Maximum Depth Rules for Total Magnetic Field Anomalies at Low Latitude

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Abstract: ***The maximum depth rules have benn derived for total magnetic field anomalies at low latitude using homogeneous sphere as model. Three different rules were derived for both N-S and E-W profiles over the centre point of magnetic sphere. For the E-W traverse, the rules are: Inflexion Point Rule (IPR); Half-Width Rule (HWR); and Amplitude-Slope Rule (ASR) and for N-S traverse, the rules are:IPR, ASR and Amplitude-Distance Rule (ADR). The reliability of these rules was tested using a set of synthetic total magnetic field data generated over 10 linear spherical magnetic source bodies of 3 – 10 m depth range. The rules give very close values to the true depth of the magnetic source bodies with maximum variation of ±1 m. On E-W profile, the correlation coefficients between computed and the real depths are 0.95, 0.96 and 0.97 respectively for IPR, HWR and ASR. Also, on N-S profile, the correlation coeffecient between computed and the real depths are 0.96, 0.97 and 0.96 respectively for IPR, ADR and ASR. The maximum depth rules are adjudged reliable on the bases of their results.***

Keywords: maximum depth, inflexion point, half-width, amplitude, slope, sphere, magnetic anomaly

1. Introduction

Depth determination to magnetic source bodies is a major parameter in the interpretation of magnetic anomalies both in the mineral industry and petroleum industry. In the mineral industry depths to magnetic source bodies determine the depth of exploratory boreholes and in the petroleum industry depths to magnetic source bodies are used to approximate the depth to basement surface, which gives a measure of the volume of sediments available in a given basin. This is made possible because most basement rocks have more magnetite and are more magnetic than nearly all sedimentary rocks, therefore for practical purposes, the magnetic effect recorded by an airborne magnetometer can be considered the same as it would be if the sediments were not present (Nettleton, 1976).

Many methods of depth to magnetic source bodies have been developed by various workers. Peter (1949) published a rather complete system of quantitative magnetic interpretation which involved downward continuation to the basement surface and also introduced the concept of the measurements of maximum slope and half slope distances as depth criteria. Bean (1966) gave a graphical method of depth estimation based on inflection and half-slope method. Hartman *et al* (1971) described a Werner Deconvolution method, that uses a vertical- gradient profile which may be measured or calculated and a calculated horizontal gradient profile in addition to the recorded total-magnetic intensity curve. Spector and Grant (1970) described a two-dimensional spectral analysis of a given map or region. Another very useful and efficient method of depth determination is modeling. This involves the calculation of magnetic field of a given model at the points of observation (usually by computer) and comparing it with the measured data. The depth and the dimensional parameters of the body are adjusted until satisfactory agreement is achieved between the calculated and the measured values.

However, the ambiguity in the interpretation of potential field anomalies has made it impossible to determine uniquely the depth to magnetic source bodies. This has made interpretation of magnetic anomalies to be very subjective. Although geologic information and experience on the part of the interpreter will help to narrow down the interpretation problems but for a survey where geologic information is limited such as magnetic reconnaissance survey over a large area for petroleum potential, interpretation then becomes more influenced. But a possible approach to the interpretation of magnetic anomalies is to approximate the maximum depths to the causative bodies of these anomalies by using a sphere model. This is because for most anomalies, a sphere model can equivalently be developed on which the depth to the center of the body can be developed.

2. Geomagnetic Field

The earth behaves like a weak magnetic body whose magnetic field can be approximated to that of a uniformly polarized magnetic dipole at the center of the earth inclined at about 11.5º to the axis of rotation (Kearey and Brooks, 1988). The origin of the geomagnetic field is not well understood but is attributed to convection currents of conducting materials circulating in the outer, fluid, part of the Earth's core (Telford *et al*, 1990). The direction of the ambient geomagnetic field is taken to be the direction a freely suspended magnetic needle at any point on the surface of the earth will settle. This direction depends on the location with respect to the magnetic poles which is measured by the location of the earth's field or the magnetic latitude. The direction of the ambient geomagnetic field is at an angle to both the vertical and geographic north. The earth therefore possesses a magnetic field, B_{abs}, which can be resolved into certain vector components: horizontal component, B_{hor}, and vertical component, B_z. The horizontal component also has components along the north, B_x and the east, B_y. All these constitute what is known as geomagnetic elements (Fig. 1). The dip of $B_{abs}$is the inclination, *I*, of the field and the horizontal angle between the geographic and the magnetic north is the declination *D*. B_{abs} varies in strength from about 25000 nT in the equatorial regions to about 70000 nT at the poles, In the northern hemisphere the magnetic field generally dips downward towards the north and becomes vertical at the north magnetic pole. In the southern hemisphere the dip is generally upwards towards the north and vertical at the south magnetic pole.

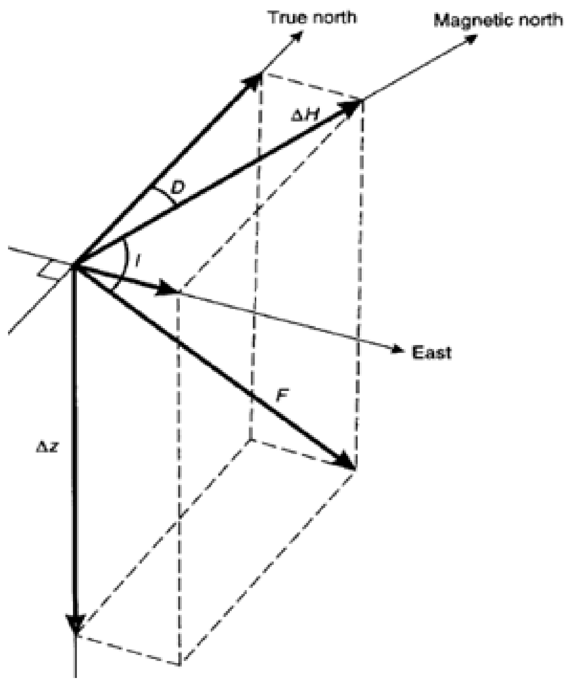

Figure1: Geomagnetic Elements (After Kearey and Brooks, 1988)

3. Magnetic Anomalies of A Sphere

A sphere as a geologic body is rare in the real situations, however, the study of its magnetic anomalies is useful in that it incorporates many features of the anomalies of bodies of complex but roughly isometric shape (Parasnis, 1994). For a homogeneous spherical body, the components of anomaly due to only induced magnetization are given by Rosler (1984) as:

$$\delta B_x(x,y,0) = \frac{\mu_0 a^3 (M_1(2x^2-y^2-d^2)+3M_2xy-3M_3xd)}{3(x^2+y^2+d^2)^{5/2}} \quad (1)$$

$$\delta B_y(x,y,0) = \frac{\mu_0 a^3 (3M_1xy+M_2(2y^2-x^2-d^2)-3M_3yd)}{3(x^2+y^2+d^2)^{5/2}} \quad (2)$$

$$\delta B_z(x,y,0) = -\frac{\mu_0 a^3 (3M_1xd+3M_2yd-M_3(2d^2-x^2-y^2))}{3(x^2+y^2+d^2)^{5/2}} \quad (3)$$

$\delta B_x(x, y,0)$,$\delta B_x(x,y,0)$ and $\delta B_x(x,y,0)$ are respectively the north-, east- and vertical components of the magnetic field measured at a position *x*, *y* on the surface, produced by a spherical, magnetized body with radius*a* and components magnetization M_1, M_2, and M_3buried at a depth*d*. $\mu_o$is the magnetic permeability of free space and is equal to $4\pi\times10^{-7}$. M_1, M_2, and M_3 are $M\cos I_m \cos D_m$, $M\cos I_m \sin D_m$ and $M\sin I_m$ respectively. I_m and D_m are respectively the inclination and the declination of geomagnetic field. The horizontaland the total magnetic field are given as:

$$\delta B_{hor} = \delta B_x \cos D_0 + \delta B_y \sin D_0 \quad (4)$$

$$\delta B_{abs} = \delta B_{hor} \cos I_0 + \delta B_z \sin I_0 \quad (5)$$

I_0 and $D_0$are the inclination and declination of magnetization vector respectively.

At low latitude where$I_m \approx 0$, the typical anomaly curves for a N-S and E-W traverses crossing the center point of a sphere are given in Fig. 2 and Fig. 3. The δB_ycomponent of the horizontal field for N-S traverse is zero and the anomalycurves for $\delta B_{hor}$and $\delta B_{abs}$are approximately the same as the δB_xcurves.Also, the typical 3-D anomaly curves including the contour maps for B_x, of a sphere is shown in Fig. 4.

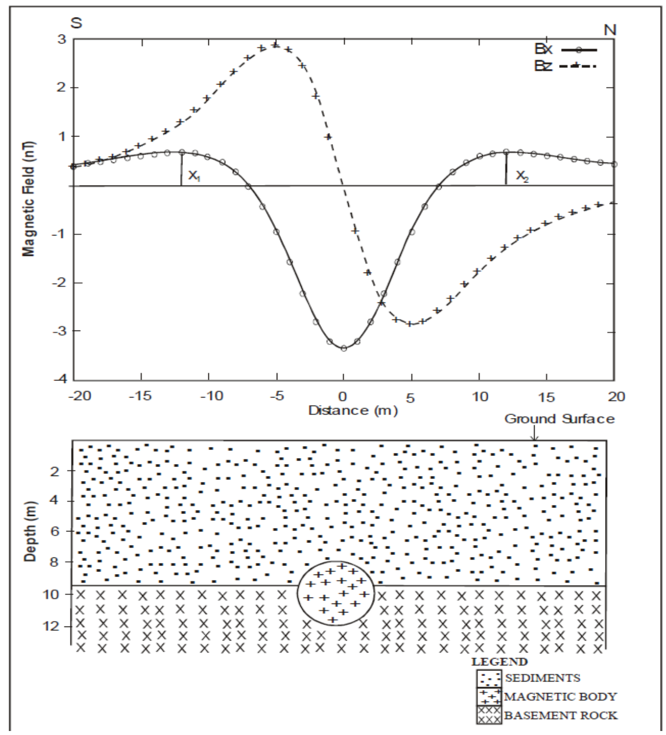

Figure2: Magnetic anomaly along a N-S profile overa sphere of radius 2 m, magnetization 1 A/m and centre depth 10 m at low latitude

4. Maximum Depth Rules at Low Latitudefor the East Componentof Magnatic Field

The homogeneous sphere is usually used as the model for the determination of the maximum depth to magnetic source bodies. At low latitude where inclination is approximately zero and assuming that declination of the geomagnetic field is zero, Equ. (1) will reduce to:

$$\delta B_x(x,y,0) = \frac{\mu_0 M a^3 (2x^2-y^2-d^2)}{3(x^2+y^2+d^2)^{5/2}} \quad (6)$$

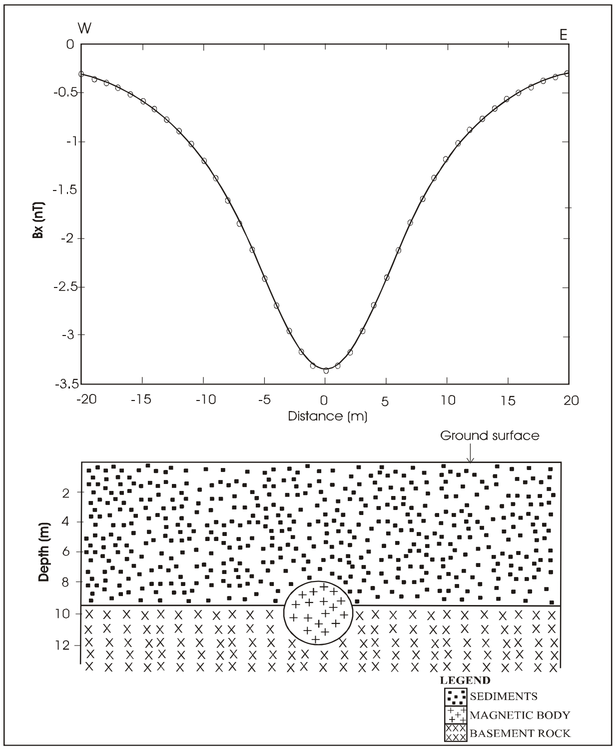

Figure 3: Magnetic anomaly along an E-W profile overa sphere of radius 2 m, magnetization 1 A/m and centre depth 10 m at low latitude

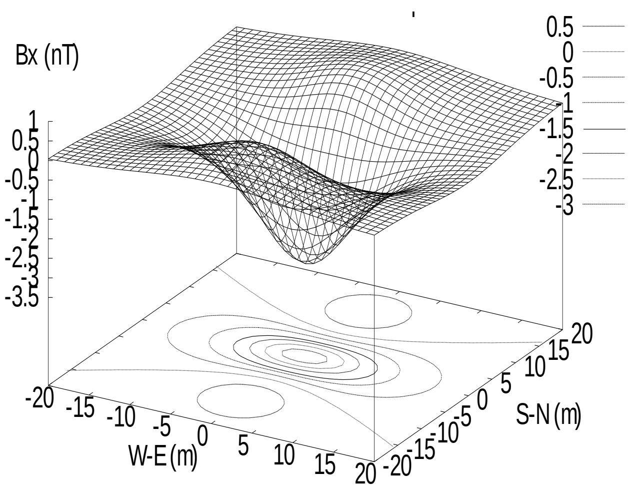

Figure 4: 3-D magnetic anomaly over a sphere of radius 2 m, magnetization 1 A\m, and centre depth 10 m at low latitude.

The depth function d to the center of a sphere has relationship with many parameters of the anomaly of the sphere. These relationships can be used to derive some formulae for the calculation of depth depending on the direction of profile. Some of the parameters are:

1) Maximum amplitude width
2) Inflexion point and
3) Amplitude and maximum slope

4.1 Maximum Depth Rules on a N-S Traverse

For a S-N traverse, at y = 0 equation (6) reduces to:

$$\delta B_x = \frac{\mu_0 M a^3 (2x^2 - d^2)}{3(x^2+d^2)^{5/2}} \qquad (7)$$

4.1.1 Amplitude-Width Rule

This rule relates the distance between the two maximum amplitudes to the center depth of a magnetic sphere. If equation (7) is differentiated

$$\delta B'_x = \frac{\mu_0 M a^3 (9x d^2 - 6x^3)}{3(x^2+d^2)^{7/2}} \qquad (8)$$

At maximum amplitude for δB_x, $\delta B'_x = 0$. Thus

$$9xd^2 - 6x^3 = 0 \qquad (9)$$

$$x = \pm\sqrt{3/2d} \qquad (10)$$

If $x_1$and x_2 represent respectively the positions of the left and the right maximum amplitudes in Fig. 2, then

$$d = \frac{x_2 - x_1}{\sqrt{6}} \qquad (11)$$

4.1.2 Inflexion Point Rule

Inflexion point rule gives the relationship between the distance from the point of maximum slope to point of minimum field and the center depth of a magnetic sphere. If equation (8) is differentiated

$$\delta B''_x = \frac{\mu_0 M a^3 (24x^4 - 72x^2 d^2 + 9d^4)}{3(x^2+d^2)^{9/2}} \qquad (12)$$

At inflexion point $\delta B^{11}_x = 0$. Hence

$$24x^4 - 72x^2d^2 + 9d^4 = 0 \qquad (13)$$

Dividing equation (13) by $24d^4$,

$$\frac{x^4}{d^4} - \frac{3x^2}{d^2} + \frac{3}{8} = 0 \qquad (14)$$

Andmaking $y = \frac{x^4}{d^4}$ equation (14) becomes

$$y^2 - 3y + \frac{3}{8} = 0 \qquad (15)$$

Therefore,

$$d = \pm\frac{x}{\sqrt{2.8693}} \qquad (16)$$

Or

$$d = \pm\frac{x}{\sqrt{0.1307}} \qquad (17)$$

Applying this to the model in Fig. 2, equation (16) is satisfied by the first and the last inflexion points from the left and equation (17) is satisfied by the second and the third inflexion points.

4.1.3 Amplitudes-Slope Rule

Amplitudes-Slope rule relates depth to the difference between maximum and minimum magnetic fields and the maximum slope. From equation (7), when x = 0, δB_x is minimum and is given as:

$$\delta B_{min} = -\frac{\mu_0 M a^3}{3d^3} \qquad (18)$$

When equation (4.11) is zero, $x = \pm\sqrt{3/2}d$ and $\delta B^{|}_{x}$ is maximum. i.e.

$$\delta B_{\max} = \frac{2\mu_0 Ma^3}{29.646\, d^3} \quad (19)$$

At $\delta B^{||}_{x} = 0$, $x = \pm\sqrt{2.8693}d$ or $x = \pm\sqrt{0.1307}d$ and $\delta B^{|}_{x}$ is maximum. $\delta B^{|}_{x}$ is given as:

$$\delta B^{|}_{\max} = \pm\frac{2.9720\ \mu_0 Ma^3}{3\times 1.54\, d^4} \quad (20)$$

or

$$\delta B^{|}_{\max} = \pm\frac{10.3848\ \mu_0 Ma^3}{3\times 113.95\, d^4} \quad (21)$$

$$d = \pm\frac{\delta B_{\max} - \delta B_{\min}}{0.62\,\delta B^{|}_{\max}} \quad (22)$$

or

$$d = \pm\frac{\delta B_{\max} - \delta B_{\min}}{13.2\delta B^{|}_{\max}} \quad (23)$$

4.2 Maximum depth rules on an E-W traverse for the δB_x components

On an E-W traverse, when x is zero, equation (6) becomes

$$\delta B_x(0, y, 0) = \frac{\mu_0 Ma^3(-y^2 - d^2)}{3(y^2 + d^2)^{5/2}} \quad (24)$$

4.2.1 Half width rule

The half width is the horizontal distance between the principal maximum or minimum of an anomaly (assumed to be over the center of source) and the point where the value of the anomaly is exactly one half the maximum amplitude.At y = 0, equation (26) becomes

$$\delta B_x(0, y, 0) = \frac{\mu_0 Ma^3}{3d^3} \quad (25)$$

If b is assumed to be equal to the minimum, then

$$b = -\frac{\mu_0 Ma^3}{3d^3} \quad (26)$$

Therefore, at half width ($y_{1/2}$)

$$\frac{1}{2}b = -\frac{\mu_0 Ma^3}{3(y^2_{1/2} + d^3)^{3/2}} \quad (27)$$

Hence

$$-\frac{\mu_0 Ma^3}{6d^3} = -\frac{\mu_0 Ma^3}{3(y^2_{1/2} + d^3)^{3/2}} \quad (28)$$

Thus

$$d \approx 1.3 y_{1/2} \quad (29)$$

4.2.2 Inflexion point rule

If equation (24) is differentiated with respect to y then

$$\delta B^{|}_{x} = \frac{\mu_0 Ma^3 y}{(y^2 + d^2)^{5/2}} \quad (30)$$

$$\delta B^{||}_{x} = \frac{\mu_0 Ma^3(d^2 + 4y^2)}{(y^2 + d^2)^{7/2}} \quad (31)$$

At inflexion point, $\delta B^{||}_{x} = 0$. Therefore:

$$d = \pm 2y \quad (32)$$

4.2.3 Amplitude-Slope Rule

This rule relates depth to the minimum magnetic field and maximum slope. At $\delta B^{||}_{x} = 0$, y = ±d/2and $\delta B^{|}_{x}$ is maximum i.e.

$$\delta B^{|}_{x} = \pm\frac{\mu_0 Ma^3}{3.49d^3} \quad (33)$$

Dividing δB_x at minimum by $\delta B^{|}_{x}$ at maximum

$$\frac{\delta B_{\min}}{\delta B^{|}_{\max}} = \pm 1.16d \quad (34)$$

Thus

$$d = \frac{\delta B_{\min}}{1.16\delta B^{|}_{\max}} \quad (35)$$

5. Testing of Maximum Depth Rules

All the derived formulae (Equ.11,17, 23, 29, 32 and 35) for the determination of depth to the centcre of spherical bodies causing magnetic anomalies were tested using synthetictotal magnetic data.

The sets of data were generated over tenlinear magnetic spherical bodiesalong E-W and N-S directionsand their respective first derivatives (Fig. 5 and Fig. 6).This was carried out in order to create a real field situation where there is interference and the magnetic effect at every point of observation gives a sum of the contributions from all themagnetic bodies in the vicinity of the point. The parameters of the spheres are shown in Table 1.

For east-west profile, three maximum depth rules were tested. The rules are: Inflexion Point Rule (IPR); Half-Width Rule (HWR); and Amplitude-Slope Rule (ASR). Also, three rules were testedon N-S profile. The rules are: IPR, ASR and Amplitude-Distance Rule (ADR).

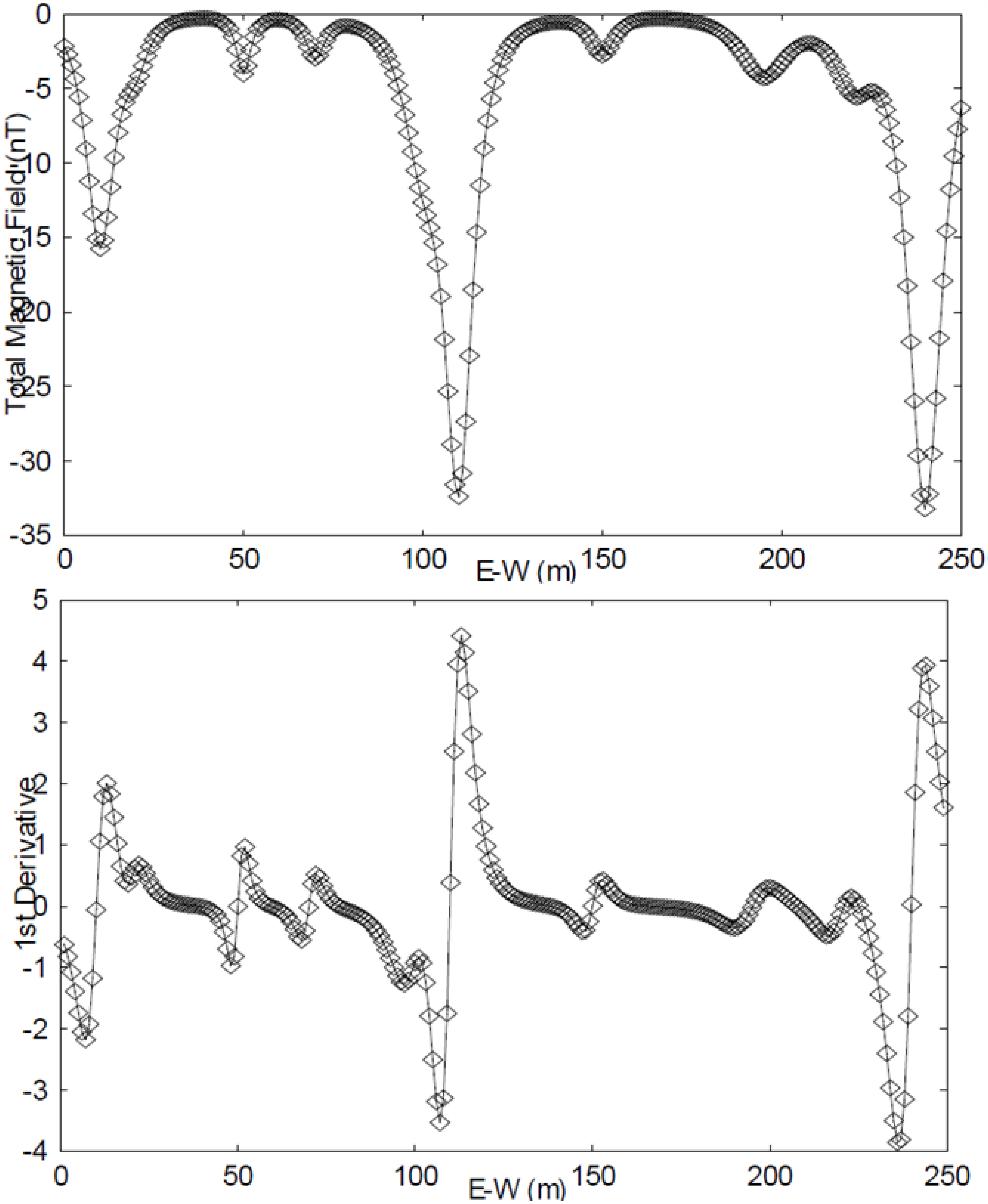

Figure 5: Total magnetic field anomaly over the center points of 10 linear spheres along east-west profile at low latitude. Magnetic field (above) and first derivative of magnetic field (below)

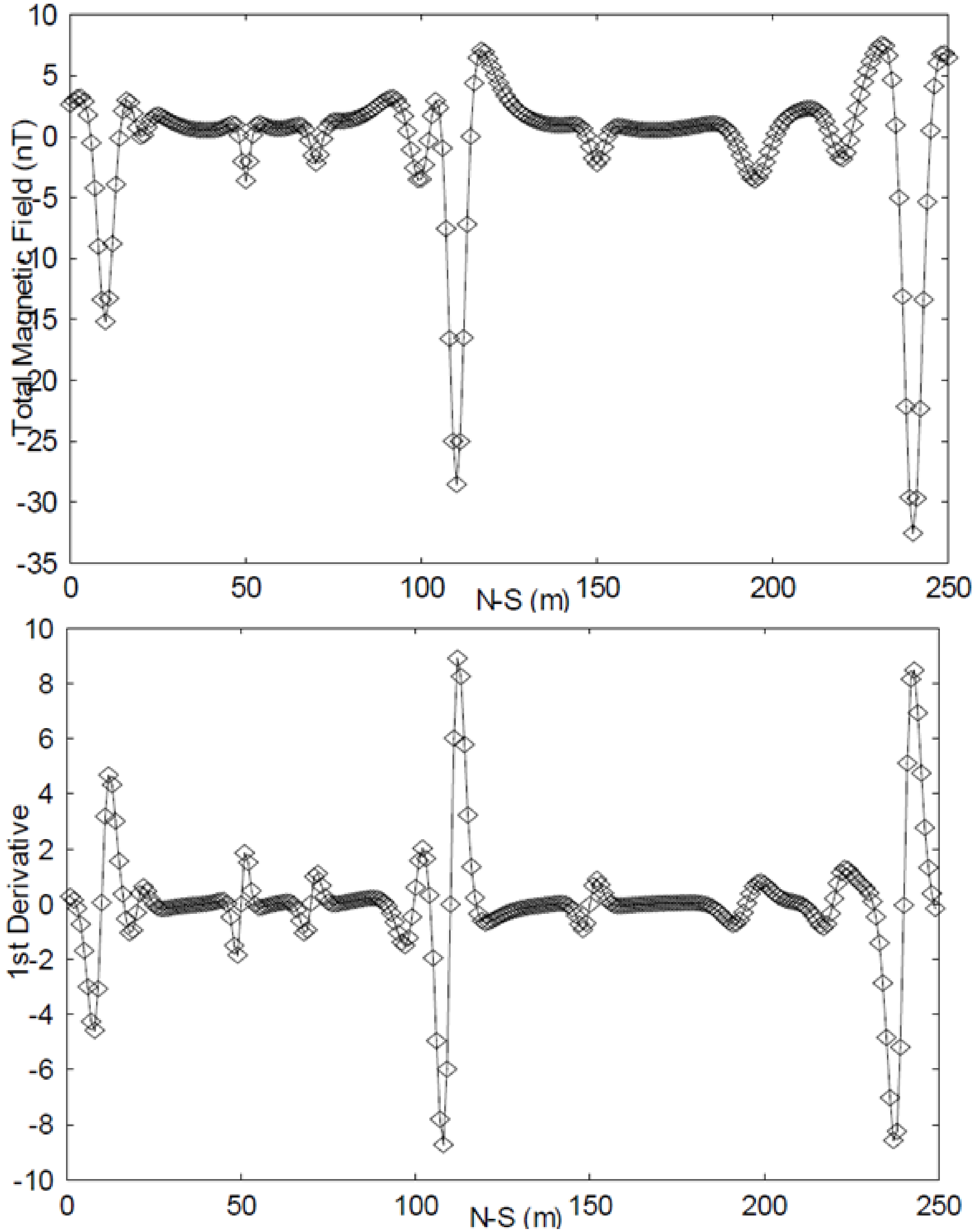

Figure 6: Total magnetic field anomaly over the center points of 10 linear spheres along north-south profile at low latitude. Magnetic field (above) and first derivative of magnetic field (below)

Table 1: Parameters of sphere used for generating synthetic total magnetic field data

S/N	Radius (m)	Depth (m)	Magnetization(A/m)	Center Points(m)
1	2.0	6.0	1.0	10
2	0.8	5.0	1.5	20
3	0.5	3.0	2.0	50
4	0.7	4.0	1.2	70
5	2.0	8.0	1.3	100
6	2.5	6.0	1,0	110
7	0.8	5.0	1.5	150
8	2.0	10.0	1.2	195
9	1.5	8.0	1.5	220
10	1.3	7.0	1.0	240

6. Results and Discussion

The results of the depth rules for east-west and north-south directions are shown in Table 2 and Table 3 respectively.On E-W profile, results were obtained for only 8 spheres and 10 spheres on N-S profile. In real field situation, some magnetic anomalies are superimposed on higher magnetic field, though, dependent on the distance between the magnetic bodies and the direction of profile. This is the case in the E-W profile, where only 8 anomalies were clearly shown and the other two spheres – 2 and 5 are not conspicuous (Fig. 5) but clearly shown in N-S profile (Fig. 6). The results show that the depth rules give very close values to the true depth of the magnetic source bodies with maximum variation of ±1 m.

Table 2: Results of maximum depth rules on E-W profile

Sphere	Obtained Depth (m)			True Depth (m)
	IPR	HWR	ASR	
1	6	6.5	6.23	6
2	Nill	Nill	Nill	5
3	4	3.25	3.59	3
4	4	4.55	4.64	4
5	Nill	Nill	Nill	8
6	6	5.85	6.33	6
7	6	5.85	5.69	5
8	10	9.75	10.47	10
9	7	9.75	10	8
10	8	7.15	7.28	7

Table 3: Results of maximum depth rules on N-S profile

Sphere	Obtained Depth (m)			True Depth (m)
	IPR	ADR	ASR	
1	5.43	5.72	6.35	6
2	5.53	4.08	4.46	5
3	2.77	3.27	4.0	3
4	5.53	4.9	5.0	4
5	8.3	7.88	7.16	8
6	5.53	5.72	6.12	6
7	4.7	4.9	5.47	5
8	11.0	9.8	10.13	10
9	8.3	8.17	7.82	8
10	6.8	7.35	7.46	7

From the results of the depth rules on E-W profile, the absolute mean values variation between the computed depth and the real depth of the magnetic source bodies are 0.5 for IPR with correlation coefficient of 0.95, 0.56 for HWR with 0.96 correlation coefficient and 0.65 for ASR with 0.97 correlation coefficient. Also, from the results of the depth rules on N-S profile, the absolute mean values variation between the computed depth and the real depth of the magnetic source bodies are 0.54 for IPR with correlation coefficient of 0.96, 0.30 for ADR with 0.97 correlation coefficient and 0.53 for ASR with 0.96 correlation coefficient.

Judging from the results of the maximum depth rules. IPR, HWRand ASR give similar results on E-W profile and also, IPR, ADR and ASR give similar results on N-S profile. An average value form the different depth rules will give a better depth valuie with minimal error.

7. Conclusion

Three maximum depth rules of magnetic source bodies on E-W profile (IPR,HWR and ASR) and N-S profile (IPR, ADR and ASR) have been developed for low latitudinalregions using homogeneous spherical magnetic source body as model. On testing, their results gave similar and close depth values to the real depths. Therefore these rules can be relied upon to calculate the maximun depth to magnetic source bodies in low latitudinal regions. For a valid result, the depth from the rules must fall within same range and the average value or the middle value can then be taken as the depth.

Therules can be applied in producing sediment thickness map in reconnaisance survey for oil exploiration and as a guide in estimating cost of drilling to magnetic bodies in mineral exploitation.

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