**International Journal of Scientific Engineering and Research (IJSER)**
**ISSN (Online): 2347-3878**
**Index Copernicus Value (2015): 56.67 | Impact Factor (2017): 5.156**

# Integral Representations of Analytical Functions Associated with Conic Regions

**D S Raju**

Associate Professor and Head, Department of Mathematics, NIE Institute of Technology, Mysuru - 570 018, India

**Abstract:** ***In this paper, we define generalized subclasses of k-uniformaly Janowski starlike and k-uniformaly Janowski convex functions associated with m-symmetric points. Interesting integral representations for functions belonging to this class are investigated.***

**2000 Mathematics Subject Classification,** 30C45

**Keywords:** Univalent functions, Janowski star like functions, Janowski convex functions and integral representations

## 1. Introduction

Let A denote the class of functions $f$ of the form

$$f(z)=z+\sum_{m=2}^{\infty} a_m z^m \qquad (1.1)$$

which are analytic in the open unit disk U = {z : |z| <1}. Furthermore S rep- resents class of all functions in A which are univalent in U Generalizing the well known class of star like functions Sakaguchi [6] introduced a class $S_s^*$of functions star like with respect to symmetric points, consisting of functions $f$∈S satisfying the inequality

$$\mathcal{R}\left(\frac{zf'(z)}{f(z)-f(-z)}\right) > 0, \ z \in U. \qquad (1.2)$$

Analogously Das and Singh [5] in 1977 extend the results of Sakaguchi to other class in U namely convex functions with respect to symmetric points. Let $C_s$ denote the class of convex functions with respect to symmetric points and satisfying the condition

$$\mathrm{R}\left(\frac{(zf'(z))'}{f;(z)-f'(-z)}\right) > 0, \qquad z \in U$$

We note that Cs and Ss∗are connected by the well-known Alexader relation. i.e., f ∈C s if and only if $zf'(z) \in S_s^*$.Chand and Singh [11] introduced class $S_s^*$of functions starlike with respect to m-symmetric points, which consists of functions f∈ S, satisfying the inequality

$$\mathcal{R}\left(\frac{zf'(z)}{f_m(z)}\right) > 0, \ z \in U. \qquad (1.3)$$

where

$$f_m(z) = \frac{1}{m}\sum_{\mu=0}^{m-1} \varepsilon^{-\mu} f(\varepsilon^{\mu} z), (\varepsilon^{\mu} = 1, \ m \in N) \ (1.4)$$

From (1.4) we can write

$$f_m(z)=\frac{1}{m}\sum_{\mu=0}^{m-1} \varepsilon^{-\mu} f(\varepsilon^{\mu} z)=\frac{1}{m}\sum_{\mu=0}^{m-1} \varepsilon^{-\mu}\left(\varepsilon^{\mu} z+\sum_{m=2}^{\infty} a_m (\varepsilon^{\mu} z)^m\right)$$

$$=z+\sum_{n=2}^{\infty} a_n b_n z^n \qquad (1.5)$$

Where

$$b_n = \frac{1}{m}\sum_{\mu=0}^{m-1} \varepsilon^{(n-1)\mu} = \begin{cases} 1, & n = lm+1 \\ 0, & n \neq lm+1, \end{cases} \qquad (1.6)$$

Where $l, m \in N, \ \ n \geq 2, \ \ \varepsilon^m = 1.$

The following equations follow directly from the above definition [1],

$$f_m(\varepsilon^{\mu} z)= \varepsilon^{\mu} f_m(z) \qquad (1.7)$$

$$f'_m(\varepsilon^{\mu} z)= f_m(z)=\frac{1}{m}\sum_{\mu=0}^{m-1} f'(\varepsilon^{\mu} z), \quad z \in U \ (1.8)$$

The convolution or Hadamard product of two power series $f(z)=z+\sum_{m=2}^{\infty} a_m z^m$ and $g(z)=z+\sum_{m=2}^{\infty} b_m z^m$

is defined as the power series $(f*g)(z)$

$$=z+\sum_{m=2}^{\infty} a_m b_m z^m.$$

Now we define the concept of subordination.

**Definition 1.1.** [8] *Let* Ω *be the set of analytic functions* $\omega$ *such that* $\omega(0) = 0$, $|\omega(z)| <1$, z ∈ U. *For any two analytic functions* $f$ *and* $g$ *defined in the unit disc* U, *we say that* $f$ *is subordinate to* $g$ *in* U *if there exists a function* $\omega \in \Omega$ *such that*

$$f(z) = g(\omega(z)) \qquad (1.9)$$
$$(z \in U).$$

*If* $f$ *is subordinate to* $g$ *on* U *we denote this by* $f \prec g$.

**Definition 1.2.** *Let* P *denote the class of analytic functions of the form*

$p(z) = 1 + \sum_{m=1}^{\infty} p_m z^m$, *defined in* U *satisfying* $p(0) = 1$ *and* R{$p(z)$} >0 (z ∈ U).

The class P can be completely characterized in terms of subordination. In fact the linear fractional transformation

$$\omega = g(z) = \frac{1+z}{1-z} \quad (z \in U)$$

maps the disc Uunivalently onto the right half plane R{$\omega$} >0and we have $g(0)$=1. It follows from the definition that a function $p$ is in P if and only if

$p(z)\prec \frac{1+z}{1-z}$. Hence any function p∈ $P$ has the representation [9],

$$p(z) = \frac{1+\omega(z)}{1-\omega(z)} \qquad (\omega \in \Omega, \ z \in U).$$

This representation for functions with positive real part in terms of analytic functions defined in U satisfying the conditions of Schwarz lemma [9] motivated Janowski[10] to define a new classP(A, B).


Licensed Under Creative Commons Attribution CC BY

**Definition 1.3.** [10] *Let* $P(A,B)(-1 \le B < A \le 1)$, *denote the class of analytic functions p defined in* U *with the representation*

$$p(z) = \frac{1+A\omega(z)}{1-B\omega(z)} \quad (\omega \in \Omega,\ z \in U).$$

Further $P(1,-1) = P$.

Kanas and Wisniowska [3, 4] introduced and studied the class *k - UCV* of *k*-uniformly convex functions and the corresponding class *k-ST* of *k*-star like functions. These classes were defined subject to the conic region $\Omega_k$, $k \ge 0$ given by

(1.10) $$\Omega_k = \{u + iv : u > k\sqrt{(u-1)^2 + v^2}\}.$$

This domain represents the right half plane for $k = 0$, hyperbola for $0 < k < 1$, a parabola for $k = 1$ and ellipse for $k > 1$.

They also extended this domain to $\Omega_{k,\sigma}$ defined by

$$\Omega_{k,\sigma} = (1-\sigma)\Omega_k + \sigma \ (0 \le \sigma < 1).$$

The function $p_{k,\sigma}$, with $p_{k,\sigma}(0) = 1$, $p'_{k,\sigma} > 0$ plays the role of extremal for the conic domain $\Omega_{k,\sigma}$ and is given by

$$p_{k,\sigma} = \begin{cases} \frac{1+(1-2\sigma)z}{1-z}, & k = 0. \\ 1 + \frac{2(1-\sigma)}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, & k = 1. \\ 1 + \frac{2(1-\sigma)}{1-k^2}\sinh^2\left[\left(\frac{2}{\pi}\arccos k\right)\operatorname{arctanh}\sqrt{z}\right], & 0 < k < 1. \\ 1 + \frac{(1-\sigma)}{k^2-1}\sin\left[\frac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}}\frac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}dx\right] + \frac{1}{k^2-1}, & k > 1. \end{cases} \quad (1.11)$$

Where $u(z) = \frac{z-\sqrt{t}}{1-\sqrt{tx}}$, $t \in (0,1)$, $z \in u$ and t is chosen such that $k = \cosh\left(\frac{\pi R'(t)}{4R(t)}\right)$, with $R(t)$ is the Legendre's complete elliptic integral of the first kind and $R'(t)$ is complementary integral $R(t)$.

Let $P_{k,\sigma}$ denote the class of all functions *p* which are analytic in U with $p(0) = 1$ and $p(z) \prec p_{k,\sigma}(z)$ for $z \in U$. Clearly, it can be seen that $P_{k,\sigma}(z) \subset P$.

**Definition 1.4.** [5] *A function p is said to be in k -* P[*A*, *B*], *if and only if,*

$$p(z) \prec \frac{(A+1)p_k(z)-(A-1)}{(B+1)p_k(z)-(B-1)}, \quad k \ge 0. \quad (1.12)$$

where $p_k(z)$ is defined by (1.11) and $-1 \le B < A \le 1$.

Geometrically, the function $p \in k$ - P[*A*, *B*] takes all values from the domain $\Omega_k[A,B]$, $-1 \le B < A \le 1$, $k \ge 0$ which is defined as

$$\Omega_k[A,B] = \left\{\omega : R\left(\frac{(B+1)\omega(z)-(A-1)}{(B+1)\omega(z)-(A+1)}\right) > k\left|\frac{(B-1)\omega(z)-(A-1)}{(B+1)\omega(z)-(A+1)} - 1\right|\right\} \quad (1.13)$$

Or equivalently, $\Omega_k[A,B] = \{u + iv : [(B^2-1)(u^2+v^2) - 2(AB-1)u + (A^2-1)]^2 > k^2[(-2(B+1)(u^2+v^2) + 2(A+B+2)u - 2(A+1))^2 + 4(A-B)^2v^2]\}$.

The domain $\Omega_k[A,B]$ retains the conic domain $\Omega_k$ inside the circular region defined by $\Omega[A,B]$. The impact of $\Omega[A,B]$ on the conic domain $\Omega_k$ changes the original shape of the conic regions. The ends of hyperbola and parabola get closer to each other but never meet anywhere and the ellipse gets the oval shape. When $A \to 1$, $B \to -1$, the radius of the circular disk defined by $\Omega[A,B]$ tends to infinity, consequently the arms of hyperbola and parabola expand and the oval turns into ellipse. We see that $\Omega_k[1,-1] = \Omega_k$, the conic domain defined by Kanas and Wisniowska [3,4].

Motivated essentially by the recent paper of Noor and Malik [7], we define some classes of analytic functions associated with conic domains as follows:

**Definition 1.5.** *A function p is said to be in the class k -* P[*A*, *B*, *σ*], *if and only if*

$$p(z) \prec \frac{(A+1)p_{k,\sigma}(z)-(A-1)}{(B+1)p_{k,\sigma}(z)-(B-1)}, \quad k \ge 0,$$

where $p_{k,\sigma}(z)$ is defined by (1.11), $0 \le \sigma < 1$ and $-1 \le B < A \le 1$.

**Definition 1.6.**. A function f ∈A is said to be in the class k-$ST_s^{(m)}[A,B,\sigma]$, $-1 \le B < A \le 1$, $k \ge 0$, if and only if

$$R\left(\frac{(B-1)\frac{zf'(z)}{f_m(z)}-(A-1)}{(B+1)\frac{zf'(z)}{f_m(z)}-(A+1)}\right) > k\left|\frac{(B-1)\frac{zf'(z)}{f_m(z)}-(A-1)}{(B+1)\frac{zf'(z)}{f_m(z)}-(A+1)} - 1\right|$$

Or equivalently

$$\frac{zf'(z)}{f_m(z)} \in k - P[A,B,\sigma] \quad (1.14)$$

where $f_m$ is defined by ( 1:5).

**Definition 1.7.**. A function f ∈A is said to be in the class k-$UCV_s^{(m)}[A,B,\sigma]$, $-1 \le B < A \le 1$, $k \ge 0$, if and only if

$$R\left(\frac{(B-1)\left(\frac{(zf'(z))'}{f_m(z)}\right)-(A-1)}{(B+1)\left(\frac{(zf'(z))'}{f_m(z)}\right)-(A+1)}\right) > k\left|\frac{(B-1)\left(\frac{(zf'(z))'}{f_m(z)}\right)-(A-1)}{(B+1)\left(\frac{(zf'(z))'}{f_m(z)}\right)-(A+1)} - 1\right|$$

Or equivalently

$$\left(\frac{(zf'(z))'}{f'_m(z)}\right) \in k - P[A,B,\sigma] \quad (1.14)$$

where $f_m$ is defined by ( 1:5).

As special cases, we get the results of Nasir Khan [12], Noor and Malik [7], Kwon and Sim [1], Kanas and Wisniowska [3], Shams, Kulkarni and Jahangiri [2], Janowski [13], Kanas and Wisniowska [4].

## 2. Integral Representation

We give two meaningful conclusions about the classes k-$ST_s^{(m)}[A,B,\sigma]$ and k−$UCV_s^{(m)}[A,B,\sigma]$.


Licensed Under Creative Commons Attribution CC BY

**Theorem 2.1.** Let $f \in$k-$ST_s^{(m)}[A,B,\sigma]$. Then $f_m \in k-ST[A,B,\sigma] \subseteq k-ST \subseteq S$.

**Proof:** If $\in$k-$ST_s^{(m)}[A,B,\sigma]$, we can obtain

$$\frac{zf'(z)}{f_m(z)} \prec \frac{(A+1)p_{k,\sigma}(z)-(A-1)}{(B+1)p_{k,\sigma}(z)-(B-1)}, \qquad (z \in U) \ (2.1)$$

Substituting z by $\epsilon^{\mu}$z respectively ($\mu$= 0,1, 2,…m -1).

$$\frac{\epsilon^{\mu} zf'(\epsilon^{\mu} z)}{f_m(\epsilon^{\mu} z)} \prec \frac{(A+1)p_{k,\sigma}(\epsilon^{\mu} z)-(A-1)}{(B+1)p_{k,\sigma}(\epsilon^{\mu} z)-(B-1)} \prec \frac{(A+1)p_{k,\sigma}(z)-(A-1)}{(B+1)p_{k,\sigma}(z)-(B-1)}, \qquad (z \in U) \qquad (2.2)$$

By definition of $f_m$ and $\epsilon = \exp\left(\frac{2\pi i}{m}\right)$, we know that $\epsilon^{-\mu} f_m(\epsilon^{\mu} z) = f_m(z)$. Then equation (2.2), becomes

$$\frac{zf_m'(z)}{f_m(z)} \prec \frac{(A+1)p_{k,\sigma}(z)-(A-1)}{(B+1)p_{k,\sigma}(z)-(B-1)}, \qquad (z \in U) \qquad (2.3)$$

Let ($\mu$ = 0, 1, 2, ....*m-1* ) in ( 2.3) , respectively and sum them to get

$$\frac{zf'(z)}{f_m(z)} \prec \frac{1}{m}\sum_{\mu=0}^{m-1} \frac{zf'(\epsilon^{\mu} z)}{f_m(z)} \prec \frac{(A+1)p_{k,\sigma}(z)-(A-1)}{(B+1)p_{k,\sigma}(z)-(B-1)}, (z \in U)$$

Thus $f_m \in k-ST[A,B,\sigma] \subseteq S$.

For $\sigma = 0$ we obtain the following Corollary, which is comparable to the result obtained by Nasir Khan [12].

**Corollary 2.2:** Let $f \in$k-$ST_s^{(m)}[A,B]$. Then $f_m \in k-ST[A,B] \subseteq k-ST \subseteq S$.

Putting $\sigma$ = 0 and $k$ = 0 we can obtain below result, which is comparable to the result obtained by Kwon and Sim[1].

**Corollary 2.3:** Let $\in ST_s^{(m)}[A,B]$. Then $f_m \in ST[A,B] \subseteq k-ST \subseteq S$.

Similar to the proof of Theorem 2.1, we can prove the following Theorem.

**Theorem 2.4.** Let $f \in$k-$UCV_s^{(m)}[A,B,\sigma]$. Then $f_m \in k-UCV[A,B,\sigma] \subseteq S$.

Now we give the integral representations of the functions belonging to the classes
k-$ST_s^{(m)}[A,B,\sigma]$ and k-$UCV_s^{(m)}[A,B,\sigma]$.

**Theorem 2.5.** Let $f \in$k-$ST_s^{(m)}[A,B,\sigma]$. Then

$$f_m(z) = z.\left(\exp(A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\epsilon^{\mu} z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right), \ (2.4)$$

Where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

**Proof:** Let $f \in ST_s^{(m)}[A,B,\sigma]$, from definition of the subordination, we can have

$$\frac{zf'(z)}{f_m(z)} \prec \frac{(A+1)p_{k,\sigma}(\omega(z))-(A-1)}{(B+1)p_{k,\sigma}((\omega(z))-(B-1)}, \qquad (z \in U) \ (2.5)$$

Where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

Substituting z by $\epsilon^{\mu} z$ respectively ($\mu = 0, 1, 2, \ m-1$), we have

$$\frac{\varepsilon^{\mu} zf'(\varepsilon^{\mu} z)}{f_m(\varepsilon^{\mu} z)} = \frac{(A+1)p_{k,\sigma}(\omega(\varepsilon^{\mu} z))-(A-1)}{(B+1)p_{k,\sigma}((\omega(\varepsilon^{\mu} z))-(B-1)}, \qquad (z \in U) \quad (2.6)$$

Using the equations (1.7) and (1.8) we have

$$\frac{zf'(z)}{f_m(z)} = \frac{1}{m}\sum_{\mu=0}^{m-1}\frac{(A+1)p_{k,\sigma}(\omega(\varepsilon^{\mu} z))-(A-1)}{(B+1)p_{k,\sigma}((\omega(\varepsilon^{\mu} z))-(B-1)}, (z \in U) (2.7)$$

Or equivalently

$$\frac{f_m'(z)}{f_m(z)} - \frac{1}{z} = \frac{1}{m}\sum_{\mu=0}^{m-1}\frac{(A-B)(p_{k,\sigma}(\omega(\varepsilon^{\mu} z))-1)}{z((B+1)p_{k,\sigma}((\omega(\varepsilon^{\mu} z))-(B-1))}, \qquad (z \in U) \quad (2.8)$$

Integrating equality (2.8), we have

$$\log\frac{f_m(z)}{z} = (A-B)\frac{1}{m}\left(\sum_{\mu=0}^{m-1}\int_0^z \frac{p_{k,\sigma}(\omega(\varepsilon^{\mu}\zeta)-1)}{\zeta(B+1)p_{k,\sigma}(\omega(\varepsilon^{\mu}\zeta))-(B-1)}d\zeta\right), (2.9)$$

$$=(A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu} z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt$$

Therefore arranging equality (2.9) for $f_m(z)$ we can obtain

$$f_m(z) = z.\left(\exp\,(A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu} z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right)$$

and so the proof of the Theorem 2.5 is complete.

For $\sigma = 0$ we obtain the following Corollary, which is comparable to the result obtained by Nasir Khan [12].

**Corollary2.6.**Let $f \in ST_s^{(m)}[A,B]$. Then

$$f_m(z) = z.\left(\exp(A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu} z}\frac{p_k(\omega(t)-1)}{t(B+1)p_k(\omega(t))-(B-1)}dt\right) (2.10)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

Putting m=1 in Theorem 2.5 we obtain the following corollary.

**Corollary 2.7.** Let $f \in$k-ST$[A,B,\sigma]$. Then

$$f(z) = z.\left(\exp\,(A-B)\int_0^z \frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right), (2.11)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

Putting $\sigma$ = 0 and k=0 in Theorem 2.5 we obtain the following corollary. Which is comparable to the result obtained by Kwon and Sim[1].

**Corollary2.8.** Let $f \in$k-$ST_s^{(m)}[A,B]$. Then

$$f_m(z) = z.\left(\exp(A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu} z}\frac{\omega(t)}{t(1+B\omega(t))}dt\right), (2.12)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.


Licensed Under Creative Commons Attribution CC BY

Putting $\sigma = 0$, m=1, A=1 and B=-1 in Theorem 2.5, we obtain the following corollary.

**Corollary2.9.** Let $f \in$k-ST. Then

$$f(z) = z.\left(\exp \int_0^z p_k(\omega(t) - 1)dt\right), \quad (2.13)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and$|\omega(z)| < 1$.

Similar to the proof of Theorem 2.5, we can prove the following Theorem.

**Theorem 2.10.** Let $f \in$k-$UCV_s^{(m)}[A,B,\sigma]$. Then

$$f_m(z) = \int_0^z \exp\left((A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu}z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right)d\zeta \quad (2.14)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

**Theorem 2.11.** Let $f \in$k-$ST_s^{(m)}[A,B,\sigma]$. Then

$$f(z) = \int_0^z \exp\left((A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu}z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right)\left(\frac{(A+1)p_{k,\sigma}(\omega(\zeta))-(A-1)}{(B+1)p_{k,\sigma}((\omega(\zeta))-(B-1)}\right)d\zeta \quad (2.15)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

**Proof:** Let $f \in$k-$ST_s^{(m)}[A,B,\sigma]$. Then from equalities (2.14) and (2.15) we have

$$f(z)=\left(\frac{f_m(z)}{z}\right)\left(\frac{(A+1)p_{k,\sigma}(\omega(\zeta))-(A-1)}{(B+1)p_{k,\sigma}((\omega(\zeta))-(B-1)}\right)$$

$$f'(z) = \exp\left((A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu}z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right)\left(\frac{(A+1)p_{k,\sigma}(\omega(\zeta))-(A-1)}{(B+1)p_{k,\sigma}((\omega(\zeta))-(B-1)}\right) \quad (2.16)$$

Integrating the equation (2.16), we have

$$f(z) = \int_0^z \exp\left((A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu}z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right)\left(\frac{(A+1)p_{k,\sigma}(\omega(\zeta))-(A-1)}{(B+1)p_{k,\sigma}((\omega(\zeta))-(B-1)}\right)d\zeta \quad (2.17)$$

And so the proof of Theorem 2.11 is completed.

For $\sigma = 0$ we get Nasir Khan [12] result. Which reads as follows.

**Corollary2.12.** Let $f \in$k-$ST_s^{(m)}[A,B]$. Then

$f(z) =$
$\int_0^z \exp\Big((A -$
$B)1m\mu=0m-10\varepsilon\mu zpk(\omega t-1)tB+1pk(\omega t)-(B-1)dtA+1pk(\omega\zeta)-A-1B+1pk(\omega(\zeta)-B-1$d$\zeta$(2.18)

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

Putting $\sigma = 0$ and k=0 in Theorem 2.11 we obtain the following corollary.

Which is comparable to the result obtained by Kwon and Sim[1].

**Corollary2.13.** Let $f \in$k-$ST_s^{(m)}[A,B]$. Then

$$f(z) = \int_0^z \exp\left((A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu}z}\frac{\omega(t)}{t(1+B\omega(t))}dt\right)\left(\frac{(1+A\omega(\zeta))}{(1+B(\omega(\zeta))}\right)d\zeta \quad (2.19)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

**Theorem 2.14.** Let $f \in$k-$USV_s^{(m)}[A,B,\sigma]$. Then

$$f_m(z) = \int_0^z\left[\frac{1}{\zeta}\int_0^{\zeta} exp\left((A-B)\frac{1}{m}\sum_{\mu=0}^{m-1}\int_0^{\varepsilon^{\mu}z}\frac{p_{k,\sigma}(\omega(t)-1)}{t(B+1)p_{k,\sigma}(\omega(t))-(B-1)}dt\right)\right]\left(\frac{(A+1)p_{k,\sigma}(\omega(\zeta))-(A-1)}{(B+1)p_{k,\sigma}((\omega(\zeta))-(B-1)}\right)d\zeta d\zeta, \quad (2.20)$$

where $\omega(z)$ analytic function U, with $\omega(0) = 0$ and $|\omega(z)| < 1$.

## References

[1] O. Kwon and Y. Sim, A certain subclass of Janowski type functions associated with k-symmetricpoints,Commun.KoreanMath.Soc.28(2013),143-154.
[2] S.Shams, S. R. Kulkarni and J. M. Jahangiri, Classes of uniformly starlike and convex functions, Int.J.Math.Math.Sci.55(2004),2959-2961.
[3] S. Kanas and A. Wisniowska, Conic domains and starlike functions, *Rev. Roumaine Math. Pures Appl.*, 45 (2000),647-657.
[4] S. Kanas and A. Wisniowska, Conic regions and k-uniform convexity, *J. Comput. Appl.Math.*,105 (1999), 327-336.
[5] R.N.DasandP.Singh,OnsubclassesofSchlichtmappings.Ind.J.Pure.App.Math.8(1977), 864-872.
[6] K.Sakaguchi,Onacertainunivalentmapping,J.Math.Soc.Japan.11(1959),72-75.
[7] K. I. Noor and S. N. Malik, On coefficient inequalities of functions associated with conic domains, *Comput. Math. Appl.*, 62 (2011),2209-2217.
[8] P.L. Duren, Univalent functions, Springer - Verlag, Newyork(1983).
[9] Z. Nehari, Conformal mapping, *Mc Graw Hill Book Company, Inc.*,(1952).
[10] W. Janowski, Extremal problems for a family of functions with positive real part and for some related families, *Annales Polonici Mathematici*, 23(1970), 159 −177.
[11] R. Chand and P. Singh. On certain Schlicht


Licensed Under Creative Commons Attribution CC BY

mapping, Ind. J. Pure App. Math. 10(1979), 1167-1174.

[12] Nasir Khan, Certain Classes of Analytic Functions Associated with Conic Domains, J. of New Theory, 24(2018),20-34.

[13] W. Janowski, Some external problem for certain families of analytic functions, J. Ann. Polon. Math, 28(1973),298-326.


Licensed Under Creative Commons Attribution CC BY