

MTSF, Profit Analysis of a Two Units Cold Standby System with Inspection for High Pressure Horizontal Die Casting Machines

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Abstract: *The Present Paper is an attempt to find out reliability of a two units standby system in which one unit is in and another is cold standby unit. In case of any failure the standby unit switch instantaneously and come in to operative mode. The repairman first repairs the unit and takes an inspection for insuring the perfection of the repair. Die casting machine is a mechanical used for assembling of parts or some specified work in industry. It works automatically without any rest and more efficiently in comparison of any other means of working. These machines are used in most of the industry applications like car part assembling, bicycle part assembling etc. Here reliability is discussed in term of MTSF, Profit analysis and Availability of the system with different parameters like failure rate of the priority unit, secondary unit etc.*

Keywords: Die Casting Machines, Reliability, Two Unit Standby System

1. Introduction

In every industry there is a steadily rising demand for high quality die castings. To meet this demand Buhler entered into technical collaboration for the manufacture of cold chamber horizontal high pressure die casting machine in the year 1969. Today there are many Buhler Pressure die casting Machines working in variety of industries, contributing to the achievement of better quality standards in pressure die castings. In this machine use alloys are Aluminium, Magnesium, Zinc, copper base alloys.

“Pressure die casting” is a process in which molten metal is forced under high pressure into a cavity in a metal die in a fraction of a second and then allowed to solidify. When the casting is solidified, the die is opened and the diecasting is removed (ejected). The process is rapid and allows complex shapes to be cast as almost finished parts, and many thousands of casting can be produced from a set of dies without significant changes in casting dimensions.

There are basic diecasting processes the hot chamber and cold chamber processes. The hot chamber process is used for diecasting metals that melt at lower temperature, such as zinc and lead. The cold chamber process is used for metals that melt at higher temperature, such as aluminium, magnesium, and brass. But the horizontal high pressure die casting machine is cold chamber.

In fact a large number of researchers in the field of reliability modelling including Nakagawa and Osaki (1975), Goel and Agnihotri (1992), Mokaddis and Labib (1997), Tuteja (2001), Sharma and Taneja (2011), Kumar and Bhatia (2011), Kumar and Rani (2013), V. Kumar and P. Bhatia with S. Ahmed (2014), etc. analyzed the one/ two unit redundant systems. Kumar and Vashistha (2001) explained the two unit redundant system with degradation and replacement of the faulty. Kumar and Bhatia (2011) discussed the behaviour of the single unit centrifuge system

considering the concepts of inspections, halt of system, degradation, minor/major faults, neglected faults, online/offline maintenances, repairs of the faults. Kumar and Rani (2013) explained the cost benefit analysis for a redundant system. V. Kumar and P. Bhatia with S. Ahmed (2014) explained in very detail the profit analysis for a two unit standby centrifuge system having a single repairman. Jain (2014) explained the different failures in a repairable redundant system.

Pressure Die casting Machine is a mechanical machine used for assembling of parts or some specified work in industry. The die casting machine are two types; 1. cold chamber 2. hot chamber but we use horizontal high pressure cold chamber machines. It works automatically without any rest and more efficiently in comparison of any other means of working. These machines are used in most of the industry applications like car part assembling, bicycle part assembling etc. Now a day, robotic machine works for replacement of a large group of people, the diecasting machine works very fast in comparison of people's work. Also the diecasting machine is less costly in comparison of work done cost by people. The work will not be affected just like human problems. The high pressure die casting machines are used more frequently in industries so there is need of analysis of robotic machine to improve the reliability of the system. As the system may be in failed state due to some problem in the machine, one solution is to use the standby unit. In this paper we explain this concept in detail.

As far as we concern with the research work on reliability, none of the researchers have analyzed such a two-unit cold standby system considering such a situation with occurrence of various faults. To fill up this gap, the Chapter discussed an analysis of a stochastic model for two-unit system with all possibilities of occurrence of failure and measures the affects in terms of MTSF and Profitability.

We are taking two high pressure die casting machines case, initially the primary unit is working and the second unit is at cold standby. We are discussing all the possibilities and all the states possible for the system. There are some states which are called up states and some down states. The up states are those states in which at least one machine is in operative mode either primary unit or secondary unit. The down states are states in which both machines are not in operative mode. So this paper chapter deals with a two unit redundant system in which one unit is operative and the other is cold standby, i.e. the standby unit is used to replace the operative failed unit instantaneously. Before the repair of the operative failed unit, it is sent for fault detection which takes a random amount of time. After the repair, the unit goes into inspection for deciding whether the repair is perfect or not. If the repair is found to be imperfect the unit is sent for post repair. The several reliability characteristics of interest such as mean sojourn time, MTSF, expected up time, expected down time of the system and busy period of the repairman are obtained using Markov processes and regenerative point technique. The graphical study technique is used to tell about reliability and profitability of the system.

2. Assumption

The main characteristics related to reliability are:

- The system will work as new after repairing
- The switching of machines is very fast as if system is not in stop state
- The repair team is totally watching the system carefully and will listen instantaneously
- The failure time distribution are exponential
- We consider stochastic modelling, so all possible random variables are considered, so all possible states of the system are considered
- The error in the machine will be siren basis or self announcing

3. Notations

X_0 : Priority unit is operative

Y_0 : Non priority unit is operative

Y_{ws} : Non priority unit is in standby mode

X_{PR} : Priority unit is sent for post repair

X_{FR} : Priority unit is failed and sent for repair

X_i : Priority unit is under inspection

Y_{FW} : Non priority is failed and waiting for repair

Y_{FR} : Non priority is failed and under repair

$H_1(.)$: c d f(Cumulative distribution function) of repair time of priority unit

$H_2(.)$: c d f(Cumulative distribution function) of post repair time of priority unit

$H_3(.)$: cdf of repair time of non priority unit

μ : Constant rate for inspection

α_1 : Parameter of failure time distribution for main unit

α_2 : Parameter of failure time distribution for secondary unit

p_1 : Probability of repaired unit in working unit

p_2 : Probability of repaired unit require post repair

r : Repair rate of priority unit

$*$: Symbol for Laplace transformation $F^*(s) = \int_0^\infty e^{-st} f(t) dt$

\sim : Symbol for Laplace Stieltjes transformation $F \sim(s) = \int_0^\infty e^{-st} dF(t)$

Ψ_i : Mean sojourn time in state S_i .

$M_i(t)$: Probability that the sojourns in state S_i upto time t .

$\Phi_i(t)$: where starting from up state S_0 cdf of time to the system.

\odot : Symbol for Laplace convolution

The states of the system

The different states of the system having all the possibilities either main unit is in operative state, failed state, repairing state, inspection state and similarly for second standby unit, the possibilities are in operative state, repairing state, inspection state, failed state, waiting state are taken into account.

$S_0 = [X_0, Y_{ws}]$; $S_1 = [X_{FR}, Y_0]$; $S_2 = [X_i, Y_0]$;

$S_3 = [X_{PR}, Y_0]$; $S_4 = [X_{FR}, Y_{FW}]$; $S_5 = [X_i, Y_{FW}]$;

$S_6 = [X_0, Y_{FR}]$; $S_7 = [X_{PR}, Y_{FW}]$;

The Model

The figure showing all the possible states of the system, some states are up states and some are down states. The states S_0, S_1, S_2, S_3, S_6 are up states and the states S_4, S_5, S_7 are down states. The all possibilities are shown in this figure:

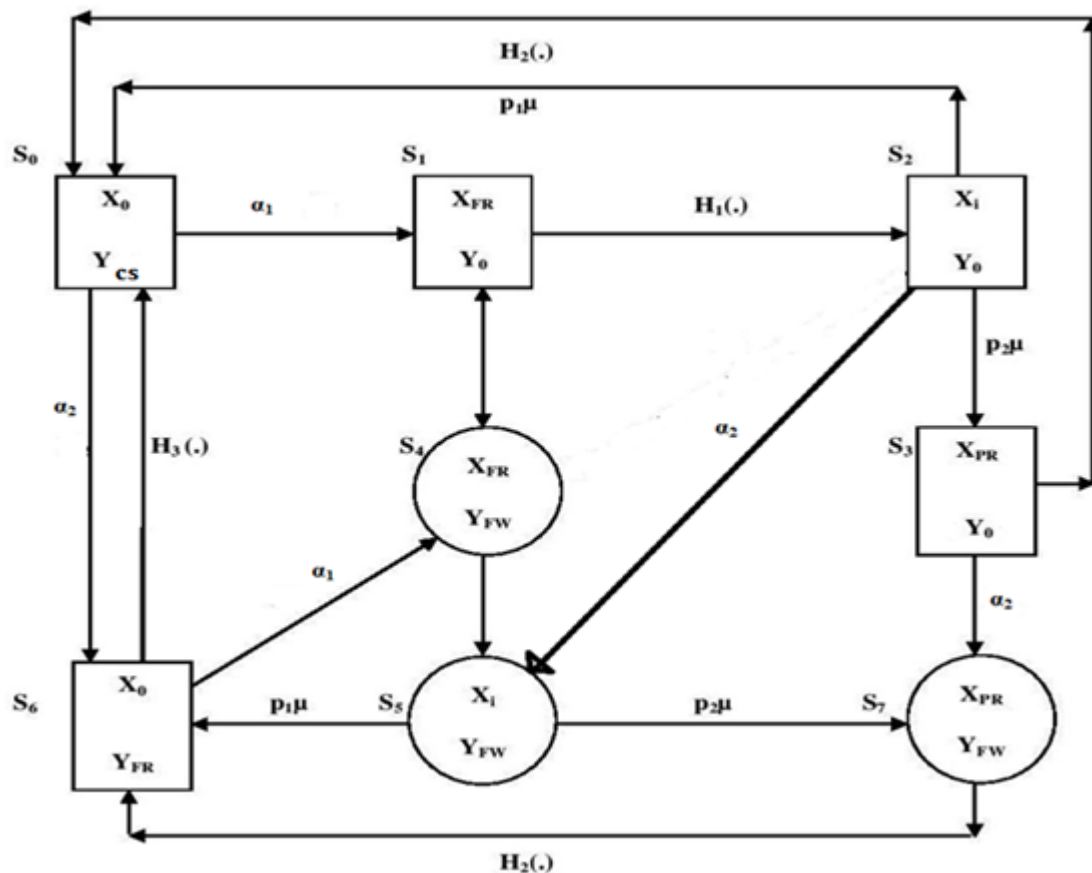
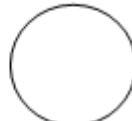


Figure 1: Transition of Different States for Two Units Standby System

Up State

Failed State

**Transition Probabilities and Mean Sojourn Times**

Here $Q_{ij}(t)$ denotes the cdf (cumulative distribution function) of transition time from state S_i to S_j in 0 to t . To determine the transition probabilities of states. Let T_0, T_1, T_2, \dots denotes the regenerative epochs. Then $\{X_n, T_n\}$ constitute a space E , set of regenerative states and $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i]$ is the semi Markov over E . The various transition probabilities are:

$$Q_{01}(t) = \alpha_1 \int_0^t e^{-(\alpha_1 + \alpha_2)t} dt \quad Q_{06}(t) = \alpha_2 \int_0^t e^{-(\alpha_1 + \alpha_2)t} dt$$

$$Q_{12}(t) = \int_0^t dH_1(t) e^{-\alpha_2 t} dt \quad Q_{20}(t) = p_1 \mu \int_0^t e^{-(\mu + \alpha_2)t} dt$$

$$Q_{15}^{(4)}(t) = \alpha_2 \int_0^t e^{-\alpha_2 t} \bar{H}_1(t) \int_u^t \frac{dH_1(t)}{\bar{H}_1(t)} = \frac{\alpha_2}{\alpha_2} \int_0^t dH_1(v) (1 - e^{-\alpha_2 t}) = \int_0^t dH_1(t) (1 - e^{-\alpha_2 t})$$

$$Q_{23}(t) = p_2 \mu \int_0^t e^{-(\mu + \alpha_2)t} dt \quad Q_{26}^{(5)}(t) = p_1 \mu \int_0^t e^{-(\mu t)} (1 - e^{-\alpha_2 t}) dt$$

$$Q_{27}^{(5)}(t) = p_2 \mu \int_0^t e^{-(\mu t)} (1 - e^{-\alpha_2 t}) dt \quad Q_{30}(t) = \int_0^t e^{-\alpha_2 t} dH_2(t)$$

$$Q_{36}^{(7)}(t) = \int_0^t (1 - e^{-\alpha_2 t}) dH_2(t) \quad Q_{45}(t) = \int_0^t dH_1(t)$$

$$Q_{56}(t) = p_1 \mu \int_0^t e^{-\mu t} dt \quad Q_{57}(t) = p_2 \mu \int_0^t e^{-\mu t} dt$$

$$Q_{60}(t) = \int_0^t e^{-\alpha_1 t} dH_3(t) \quad Q_{64}(t) = \int_0^t dH_3(t) (1 - e^{-\alpha_1 t})$$

$$Q_{76}(t) = \int_0^t dH_2(t)$$

Steady State Transition Probabilities

We generally take limit from 0 to t while calculating the cumulative distribution function (cdf) for the system. Here we are taking in steady state the limit of t tends to infinity. And then the transition probabilities are calculated.

$$P_{01} = \frac{\alpha_1}{\alpha_1 + \alpha_2} \quad P_{06} = \frac{\alpha_2}{\alpha_1 + \alpha_2}$$

$$P_{12} = H_1(\alpha_2) \quad P_{15}^{(4)} = 1 - H_1(\alpha_2)$$

$$P_{20} = p_1 \mu / (\mu + \alpha_2) \quad P_{23} = p_2 \mu / (\mu + \alpha_2)$$

$$P_{26}^{(5)} = p_1 \alpha_2 / (\mu + \alpha_2) \quad P_{27}^{(5)} = p_2 \alpha_2 / (\mu + \alpha_2)$$

$$P_{30} = H_2(\alpha_2) \quad P_{36}^{(7)} = 1 - H_2(\alpha_2)$$

$$P_{45} = 1 \quad P_{56} = p_1$$

$$P_{57} = p_2 \quad P_{60} = H_3(\alpha_1)$$

$$P_{64} = 1 - H_3(\alpha_1) \quad P_{76} = 1$$

By these transition probabilities, it can be verified that

$$P_{01} + P_{06} = 1; \quad P_{12} + P_{15} = 1; \quad P_{20} + P_{23} + P_{26} + P_{27} = 1;$$

$$P_{30} + P_{36} = 1; \quad P_{56} + P_{57} = 1; \quad P_{60} + P_{64} = 1;$$

$$P_{45} = 1; \quad P_{76} = 1$$

Mean Sojourn time

The mean sojourns time S_i denoted by Ψ_i which is the time spent in a particular state before going to another state. To obtain mean sojourn time Ψ_i , we observe the system work in the S_i (state) to any other state. The sojourn times are $\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5, \Psi_6, \Psi_7$ and they are calculated as:

$$\begin{aligned}\Psi_i &= E[T_i] = \int P(T_i > t) dt \\ \Psi_0 &= \int_0^\infty e^{-(\alpha_1 + \alpha_2)t} dt = 1/(\alpha_1 + \alpha_2) \quad \Psi_1 = \{1 - \tilde{H}_2(\alpha_2)\}/\alpha_2 \\ \Psi_2 &= \int_0^\infty e^{-(\mu + \alpha_2)t} dt = 1/(\mu + \alpha_2) \quad \Psi_3 = \{1 - \tilde{H}_2(\alpha_2)\}/\alpha_2 \\ \Psi_4 &= \int_0^\infty H_1 dt \quad \Psi_5 = \int_0^\infty e^{-\mu t} dt = 1/\mu \\ \Psi_6 &= \{1 - \tilde{H}_3(\alpha_1)\}/\alpha_1 \quad \Psi_7 = \int_0^\infty H_2 dt\end{aligned}$$

Mean Time to System Failure

Let random variables T_i denotes the time to system failure. $\phi_i(t)$ is the c.d.f of system failure for first time when $E_0 = E_i \in E$. The arguments of regenerative point processes used to obtain the expressions of $\phi_i(t)$. First we will determine $\phi_0(t)$ and another may similar.

$$\begin{aligned}\text{MTSF} = E(T) &= \frac{\Psi_0 + \Psi_1 p_{01} + \Psi_2 p_{01} p_{12} + \Psi_3 p_{01} p_{12} p_{23} + \Psi_0 p_{06}}{1 - p_{01} p_{12} (p_{20} + p_{23} p_{30}) - p_{06} p_{60}} \\ \text{Putting the values of } p_{ij} \text{'s and } \Psi_{ij} \text{'s in above equation} \\ &= \frac{\alpha_1 \alpha_2 (\mu + \alpha_2) + \alpha_1^2 (\mu + \alpha_2) + \alpha_1^2 \alpha_2 \tilde{H}_1(\alpha_2) + p_{23} \mu \alpha_1^2 \tilde{H}_1(\alpha_2) [1 - \tilde{H}_2(\alpha_1)] + \alpha_2^2 (\mu + \alpha_2) [1 - \tilde{H}_3(\alpha_1)]}{\alpha_1 \alpha_2 \{(\alpha_1 + \alpha_2)(\mu + \alpha_2) - \alpha_1 \tilde{H}_1(\alpha_2) [p_{1\mu} + p_{2\mu} \tilde{H}_2(\alpha_2)] - \alpha_2 (\mu + \alpha_2) \tilde{H}_3(\alpha_1)\}} \\ \text{Expected up time } A_0 &= N_1/D_1\end{aligned}$$

The steady state probabilities the system will up in the long run $= N_2/D_2$

Busy time for repair time $B_0 = N_3/D_3$

Expected number of visits by repairman $V_0 = N_4/D_4(0)$

Where $N_1(0) = \Psi_0 + \Psi_1 p_{01} + \Psi_2 p_{01} p_{12} + \Psi_3 p_{01} p_{12} p_{23} + \Psi_6 p_{06}$

$$\begin{aligned}D_1(0) &= 1 - p_{01} p_{12} (p_{20} + p_{23} p_{30}) - p_{06} p_{60} \\ N_2(0) &= [\Psi_0 + p_{01} \Psi_1 + p_{01} p_{12} (\Psi_2 + p_{23} \Psi_3)] C_1 \\ &\quad + p_{01} p_{12} (p_{23} p_{36}^{(7)} + p_{26}^{(5)} + p_{27}^{(5)} p_{76}) \\ &\quad + \Psi_6 + p_{01} p_{15}^{(4)} \Psi_6 + p_{06} \Psi_6\end{aligned}$$

$$\begin{aligned}D_2(0) &= C_1 - p_{01} p_{12} [p_{20} + p_{23} p_{30}] C_1 + (p_{23} p_{36}^{(7)} + p_{26}^{(5)} \\ &\quad + p_{27}^{(5)} p_{76}) p_{60} - p_{01} p_{15}^{(4)} (p_{56} \\ &\quad + p_{57} p_{76}) p_{60} - p_{06} p_{60}\end{aligned}$$

$$\begin{aligned}N_3(0) &= p_{01} [\Psi_1 + p_{12} (\Psi_2 + p_{23} \Psi_3 + p_{27}^{(5)} \Psi_7)] p_{60} \\ &\quad + p_{01} p_{12} (p_{23} p_{36}^{(7)} + p_{26}^{(5)} + p_{27}^{(5)} p_{76}) \\ &\quad + p_{06} (\Psi_4 p_{64} \\ &\quad + \Psi_5 p_{64} p_{45} + \Psi_6 + \Psi_7 p_{64} p_{45} p_{57}) \\ &\quad + [\Psi_4 p_{64} + \Psi_5 + \Psi_6 + \Psi_7 p_{57}]\end{aligned}$$

$$\begin{aligned}D_3(0) &= C_2 - p_{01} p_{12} [(p_{20} + p_{23} p_{30}) C_1 + (p_{23} p_{36}^{(7)} \\ &\quad + p_{26}^{(5)} + p_{27}^{(5)} p_{76}) p_{60}] - p_{01} p_{15}^{(4)} (p_{56} \\ &\quad + p_{57} p_{76}) p_{60} - p_{06} p_{60} \\ N_4(0) &= p_{60}\end{aligned}$$

$$\begin{aligned}D_4(0) &= (\Psi_0 + \Psi_1 p_{01}) p_{60} + (\Psi_2 + \Psi_3 p_{23}) p_{01} p_{12} p_{60} + \\ &\quad (\Psi_4 + \Psi_5 + \Psi_7 p_{57}) p_{64} + \Psi_6 [1 - p_{01} p_{12} (p_{20} + p_{23} p_{30})] + \\ &\quad \Psi_5 p_{01} p_{15}^{(4)} p_{60} + \Psi_7 \Psi_6 p_{01} p_{60} (p_{12} p_{27}^{(5)} + p_{57} p_{15}^{(4)})\end{aligned}$$

Profit Analysis

The profit analysis of the system can be carried out by considering the all the factors in time period $(0, t)$. Therefore, the expected profit of system is:

$P(t)$ = expected total revenue in $(0, t)$ - expected total expenditure in $(0, t)$

$$\begin{aligned}M_{ij} &= -Q_{ij}(0) = -\frac{d}{ds} \int_0^\infty e^{-st} dQ_{ij}(t) |_{s=0} \\ \sum_j m_{ij} &= \Psi_i, \text{ for different values of } i \text{ and } j\end{aligned}$$

m_{ij} is the mean elapsed time of the system in the state S_i to any other regenerative state S_j

4. Other Measures of System Effectiveness

The average first passage time to the failed state or expected life time of the system is known mean time to system failure (MTSF). By using the sojourn time and transition probability, the MTSF is given by the following relation:

The mean time to system failure (MTSF) and other parameters for the system

MTSF is the mean time between the failures of the system and it is also the time elapsed between the failed states of the system.

In steady state, expected no of profit per unit time

$$P = \lim_{t \rightarrow \infty} [P(t)/t] = \lim_{s \rightarrow 0} s^2 P^*(s)$$

$$P = H_0 A_0 - H_1 B_0 - H_2 V_0$$

H_0 = Revenue per unit for up state of system

H_1 = is the cost per unit time for which repair man is busy in repair of the failed unit.

H_2 = Cost per unit for repair

Graphical interpretations and conclusion: For more study of system behaviour, we plot MTSF, Availability, profit function with respect to failure rate of primary unit (α_1) and other values of repair rate of secondary unit (λ_1).

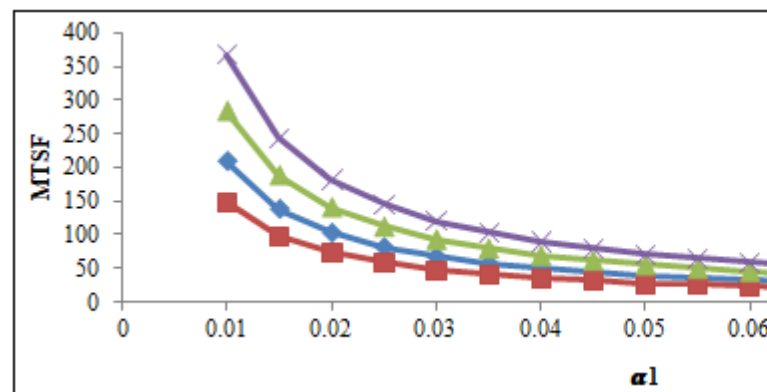


Figure 2: Shows the variations in MTSF in respect of failure rate (α_1) values (0.01 to 0.95) of primary unit and other values repair rate (λ_1) primary unit as 0.25, 0.45, 0.65, 0.85 and other parameters are fixed as $\lambda_2 = 0.55, \mu = 0.30, \alpha_2 = 0.80, \lambda_3 = 0.50$. It observed the graph that MTSF decrease with the increase in the failure parameter α_1 and increase with increase repair rate λ_1 .

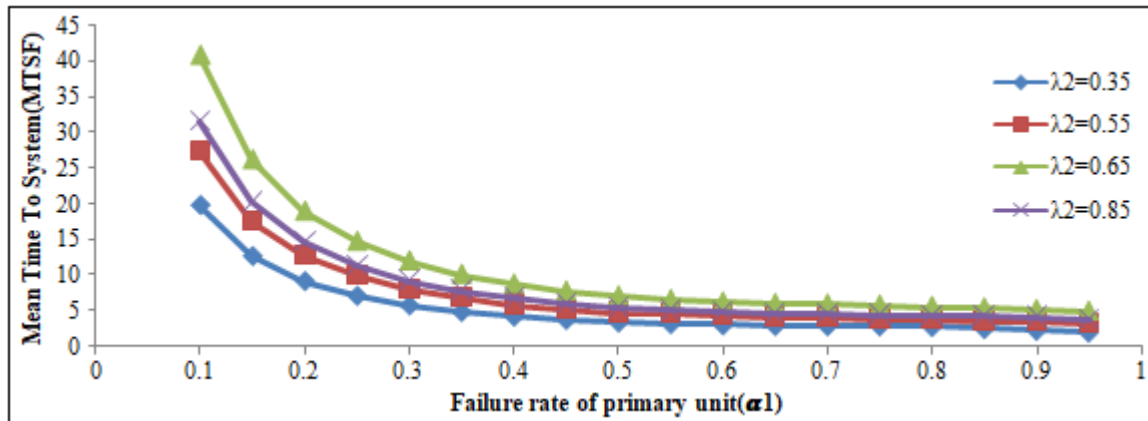


Figure 3: shows the variations in MTSF in respect of failure rate (α_1) values (0.01 to 0.95) of primary unit and other values repair rate (λ_2) non primary unit as 0.25, 0.45, 0.65, 0.85 and other parameters are fixed as $\lambda_2=0.55$, $\mu=0.30$, $\alpha_2=0.80$, $\lambda_3=0.50$. It is observed from the graph that MTSF decreases with the increase in the failure parameter α_1 and increases with an increase in the repair rate λ_1 .

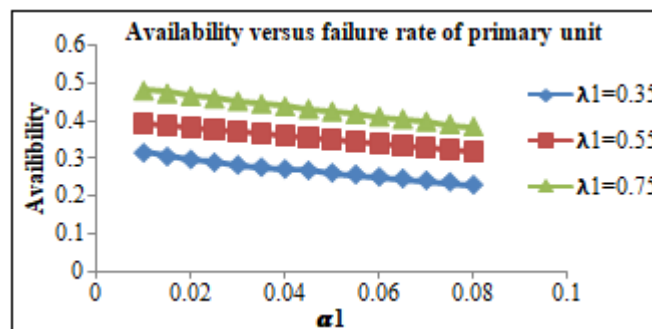


Figure 4: Shows the variations in Availability in respect of failure rate (α_1) values (0.01 to 0.95) of primary unit and other values repair rate (λ_1) primary unit as 0.35, 0.55, 0.75, and other parameters are fixed as $\lambda_2=0.55$, $\mu=0.30$, $\alpha_2=0.80$, $\lambda_3=0.50$. It is observed from the graph that Availability decreases with the increase in the failure parameter α_1 and increases with an increase in the repair rate λ_1 .

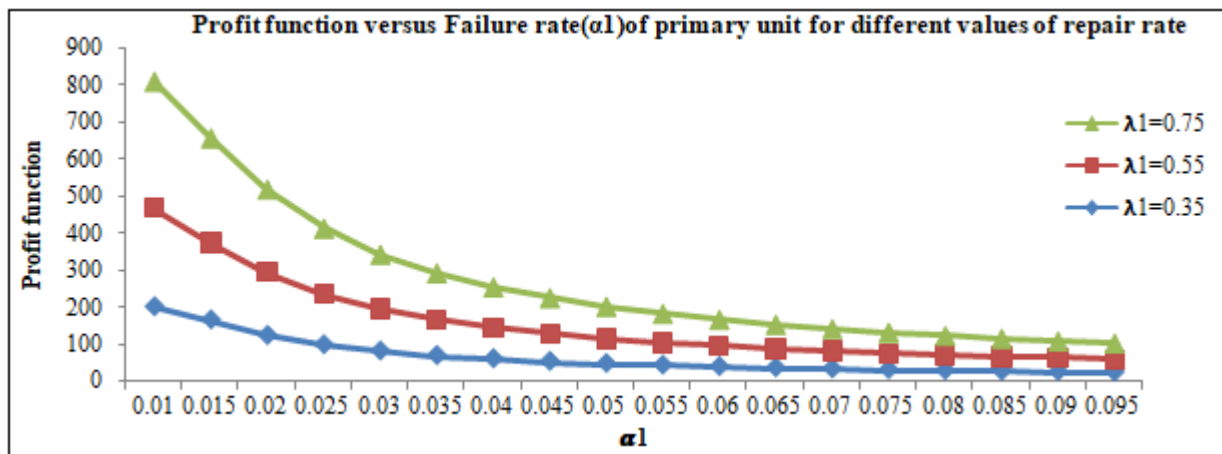


Figure 5: Shows the variations in Profit function in respect of failure rate (α_1) values (0.01 to 0.95) of primary unit and other values repair rate (λ_1) primary unit as 0.35, 0.55, 0.75, and other parameters are fixed as $\lambda_2=0.55$, $\mu=0.30$, $\alpha_2=0.80$, $\lambda_3=0.50$. It is observed from the graph that Profit function decreases with the increase in the failure parameter α_1 and increases with an increase in the repair rate λ_1 .

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