

# Reliability Estimate of Technical Systems Containing composite Materials through Analysis of the Concurrent Risks

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**Abstract:** *The technical systems containing composite materials are risk te-chnical systems (RTS). The investigation of use reliability of RTS is correlation with analyze of the exist mutual concurrent risks. This analyze demand collecting of the statistical information (sample of data) for this conduct. In row cases howe-ver this sample is determinated censorship (unfully). The general situation with random censorship (unfully) extract is characterized for RTS. This is provoked by fact that observed at technical usage article, could be a failure by two or more independence reason. Whit other words, because of article have take  $r$  indepen-dence motive provoke failures (concurrent risks), but is display one of them. At this the presence of aunfully sample could be get, while same at different kind failures. Therefore, obseved time a failure of article by  $j$ -th kind ( $j = 1, \dots, r$ ) is determi-nated by realization of convention random quantity (by condition, at isn't step a fai-lure at other kind).*

**Keywords:** reliability estimate; technical systems; composite materials

## 1. Introduction

In this study it is offered an estimate of the probability, article serviceability in moment  $t_i$ , to failure in interval of time  $(t_i, t_{i+1})$  by  $i$ -th cause under action of the catastrophic functional failure provoking by a risks. In reference [6] this probability is called gross probability. Let in first moment of time  $t_0$  under observer is find technical systems from  $n_0$  on regeneration article has probability for reliability work  $P_{0i}$ . For  $i$ -th time interval  $(t_{i-1}, t_i)$  with continuation  $\Delta t = t_i - t_{i-1}$ , every a article, non failure earlier, can be faultless by  $j$ -th reason with probability  $Q_{ij}(\Delta t)$  and keep the serviceability with **probability reliability work** (PRW)  $P_{BPi}(\Delta t)$  de-terminate by equation:

$$P_{RWi}(\Delta t) = 1 - \sum_{j=1}^r Q_{ij}(\Delta t). \quad (1)$$

## 2. Presentation

In every interval  $\Delta t$  advance  $d_{ij}$  failures with general case  $d_i = \sum_{j=1}^r d_{ij}$ , like as that number of articles  $n_{j+1}$ ,

remains serviceability in start of the  $(i+1)$ -th interval is

$$n_{i+1} = n_0 - \sum_{j=1}^i d_j \text{ or } n_i = \sum_{j=1}^r d_{ij} + n_{i+1}.$$

Consequently, at  $n_i > 0$  articles, serviceability by the moment  $t_i$  is characterized: "Conditional distribution with the number of failures in interval  $\Delta t$  and number of articles  $n_{i+1}$ , serviceability in the end of interval, is polinomiality" [1].

The distribution of probability is according[4]:

$$P\{d_{ij}, n_{i+1} | n_i\} = \left[ \frac{n_i!}{n_{i+1}! \cdot \prod_{j=1}^r d_{ij}} \right] \cdot P_i^{n_{i+1}} \cdot \prod_{i=1}^r Q_{ij}^{d_{ij}}. \quad (2)$$

The estimate of probability for failure  $\hat{Q}_{ij}(\Delta t)$  and PRW  $\hat{P}_{BPi}(\Delta t)$ , are de-terminated by the results for observer of articles working in time intervals  $W$  of number ( $W$  is determinate by condition  $n_w > 0$  and  $n_{w+1} \geq 0$ ), getting with **functional of reality like**  $L$  has been case:

$$L = \prod_{i=1}^W P\{d_{ij}, n_{i+1} | n_i\}. \quad (3)$$

The estimate of this value is:

$$\hat{P}_{BPi}(\Delta t) = n_{i+1}/n_i; \quad \hat{Q}_{ij}(\Delta t) = d_{ij}/n_i. \quad (4)$$

The mathematical expectation of the value  $\hat{Q}_{ij}(\Delta t)$  and  $\hat{P}_{RW,i}(\Delta t)$ , could be obtained with calculation:

$$M(\hat{Q}_{ij}) = M(d_{ij}/n_i) = M\left(\frac{1}{n_i} \cdot M(d_{ij}|n_i)\right) - M\left(\frac{1}{n_i} \cdot Q_{ij} \cdot n_i\right) = Q_{ij};$$

$$M(\hat{P}_{RWi}) = P_{RWi} \quad (5)$$

By conditional calculations, it could be obtained the dispersion of the value:

$$M(\hat{Q}_{ij}^2) = M(d_{ij}^2/n_i^2) = M\left(\frac{1}{n_i^2} \cdot M(d_{ij}^2/n_i)\right) = M\left\{\frac{1}{n_i^2} \cdot [n_i^2 \cdot Q_{ij}^2 + n_i \cdot Q_{ij} \cdot (1 - Q_{ij})]\right\} =$$

$$= Q_{ij}^2 + M\left(\frac{1}{n_i}\right) \cdot Q_{ij} \cdot (1 - Q_{ij}). \quad (6)$$

From equation (6) with (5) it is obtained the following new equation by the kind:

$$D(\hat{Q}_{ij}) = M\left(\frac{1}{n_i}\right) \cdot Q_{ij} \cdot (1 - Q_{ij}). \quad (7)$$

Like this it is determined the dispersion for value of  $\hat{P}_{RWi}$

$$D(\hat{P}_{RWi}) = M\left(\frac{1}{n_i}\right) \cdot P_{RWi} \cdot (1 - P_{RWi}). \quad (8)$$

The expression  $M(1/n_i)$  in equations (7) and (8) insists on approximation. For doing this approximation it is obtained the following estimation. Let examine the equation:

$$n_i = M(n_i) \cdot \{1 + [n_i - M(n_i)]/M(n_i)\} = M(n_i) \cdot (1 + \Delta) \quad (9)$$

correctly with execution of a condition  $M(\Delta) = 0$ . From equality (9) follows after condition transformation new equation:

$$M\left(\frac{1}{n_i}\right) = \frac{1}{M(n_i)} \cdot M\left(\frac{1}{1 + \Delta}\right). \quad (10)$$

Let use identity  $\frac{1}{1 + \Delta} \equiv 1 - \Delta + \Delta^2/(1 + \Delta)$  [4], and to put in equation (10). After transformation of the equation (10) it is obtained the inequality:

$$M\left(\frac{1}{n_i}\right) = \frac{1}{M(n_i)} \cdot \{1 + M[\Delta^2/(1 + \Delta)]\} \geq \frac{1}{M(n_i)} \quad (11)$$

in which random quantity  $\Delta^2$  and  $1 + \Delta$  take only one negative and positive value.

The expand of the function  $1/n_i$  in Taylor's series in neighborhood of a point  $M(n_i)$  with accuracy by first member and respective mathematical expectation, determines the equation:

$$M\left(\frac{1}{n_i}\right) = \frac{1}{M(n_i)} \cdot \{1 + D(n_i)/M^2(n_i) + v[D(n_i)/M^2(n_i)]\} \quad (12)$$

where second member in brackets is relative dispersion of  $n_i$ , and third member is quantity of the higher ordinate.

The formulas for mathematical expectation and dispersion, have kind according [2, 3]:

$$M(n_i) = n_0 \cdot \prod_{j=1}^{i-1} P_{RWj} = n_0 \cdot P_{oi},$$

$$D(n_i) = n_0 \cdot P_{oi} \cdot (1 - P_{oi}),$$

following new equation:

$$D(n_i)/M^2(n_i) = (1 - P_{oi})/(n_0 \cdot P_{oi}). \quad (13)$$

With high values of  $n_0$ , equation (13) will be striving by a value zero. Because of this, the equations (11) and (13) the mathematical dependence can be shown by the next expression:

$$\lim_{n \rightarrow \infty} M(1/n_i) = 1/M(n_i),$$

following, that the equation (7) and (8) could be approximated with expressions:

$$D(\hat{Q}_{ij}) \approx (1/n_0 \cdot P_{oi}) \cdot Q_{ij} \cdot (1 - Q_{ij}), \quad (14)$$

$$D(\hat{P}_{RWi}) \approx (1/n_0 \cdot P_{oi}) \cdot P_{oi} \cdot (1 - P_{oi}). \quad (15)$$

For determination of the efficient of the estimate  $\hat{Q}_{ij}$  it is necessary to finding the low limit of the their dispersion. By the inequality of the Rao-Kramer following, that

$$D(\hat{Q}_{ij}) \geq -[M(\partial^2 \ln L / \partial Q_{ij}^2)]^{-1}.$$

Simultaneously by the equations (14) and (15) it is determined after respective transformation equality:

$$-[M(\partial^2 \ln L / \partial Q_{ij}^2)] = n_0 \cdot P_{oi} \cdot (P_{oi} + Q_{ij}) / P_{oi} \cdot Q_{ij}. \quad (16)$$

By the equation (16) following, that *low limit*  $V_{LL}(\hat{Q}_{ij})$  of an estimate of the dispersion is equal to:

$$V_{LL}(\hat{Q}_{ij}) = P_{RWi} \cdot Q_{ij} / \left[ n_0 \cdot P_{oi} \cdot (P_{RWi} + Q_{ij}) \right] \quad (17)$$

After confront of the equation (14) and (17) it is obtained as a final result, the equation:

$$V(\hat{Q}_{ij}) - V_{LL}(\hat{Q}_{ij}) = (Q_{ij}/n_0 \cdot P_{oi}) \cdot (1/(P_{RWi} + Q_{ij}) - 1) \geq 0 \quad (18)$$

From the equation (18) follows, that the estimate  $\hat{Q}_{ij}$  is effective only in case, when it is possible the condition  $P_{RW,i} + Q_{ij} = 1$ , i.e. when actions only one risk. At the better values of  $n_0$  the left side of the equation (18) is striving to zero. Consequently, the estimate  $\hat{Q}_{ij}$  is the asymptotically effective.

### 3. Conclusion

- 1) In the present study it is determinate analysis for use reliability of the RTS containing composite materials.
- 2) This analysis is correlation with estimation of the existence mutual con-current risks.
- 3) It is proven, that the estimate of the probability reliability work  $\hat{P}_{RW,i}$  and probability of failure  $\hat{Q}_{ij}$  are only estimates, on removing by the real value.

### References

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