.

To explore the knowledge of universe many researchers inspired to enter into the field of higher dimensional theory. Weinberg[29] studied the unification of fundamental forces with gravity which reveals that the space-time should be different from four dimensions. Wesson [30, 31] and D.R. K. Reddy [32] have studied several aspects of five dimensional space-time in variable mass theory and bi-metric theory of relativity respectively. Lorentz and Petzold [33]Ibanez and Verdaguer [34], A. Pradhan et al.[35], S. D. Katore et al.[36] and Adhav et. al. [37] have also studied the higher dimensional cosmological models in general relativity and in other modified theories of gravitation. Recently, Ladke, L. S. et. al. [38] studied higher dimensional plane symmetric solutions in *f(R)* theory of gravitation. Shamir and Jhangeer [39] investigated the static plane symmetric vacuum solutions in *f*(*R*) theory of gravity for (*n+1*) dimensional space-time. Sheykhi, Ahmad [40] analised higher-dimensional charged *f (R)* black holes. Pandey, S. N., and A. M. Mishra [41] solution of an *f (R)* theory of gravitation in cylindrical symmetric godel space-time.

Delgaty, M. S. R. and Kayll Lake, [1] discussed physical acceptability of isolated, static, spherically symmetric, perfect fluid solutions of Einstein's equations satisfying physical acceptability criteria. M Sharif and Sadia Arif [2] extended this work to cylindrically symmetric space-time and constructed some static cylindrically symmetric interior solutions in the *f(R)* theory of gravity by using following two physical acceptability conditions.

1) Isotropy of pressure,
2) Subluminal sound speed, i.e. ${v_s}^2 < 1$.

In this paper, we extend this work to five dimensional space-time and obtained some five dimensional cylindrically symmetric interior solutions for perfect fluid in *f(R)* theory of gravity. These solutions are obtained by considering three assumptions and discussed by using above two physical acceptability conditions. We explore these five dimensional solutions in terms of functions of Ricci scalar, energy density and pressure.

2. $f(R)$ Theory of Gravity

The action for $f(R)$ theory of gravity are given by

$$S = \int\left(\frac{1}{16\pi G} f(R) + L_m\right)\sqrt{-g}\; d^5x, \quad (1)$$

Where L_m is the matter Lagrangian.

Now by varying the action S with respect to g_{ij}, we obtain the field equations in $f(R)$ theory of gravity as

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i \nabla_j F(R) + g_{ij}\Box F(R) = kT_{ij}, \text{ (i, j=1, 2, 3, 4, 5) } (2)$$

Where $F(R) = \frac{df(R)}{dR}$ and $\Box \equiv \nabla^i \nabla_i$,

∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.

Contracting the above field equations (2), we have

$$f(R) = \frac{2}{5}\left[-KT + 4\Box F(R) + F(R)R\right]. \quad (3)$$

Using equations (2) and (3), the field equations take the form

$$\frac{F(R)R_{ij} - \nabla_i \nabla_j F(R) - 8\pi T_{ij}}{g_{ij}} = \frac{1}{5}[F(R)R - \Box F(R) - KT]. \quad (4)$$

It follows that the equation (4) is not depend on the index i,

$$F(R)R_{ij} - \frac{1}{5}[F(R)R - \Box F(R)]\, g_{ij} - \nabla_i \nabla_j F(R) = K\left[T_{ij} - \frac{1}{5}Tg_{ij}\right] \quad (5)$$

3. Metric & the Field equations

The line element of cylindrical symmetric space-time

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2\left(d\theta^2 + \alpha^2 dz^2 + \beta^2 du^2\right) \quad (6)$$

Where A, B and C are functions of r and α, β has dimensions of $1/r$

The Ricci scalar in this case is

$$R = \frac{1}{X}\left[\left(\frac{\dot{A}}{2} - \frac{3A}{r}\right)\frac{\dot{X}}{X} - \frac{4A}{r^2} - \ddot{A}\right] \quad (7)$$

Where $X = AB$ and dot denotes derivative with respect to r .

The Stress energy tensor can be written in the simple form

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad (8)$$

Where $p = p(r)$ $\rho = \rho(r)$ and u_i is four velocity

The matter density is given by the scalar function ρ and Pressure p

Equation (5) leads to

$$-2r\ddot{F} + r\dot{F}\frac{\dot{X}}{X} - \frac{1}{2}\frac{F\dot{X}}{X} = \frac{krX}{A^2}(p + \rho) \quad (9)$$

$$\frac{1}{2}\frac{\ddot{A}}{A} - \frac{1}{5A}\left(\frac{\dot{X}}{X} - \frac{2\dot{F}}{F}\right)\left(\frac{2A}{r} - \dot{A}\right) - \frac{2}{r^2} = \frac{-KX(p + \rho)}{A^2 F} \quad (10)$$

By using conservation equation $T_{i\;;j}^{\;j} = 0$,

We get,

$$\frac{\dot{A}}{A} = \frac{-2\dot{p}}{\rho + p} \quad (11)$$

We are using three types of assumptions to solve the field equations

$X = X_0 r^a$, $F = F_0$

$X = X_0$, $F = F_0 r^b$

$X = X_0 r^a$, $F = F_0 r^b$

Where X_0, F_0, a & b are arbitrary constant

Solutions of Type I

Here we assume $X = X_0 r^a$, $F = F_0$ in Equations (9) and (10), we obtain

$$\frac{-a}{2r^2} = \frac{KX_0 r^a (p+\rho)}{A^2 F_0} \quad (12)$$

$$\frac{1}{2}\frac{\ddot{A}}{A} + \frac{2a}{5r^2} - \frac{1}{5}\frac{a}{r}\frac{\dot{A}}{A} - \frac{2}{r^2} = \frac{-KX_0 r^a (p+\rho)}{A^2 F_0} \quad (13)$$

Equating equation(12) and (13), we get

$$\frac{1}{2}\frac{\ddot{A}}{A} - \frac{1}{5}\frac{a}{r^2} - \frac{1}{5}\frac{a}{r}\frac{\dot{A}}{A} - \frac{2}{r^2} = 0 \quad (14)$$

Its general solution is

$A = A_0 r^b$, where A_0 is constant (15)

Solving equation (14) using (15), we get

$$b = (1+a) \pm \sqrt{a^2 + 3a + 3} \quad (16)$$

Usingthis value in equation (11), we get

$$(p+\rho) = \frac{-2\dot{p}r}{a} \quad (17)$$

Solving equation (17) & (12), we get

$$P = P_0 + P_1 r^{2b-a-2} \text{ Where } P_1 = \frac{1}{4}\frac{abA_0^{\ 2} F_0}{KX_0 (2b-a-2)} \quad (18)$$

Where P_0 is constant.

Substituting the value of equation (18) in equation (17), we get

$$\rho = -p_0 + \rho_1 r^{2b-a-2} \text{ Where } \rho_1 = \frac{-aA_0^{\ 2} F_0}{4KX_0}\left(\frac{5b-2a-4}{2b-a-2}\right) \quad (19)$$

Using equation (15) & (7), we get

$$R = R_1 r^{b-a-2} \text{ Where } R_1 = \frac{A_0}{2X_0}\left(ab - 6a - 8 - 2b^2 + 2b\right) \quad (20)$$

Table 1: Solutions for assumption $X = X_0 r^m$, $F = F_0$

Sr. No.	a	b	$A(r)$	$p(r)$	$\rho(r)$	$R(r)$	$f(R)$	v_s^2
I	1	−1	$A_0 r^1$	$p_0 - \frac{A_0^2 F_0}{4KX_0} r$	$-p_0 + \frac{3A_0^2 F_0}{4KX_0} r$	$\frac{-3A_0}{2X_0} r^0$	$\frac{-3F_0 A_0}{2X_0} r + f_0$	$\frac{-1}{3}$
II	0	−2	$A_0 r^0$	p_0	$-p_0$	$\frac{2A_0}{X_0}$	$\frac{-1F_0 A_0}{2X_0} + f_0$	0
III	−2	−2	$A_0 r^{-2}$	$p_0 - \frac{A_0^2 F_0}{4KX_0} r^{-4}$	$-p_0 + \frac{5A_0^2 F_0}{4KX_0} r^{-4}$	$\frac{-2A_0}{X_0} r^{-2}$	$\frac{-2F_0 A_0}{X_0} r^{-2} + f_0$	$\frac{-1}{5}$
IV	−1	−1	$A_0 r^{-1}$	$p_0 - \frac{A_0^2 F_0}{12KX_0} r^{-3}$	$-p_0 + \frac{7}{12} \frac{A_0^2 F_0}{KX_0} r^{-3}$	$\frac{-5}{2} \frac{A_0}{X_0} r^{-2}$	$\frac{-5}{2} \frac{A_0}{X_0} \frac{F_0}{r^2} + f_0$	$\frac{-1}{7}$
V	$-0 \cdot 6055$	2	$A_0 r^{-0.6055}$	$p_0 + \frac{0.0580 A_0^2 F_0}{KX_0 r^{5.211}}$	$-p_0 - \frac{1.0580 A_0^2 F_0}{KX_0} r^{-5.211}$	$-11.577 \frac{A_0}{X_0} r^{-4.6055}$	$-11.577 \frac{A_0}{X_0} r^{-4.6055} + f_0$	−0.0548
VI	−0.732	0	$A_0 r^{-0.732}$	p_0	$-p_0$	$-5.267 \frac{A_0}{X_0} r^{-2.732}$	$-5.267 \frac{A_0}{X_0} \frac{F_0}{r^{2.732}} + f_0$	0
VII	2.732	0	$A_0 r^{2.732}$	p_0	$-p_0$	$-8.728 \frac{A_0}{X_0} r^{0.732}$	$-8.728 \frac{A_0}{X_0} \frac{F_0}{r^{0.732}} + f_0$	0

In Table-I of Type-I assumption,

It is found that the solutions (I), (III), (IV) and (V) are not physically acceptable as the speed of sound is not subluminal due to negative squared sound speed.

In Solutions (II), (VI) and (VII) pressure and density are constant and hence not physically acceptable.

Solutions of Type II

Here we assume $X = X_0$, $F = F_0 r^b$ using this value in Equations (9) and (10), we get

$$\frac{-2b(b-1)}{r^2} = \frac{KX_0(p+\rho)}{A^2 F_0 r^b} \quad (21)$$

$$\frac{1}{2}\frac{\ddot{A}}{A} + \frac{1}{5A}\left(\frac{2b}{r}\right)\left(\frac{2A}{r} - \dot{A}\right) - \frac{2}{r^2} = \frac{-KX(p+\rho)}{A^2 F_0 r^b} \quad (22)$$

Equating equation (21) & (22), we get

$$\frac{1}{2}\frac{\ddot{A}}{A} + \frac{4b}{5r^2} - \frac{2}{5}\frac{\dot{A}}{A}\frac{b}{r} - \frac{2}{r^2} - \frac{2b(b-1)}{r^2} = 0 \quad (23)$$

Its general solution is

$A = A_0 r^a$, where A_0 is constant (24)

Using equation (24) in equation (21), we get

$$a = 1 + b \pm \sqrt{3b^2 - 4b + 3} \quad (25)$$

Using equation (24) in equation (11), we get

$$(p+\rho)=\frac{-2\dot{p}r}{a} \quad (26)$$

Using the value of $(p+\rho)$ in equation (21), we get

$$P=P_0+P_1r^{2a+b-2} \text{ Where } P_1=\frac{ab(b-1){A_0}^2F_0}{KX_0(2a+b-2)} \quad (27)$$

Where P_0 is constant.

From equation (26) and (27), we get

$$\rho=-P_0+\rho_1r^{2a+b-2} \quad (28)$$

Where $\rho_1=\dfrac{-b(b-1){A_0}^2F_0(5a+2b-4)r^{2a+b-2}}{KX_0(2a+b-2)}$ (29)

Put the value of equation (24) in equation (7), we get

$$R=R_1r^{a-2} \quad (30)$$

Where $R_1=-\dfrac{A_0}{X_0}(a^2-a+4)$ (31)

Table 2: Solutions for assumption $X=X_0,\ F=F_0r^b$

Sr. No.	a	b	$A(r)$	$p(r)$	$\rho(r)$	$R(r)$	$f(R)$	${v_s}^2$
I	0.1771	$\frac{1}{2}$	$A_0r^{0.1771}$	$p_0+\frac{0.03826{A_0}^2F_0}{KX_0}r^{-1.1}$	$-p_0+\frac{0.4568{A_0}^2F_0}{KX_0}r^{-1.1}$	$\frac{-3.85A_0}{X_0}r^{-1.8}$	$-1.4390F_0\left(\frac{A_0}{X_0}\right)^{0.27}R^{0.7}+f_0$	0.0837
II	0.3038	$\frac{3}{5}$	$A_0r^{0.303}$	$p_0+\frac{0.9201{A_0}^2F_0}{KX_0}r^{-0.792}$	$-p_0+\frac{0.3879{A_0}^2F_0}{KX_0}r^{-0.7}$	$\frac{-3.7882A_0}{X_0}r^{-1.69}$	$-1.6477F_0\left(\frac{A_0}{X_0}\right)^{0.375}R^{0.}+f_0$	0.23715
III	0.40	$\frac{2}{5}$	$A_0r^{0.40}$	$p_0+\frac{0.004491{A_0}^2F_0}{KX_0}r^{-1.}$	$-p_0+\frac{0.4755{A_0}^2F_0}{KX_0}r^{-1.54}$	$\frac{-3.971A_0}{X_0}r^{-1.9711}$	$-1.3230F_0\left(\frac{A_0}{X_0}\right)^{0.2030}R^{0.797}+f_0$	0.00944
IV	–0.7320	0	$A_0r^{-0.7320}$	p_0	$-p_0$	$\frac{-5.267A_0}{X_0}r^{-2.732}$	$-5.267F_0\left(\frac{-6A_0}{X_0}\right)r^{-2.732}+f_0$	0
V	0.586	1	$A_0r^{0.586}$	p_0	$-p_0$	$\frac{-3.757A_0}{X_0}r^{-1.414}$	$-2.5499F_0\left(\frac{-4A_0}{X_0}\right)R^{0.292}+f_0$	0
VI	2.732	0	$A_0r^{2.732}$	p_0	$-p_0$	$\frac{-8.731A_0}{X_0}r^{0.732}$	$-8.732F_0\left(\frac{A_0}{X_0}\right)r^{0.732}+f_0$	0

From Table-II of Type-II assumption, we observed that

Solutions (I), (II) & (III) are physically acceptable as they satisfy the pressure and density decreases with increasing of r and $0<v_s^2<1$.

& Solutions (IV), (V), (VI) are not physically acceptable as pressure and density are constant

Solutions of Type III

Here we assume $X = X_0 r^a$, $F = F_0 r^b$ for both for X and F

Substituting these values in Eqs. (9) and (10), it gives

$$\frac{-2b(b-1)}{r^2} + \frac{ab}{r} - \frac{1}{2}\frac{a}{r} = \frac{KrX(p+\rho)}{A^2 F_0} \quad (32)$$

$$\frac{1}{2}\frac{\ddot{A}}{A} + \left(\frac{a}{r} + \frac{2b}{r}\right)\left(\frac{2}{5r} - \frac{\dot{A}}{5A}\right) - \frac{2}{r^2} = \frac{-KX(p+\rho)}{A^2 F_0} \quad (33)$$

Comparing equation (32) and (33), we get

$$\frac{1}{2}\frac{\ddot{A}}{A} - \frac{a}{10r^2} - \frac{1}{5}\frac{a}{r}\frac{\dot{A}}{A} - \frac{2}{r^2} = 0 \quad (34)$$

Assuming the general solution to equation (34) as

$$A = A_1 r^{c_1} + A_2 r^{c_2} \quad (35)$$

Where A_1, A_2 are constants.

Obtaining c_1 & c_2 using (35) &(34), we get

$$c_1 = a - 2b + \sqrt{2b^2 - 6b - ab + \frac{3}{2}a + 2(a-2b)^2} \quad (36)$$

$$c_2 = a - 2b - \sqrt{2b^2 - 6b - ab + \frac{3}{2}a + 2(a-2b)^2} \quad (37)$$

Putting the values of (35) in equation (11)

$$p + \rho = \frac{-2\dot{p}r}{c_1} \text{ and } p + \rho = \frac{-2\dot{p}r}{c_2} \quad (38)$$

Put the values of (36) in equation (32)

We get

$$p = p_0 + p_1 r^{2c_1+b-a-2} + p_2 r^{2c_2+b-a-2} \quad (39)$$

With

$$p_1 = \frac{\left[2b(b-1) - ab + \frac{1}{2}a\right] F_0 c_1 A_1^2}{2KX_0(2c_1 + b - a - 2)}, \quad (40)$$

(41)

$$p_2 = \frac{\left[2b(b-1) - ab + \frac{1}{2}a\right] F_0 c_2 A_2^2}{2KX_0(2c_2 + b - a - 2)}$$

Put the value of equation (39) in equation (38), we get

$$\rho = -p_0 + \rho_1 r^{2c_1+b-a-2} + \rho_2 r^{2c_2+b-a-2} \quad (42)$$

Where,

$$\rho_1 = -\frac{\left[2b(b-1) - ab + \frac{1}{2}a\right]}{2KX_0}\left(\frac{5c_1 + 2b - 2a - 4}{2c_1 + b - a - 2}\right)F_0 A_1^{\,2} \quad (43)$$

$$\rho_2 = -\frac{\left[2b(b-1) - ab + \frac{1}{2}a\right]}{2KX_0}\left(\frac{5c_2 + 2b - 2a - 4}{2c_2 + b - a - 2}\right)F_0 A_2^{\,2} \quad (44)$$

Using equation (35)in (7), we get

The corresponding Ricci scalar is

$$R = R_1 r^{c_1-a-2} + R_2 r^{c_2-a-2} \quad (45)$$

Where

$$R_1 = \frac{A_1}{X_0}\left[\frac{ac_1}{2} - 3a - 4 - c_1(c_1 - 1)\right] \quad (46)$$

&

$$R_2 = \frac{A_2}{X_0}\left[\frac{ac_2}{2} - 3a - 4 - c_2(c_2 - 1)\right] \quad (47)$$

Table 3

Sr. No.	a	b	c_1	$A(r)$	$p(r)$	$\rho(r)$	$R(r)$	$f(R)$	$v_s^{\,2}$
I	–1	0	0.2247	$A_1 r^{0.2247}$	$p_0 - \frac{0.10203 A_1^2 F_0}{KX_0 r^{0.5506}}$	$-p_0 + \frac{0.3979 A_1^2 F_0}{KX_0 r^{0.5506}}$	$\frac{-0.93814 A_1}{X_0 r^{-0.7753}}$	$\frac{-0.93814 A_1 F_0}{X_0 r^{0.7753}} + f_0$	0.2563
II	–2	1	–0.3944	$A_1 r^{-0.3944}$	$p_0 - \frac{0.9337 A_1^2 F_0}{KX_0 r^{-0.2112}}$	$-p_0 - \frac{0.06629 A_1^2 F_0}{KX_0 r^{-0.2112}}$	$\frac{1.8444 A_1}{X_0 r^{0.3944}}$	$4.723 F_0\left(\frac{A_1}{X_0}\right)^{2.5354} R^{1.535} + f_0$	14.08
III	–3	1	–0.3632	$A_1 r^{-0.3632}$	$p_0 - \frac{0.2138 A_1^2 F_0}{KX_0 r^{-1.2736}}$	$-p_0 - \frac{1.2861 A_1^2 F_0}{KX_0 r^{-1.2736}}$	$\frac{5.0494 A_1}{X_0 r^{-0.6368}}$	$0.07867 F_0\left(\frac{X_0}{A_0}\right)^{1.5703} R^{2.5703} + f_0$	0.166
IV	–1	2	–0.15232	$A_1 r^{-0.1523}$	$p_0 - \frac{0.16428 A_1^2 F_0}{KX_0 r^{-0.6953}}$	$-p_0 - \frac{1.3357 A_1^2 F_0}{KX_0 r^{-0.6953}}$	$\frac{-1.0993 A_1}{X_0 r^{1.1523}}$	$1.1787 F_0\left(\frac{A_1}{X_0}\right)^{1.7356} R^{-0.7356} + f_0$	0.123
V	$-\frac{1}{2}$	0	0.7247	$A_1 r^{0.7247}$	$p_0 - \frac{0.7927 A_1^2 F_0}{KX_0 r^{0.05051}}$	$-p_0 - \frac{1.5430 A_1^2 F_0}{KX_0 r^{0.05051}}$	$\frac{-2.4816 A_1}{X_0 r^{0.7753}}$	$\frac{-2.4816 A_1 F_0 R}{X_0 r^{0.7753}} + f_0$	–1.1623
VI	1	–1	–0.4504	$A_1 r^{-0.4504}$	$p_0 + \frac{0.1249 A_1^2 F_0}{KX_0 r^{0.9009}}$	$-p_0 - \frac{0.6248 A_1^2 F_0}{KX_0 r^{0.9009}}$	$\frac{-1.431 A_1}{X_0 r^{1.4504}}$	$-1.28032 F_0\left(\frac{A_1}{X_0}\right)^{0.6894} R^{0.3106} + f_0$	–0.2

Solutions for assumption $X = X_0 r^a F$, $F = F_0 r^b$ and for c_1

In Table-III of Type-III assumption, it is observed that

In Solutions (I), (III) and (IV), pressure and density are decreasing with increasing r and sound speed is subluminal i.e. $0 < {v_s}^2 < 1$ hence these solutions are physically acceptable.

Solutions (V) and (VI) are not physically acceptable due to negative squared sound speed.

In solution (V) subluminal velocity is not lies between 0 and 1hence this solution is not physically acceptable.

Table 4: Solutions for assumption $X = X_0 r^m$, $F = F_0 r^n$ & for value c_2

Sr. No.	a	b	c_2	$A(r)$	$p(r)$	$\rho(r)$	$R(r)$	$f(R)$	${v_s}^2$
I	$\frac{-1}{2}$	0	−1.7247	$A_2 r^{-1.7247}$	$p_0 - \frac{0.04357 {A_2}^2 F_0}{K X_0 r^{4.948}}$	$-p_0 + \frac{0.2935 {A_2}^2 F_0}{K X_0 r^{4.948}}$	$\frac{-6.768 A_2}{X_0 r^{3.2247}}$	$-6.7680 F_0 \left(\frac{A_2}{X_0}\right)^{0.17614} r^{-3.2247} + f_0$	−0.148
II	1	$\frac{1}{2}$	0.7071	$A_2 r^{-0.7071}$	$p_0 - \frac{0.04515 {A_2}^2 F_0}{K X_0 r^{3.9142}}$	$-p_0 - \frac{0.5451 {A_2}^2 F_0}{K X_0 r^{3.9142}}$	$\frac{-8.5605 A_2}{X_0 r^{3.7071}}$	$-1.3356 F_0 \left(\frac{A_2}{X_0}\right)^{0.1348} R^{0.8650} + f_0$	−0.8345
III	−1	$\frac{1}{2}$	−3.8094	$A_2 r^{-3.8094}$	$p_0 - \frac{0.1181 {A_2}^2 F_0}{K X_0 r^{9.1904}}$	$-p_0 - \frac{0.6181 {A_2}^2 F_0}{K X_0 r^{7.7188}}$	$\frac{-14.8570 A_2}{X_0 r^{4.6094}}$	$-1.2987 F_0 \left(\frac{A_2}{X_0}\right)^{0.0965} R^{0.9035} + f_0$	−0.1856
IV	0	−1	−1.7416	$A_2 r^{-1.7416}$	$p_0 - \frac{0.53726 {A_2}^2 F_0}{K X_0 r^{6.4832}}$	$-p_0 - \frac{4.5372 {A_2}^2 F_0}{K X_0 r^{6.4832}}$	$\frac{-8.7747 A_2}{X_0 r^{3.7416}}$	$-0.5596 F_0 \left(\frac{A_2}{X_0}\right)^{0.2672} R^{1.2672} + f_0$	−0.1183
V	2	−1	−1.5677	$A_2 r^{-1.5677}$	$p_0 - \frac{0.6747 {A_2}^2 F_0}{K X_0 r^{8.1354}}$	$-p_0 - \frac{7.6743 {A_2}^2 F_0}{K X_0 r^{8.1354}}$	$\frac{-15.593 A_2}{X_0 r^{5.5677}}$	$-0.6105 F_0 \left(\frac{X_0}{A_2}\right)^{0.1796} R^{1.1796} + f_0$	−0.08791
VI	$\frac{1}{4}$	0	−1.3112	$A_2 r^{-1.3112}$	$p_0 - \frac{0.01682 {A_2}^2 F_0}{K X_0 r^{4.8724}}$	$-p_0 - \frac{0.14181 {A_2}^2 F_0}{K X_0 r^{4.872}}$	$\frac{-7.9443 A_0}{X_0 r^{3.5612}}$	$-7.9443 F_0 \left(\frac{A_2}{X_0}\right) r^{-3.5612} + f_0$	−0.1186

In Table-IV of Type-III assumption, it is seen that all the solutions in the above table are having negative squared sound speed and hence no one is physically acceptable.

4. Discussion & Conclusion

In this paper, we study some five dimensional cylindrically symmetric interior solutions for perfect fluid in *f(R)* theory of gravity by considering some assumptions. To check physical acceptable solutions, physical acceptable criteria is applied and obtained some physical acceptable solutions .*f(R)* Theory is used to study the problems of dark matter and dark energy and accelerated expansion of the universe. Physically acceptable solutions obtained here in *f(R)* theory of gravity might be useful to solve the above problems.

First type of assumptions provide some solutions and it is observed that all the solutions do not satisfies the physically acceptability criteria and hence all solutions are not physically acceptable

Second type of assumption give some solutions given in Table-II, out of which solutions (I), (II) and (III) are physically acceptable where as solutions (IV), (V), (VI) are not physically acceptable as they do not satisfies the physical acceptability conditions.

In third type of assumptions, we construct two tables corresponding to parameters c_1 and c_2

In table third, we obtain three solutions which are physically acceptable and other solutions are not physically acceptable. Solutions obtained in table-IV no one solution satisfies the physical acceptable criteria hence all solutions are not physically acceptable. In this way, we obtained six physically acceptable solutions.

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