

Mathematical Investigation and Comparison of Diesel Engine Frictional Torque and Rotational Moment of Inertia with Novel and Willians Line Method

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Abstract: *Mechanical engineering involves the combination of analysis with hands-on discovery. The influence of engine design and operating parameter have a lot to contribute when the economics and efficiency is concerned. One such parameter which primarily contributes to the engine performance is rotational moment of inertia. Many methods both theoretical and practical have evolved in the recent past for the determination of this parameter. Since rotational moment of inertia is one such important parameter that need to meant and regulated for smooth hustle free and optimum performance of engine. The project involves the comparison of the existing method for determination of the parameter to a novel method whose scope of operation is much higher and efficient.*

Keywords: Rotational moment of inertia, frictional torque, frictional power

1. Introduction

Design of Engines that are more efficient that run on hybrid fuels the study of the Fuel-Air Cycles that predict their behaviour is the most researched area presently in the field of Thermal Engineering. Engine Design predominantly caters the study of Engine parameters that have effect on the overall efficiency of the Engine. One such parameter that is of keen interest to the designers is the rotational moment of inertia, the frictional torque and the frictional power that will be expended. Engine research has led to new methods for the determination of rotational moment of inertia of engine. The methods that are used in the industry for determining the same have stood the test of practicality over the years. The Willian's Line method is by and more the most industry preferred method owing to its ease and reasonable accuracy. In our project we have closely analysed a novel method. The novel method is essentially retardation test of the motored engine to find the deceleration of engine in the regime of engine speed it will not stall. This method is a more practical method because it takes into the consideration the actual working scenario of the Engine while in use.

2. Existing method: Willian's line method

This method is also known as fuel rate extrapolation method. In this method a graph of fuel consumption (vertical axis) versus brake power (horizontal axis) is drawn and it is extrapolated on the negative axis of brake power .the intercept of the negative axis is taken as the friction power of the engine at that speed.

In most of the power range the relation between the Fuel consumption and brake power is linear when speed of the engine is held constant and this permits extrapolation. Further when the engine does not develop power, i.e. Brake Power = 0, it consumes a certain amount of fuel. This energy in the fuel would have been spent in

overcoming the friction. Hence the extrapolated negative intercept of the horizontal axis will be the work representing the combined losses due to friction, pumping and as a whole is termed as the frictional loss of the engine. This method of measuring friction power will hold good only for a particular speed and is applicable mainly for compression ignition engines.

The main drawback of this method is the long distance to be extrapolated from data between 5 and 40 % load towards the zero line of the fuel input. The directional margin of error is rather wide because the graph is not exactly linear.

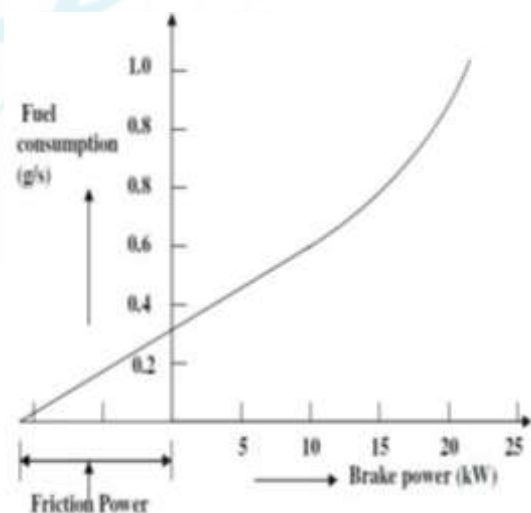


Figure 1: Frictional Power by Willian's Line Method

As shown in the figure, in most of the power range the relation between the fuel consumption and brake power is linear when speed of the engine is held constant and this permits extrapolation. Further when the engine does not develop power, i.e. brake power = 0.

Drawbacks

- The main drawback of this method is the long distance to be extrapolated from data measured between 5% and 40% load towards the zero line of fuel input.
- The directional margin of error is rather wide because of the graph which may not be a straight line many times.
- The changing slope along the curve indicates part efficiencies of increments of fuel. The pronounced change in the slope of this line near full load reflects the limiting influence of the air-fuel ratio and of the quality of combustion.
- Similarly, there is a slight curvature at light loads. This is perhaps due to difficulty in injecting accurately and consistently very small quantities of fuel per cycle.
- Therefore, it is essential that great care should be taken at light loads to establish the true nature of the curve.
- The Willian's line for a swirl-chamber CI engine is straighter than that for a direct injection type engine.
- The accuracy obtained in this method is good and compares favourably with other methods if extrapolation is carefully done.

3. Novel Method**Method for Determining Engine Moment of Inertia**

A method for determining the rotational moment of inertia of an engine, comprising the steps of carrying out engine deceleration tests, first with the engine un loaded and later with the engine sufficiently loaded to provide a recordable difference in deceleration rate (or elapsed time). Moment of inertia is determined as the numerical value of the added load divided by the difference in deceleration rates, before and after the load is applied.

Background and Summary of Invention

The present invention contemplates the use of a dynamometer for applying predetermined loads on the engine during one of the test runs. However a feature of the invention is that the load-applying mechanism (dynamometer) is a low cost relatively small capacity unit. For example, the applied load need only be approximately 2 per cent of the engine torque capability. With a large engine of approximately 1000 horsepower rating the selected dynamometer could be a relatively small 20 horsepower unit. Such a unit possesses advantages in such respects as size, cost and convenience.

This invention can be practiced using conventional state-of-the art apparatus. Therefore no drawings are included herein. Test procedures disclosed in aforementioned applications, include the steps of running an unloaded engine up to some safe operating speed, e.g. 1800 r.p.m, and de-energizing the engine to produce a controlled deceleration to some lower speed, e.g. 1000 r.p.m. Elapsed time to go from the high speed to the low speed is measured, to provide an indication of the average deceleration rate. The torque developed by the Engine is given by the relation,

Torque Impressed due to Force = (Moment of Inertia).
(Angular Acceleration)

$$\Rightarrow \tau = I . \alpha$$

When the Engine is run un-loaded and de-energised, the frictional forces alone impress a frictional torque, which decelerates the system, so

$$\tau_f = I_0 . \alpha_{NL} \dots\dots (1)$$

Where,

τ_f is the friction torque

I_0 is Moment of inertia of System

α_{NL} is deceleration rate at no load

Next a known external load is applied. The external load and the torque impressed by it, adds to the Engine friction torque, causing the Engine to decelerate at a different rate during the load test. Thus the impressed external load, imparts an additional torque in addition to the friction torque, that result in a different deceleration rate, given as

$$(\tau_f + \tau_a) = I_0 . \alpha_d \dots\dots (2)$$

Where,

τ_a is the impressed external torque

α_d is deceleration rate at impressed load

Substituting from the equation (1), we have

$$I_0 = \frac{\tau_a}{(\alpha_d - \alpha_{NL})}$$

The three quantities on the right side of this equation are known or can be determined by running successive deceleration tests with the engine loaded and then unloaded. The added external load is preferably and desirably quite small when compared to the torque output of the power plant and its friction torque. For example, if the friction torque is approximately 20 percent of the indicated torque then the added load F_A can be only about 2 percent of the indicated torque. One advantage (or perhaps requirement) is using a low added load is that it does not unduly reduce the deceleration rate. The added load must not be so large as to unduly shorten deceleration elapsed time beyond the test equipment capability.

The test runs should be carried out with a reasonably large differential between the starting high engine speed and ending low engine speed. However the operational speeds must not exceed safe operating values. For many power plants the high starting speed would be about 2500 rpm, and the low final speed would be about 1000rpm.

4. Procedure of novel method

Engines manufactured in India have to undergo yearly Conformity of Production (COP) testing for emission standards set from time to time by Automotive Research Association (ARAI). Currently this model of engine conforms to TREM-III norms. One such Engine submitted for COP is taken as our reference for this project.

Fuel Consumption with Load

The engine is started up to fly up speed of 2000 rpm and time taken for consuming 25 ml of fuel is noted using stopwatch. This procedure is repeated with different loads such as 200, 300, 400, 500, 600, 800 and 1000 grams respectively.

Deceleration Test With/Without Load

As mentioned above the engine is again started to fly up speed of 2000rpm. Using tachometer the rpm of the engine is ensured that it is 2000rpm. Now the engine is held for stabilizing the operating temperature. Then the fuel is cut off and the time for deceleration is noted for the speed range of 1800-1000 rpm using analogue tachometer and stopwatch. The engine is fired up again and is repeated without load for several times for an accurate result.

The above mentioned procedure is again done in load test with varying loads such as 200, 300, 400, 500, 600, 800

and 1000 grams respectively. The gathered results are noted and angular deceleration is using the equations in load test and no-load test.

Apparatus Required

1. Test Engine.
2. Prony Brake System
3. Fuel Cut-Off Apparatus.
4. Tachometer [Analog].
5. Stopwatch [Digital].
6. Fuel Consumption Measuring Apparatus.

5. Experimental Result

Table 1: Measurement of moment of inertia and frictional torque

ENGINE MODEL		ER-90							MAX. TORQUE (Nm)			44.4			
ENGINE SERIAL		1180280017							RATED POWER (kW)			8.8			
Moment of Applied Load, R (m)			0.286				Dead Weight, W(g)			250					
SL.No.	Speed Range (rpm)		NO-LOAD TEST		LOAD TEST				α_d (rad/s ²)	I_0 (kg-m ²)	τ_f (Nm)	FC (kg/hr)	P (W)	P/N	
	N_f	N_i	T_{NL} (sec)	α_{NL} (rad/s ²)	T_{fuel} (sec)	w (g)	$F_a = W + w$ (kg)	T_a (sec)							
1	1800	1000	4.51	18.58	76.98	200.00	0.4500	3.84	21.82	0.376	6.94	0.9727	264.43	0.1322	
2	1800	1000	4.56	18.37	76.64	300.00	0.5500	3.68	22.77	0.358	6.61	0.9770	323.19	0.1616	
3	1800	1000	4.53	18.49	76.20	400.00	0.6500	3.54	23.67	0.350	6.47	0.9827	381.95	0.1910	
4	1800	1000	4.52	18.53	75.92	500.00	0.7500	3.41	24.57	0.344	6.36	0.9863	440.71	0.2204	
5	1800	1000	4.55	18.41	75.55	600.00	0.8500	3.29	25.46	0.340	6.28	0.9911	499.47	0.2497	
6	1800	1000	4.56	18.37	74.93	800.00	1.0500	3.07	27.29	0.334	6.16	0.9993	617.00	0.3085	
7	1800	1000	4.54	18.45	74.26	1000.00	1.2500	2.88	29.09	0.330	6.09	1.0083	734.52	0.3673	
Mean No-Load Deceleration			18.46						Mean, ΣI_0		0.348	6.42			

6. Design Calculation

Sample calculations for observation No: 1

6.1. To Find Angular Deceleration at No Load

$$\alpha_{NL} = \frac{2\pi(N_f - N_i)}{60 T_{NL}}$$

N_i, N_f – Initial & Final rpm
 T_{NL} – Time Taken at No Load

$$\alpha_{NL} = \frac{2\pi(N_f - N_i)}{60} \times \frac{2\pi \times 800}{4.51} = 18.5755 \text{ rad/s}^2$$

6.2. To Find Angular Deceleration at Load Test

$$\alpha_d = \frac{2\pi \times (N_f - N_i)}{60 T_a}$$

T_a – Time Taken at Load Test

$$\alpha_d = \frac{2\pi \times (N_f - N_i)}{60} \times \frac{2\pi \times 800}{3.84} = 21.8166 \text{ rad/s}^2$$

6.3. To Find The Moment Of Inertia:

$$I_0 = \frac{\tau_a}{\alpha_d - \alpha_{NL}}$$

I_0 – moment of inertia
 τ_a – applied torque

$$I_0 = \frac{\tau_a}{\alpha_d - \alpha_{NL}} = \frac{1.262547}{3.24105} = 0.38954 \text{ kg-m}^2$$

$$FC = \frac{F}{T} \times 3600$$

6.4. To Find The Frictional Torque:

$$\tau_f = I_0 \times \alpha_{NL}$$

τ_f —frictional torque,
 I_0 —moment of inertia

$$\tau_f = I_0 \times \alpha_{NL} = 0.3895 \times 18.5755 = 7.375 \text{ Nm}$$

$$F = \frac{25\text{ml} \times 0.832}{1000} \quad (0.832\text{—density of diesel})$$

T – Time Taken For Consuming 25ml of Fuel

$$FC = \frac{25\text{ml} \times 0.832}{1000 \times T} \times 3600 = \frac{25\text{ml} \times 0.832}{1000 \times 76.98} \times 3600$$

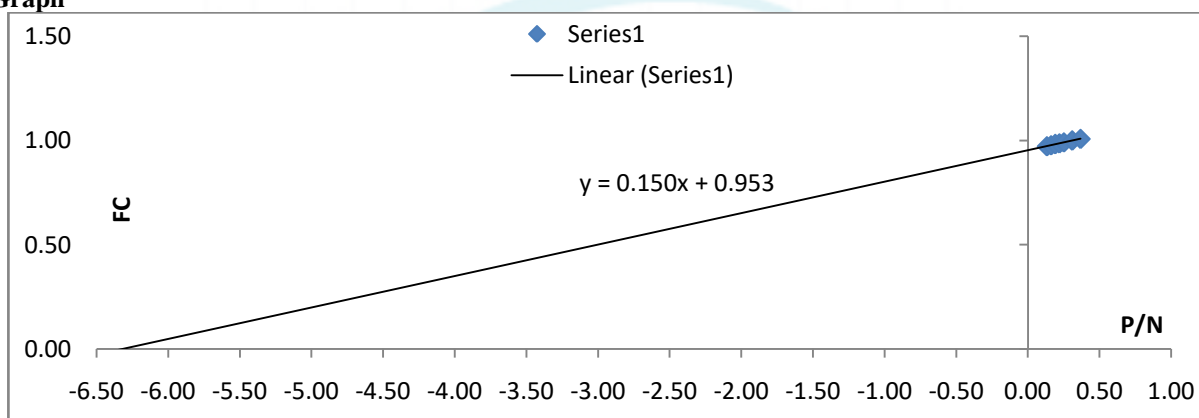
$$= 0.9727 \text{ kg/hr}$$

6.6. To find Brake Power

$$P = \frac{15860.1420}{60} = 264.3357 \text{ W}$$

6.5. To Find Fuel Consumption for Load Test

7. Graph



7. Mathematical Modelling

Flywheel Calculation

1 metric HP = 75 kg F/s
 1 kg F = 9.8 IN
 = 75x9.81 = 735.49875 watts
 $P = \frac{2\pi NT}{60} \text{ Nm/s (or) watts}$

$$T = \frac{P \times 60}{2\pi N}$$

$$T = \frac{75 \times 9.81 \times 60}{2 \times \pi \times 2000} = 3.512 \text{ N-m}$$

Torque = F x R

$$\therefore R = F/T$$

Force = 12.25 N

$$R = \frac{12.25}{3.512} = 0.2866 \text{ m}$$

R = 0.2866 m

Mathematical Model for Rotational Moment Of Inertia

The rotating moment of inertia of engine depend on factors like

- Fly Wheel Speed
- Fly Wheel torque
- Time Taken for rotation
- Angular deceleration

The relationship between rotating moment of inertia of engine and various variables constitute the mathematical model of engine moment of inertia

Various Variable used in this mathematical model are,

- | | |
|--------|-------------------------|
| ρ | -Density of the solid |
| t | -Thickness of Fly wheel |
| V | -Volume of Fly wheel |
| M | -Loud applied in kg |
| R | -Radius of Fly wheel. |

From parallel axis theorem,

Moment of inertia of an object about an axis of rotation that passes through its centre of mass (com) is known, then moment of inertia of this object about any axis parallel to this axis can be found using the following equation,

$$I = I_{\text{centre of mass}} + AH^2$$

$$\therefore I = I_{\text{center of mass}} + AH^2$$

To find the moment of inertia of a flywheel of engine radius R, thickness t, total mass M and total Volume V about its central axis.

$$I_{\text{Flywheel}} = \frac{1}{2} MR^2$$

The fly wheel can be divided into a very large number of thin rings of Thickness 't' and differential width 'dr' the volume of one of these rings of radius 'r' can be written as

$$dv = 2\pi r t dr$$

Mass can be written as,

$$dm = \rho dv$$

Where, $\therefore \rho = m/v$

$$m = \rho v$$

$$dm = \rho dv$$

Where,

ρ – Density of solid

Since every particle in the ring is located at the same distance r from the axis of rotation,

The moment of inertia of the ring can be written as,

$$dI = \rho r^2 dv$$

Where,

$$\therefore I = mr^2$$

$$dI = dmr^2$$

$$dI = \rho r^2 dv$$

Integrating for the entire engine flywheel gives,

$$I_{\text{FLYWHEEL}} = \int_0^R dI = \int_0^R \rho \cdot r^2 \cdot (2\pi r) \cdot t \cdot dr$$

$$[dr]_0^R = 2\pi \rho t [r^2 \cdot r dr]_0^R$$

$$I = 2\pi \rho t [r^3 \cdot dr]_0^R$$

$$I = 2\pi \rho t \left[\frac{r^4}{4} \right]_0^R$$

$$I = 2\pi \rho t \left[\frac{R^4}{4} - 0 \right]$$

$$I_{\text{Flywheel}} = \frac{1}{2} \rho \pi t R^4 \quad \longrightarrow \quad (1)$$

Here,

$$V = \pi t R^2$$

$$\rho = M/V$$

$$M = \rho V$$

$$M = \rho \pi t R^2 \quad \longrightarrow \quad (2)$$

Substitute Equation (2) in (1)

$$I_{\text{Flywheel}} = \frac{1}{2} \rho \pi t R^2 \cdot R^2$$

\therefore The rotational moment of inertia of Flywheel of Engine,

8. Result

The standard frictional torque calculated at ARAI (Automotive Research Association of India) has found to be that the value is **6.72 Nm**.

Value of Friction Torque as obtained by the extrapolation plot of Willian's Line of the test Engine gives the Frictional Torque to be **6.32 Nm**

The patented method gives the friction torque by calculation as **6.42 Nm**.

9. Conclusion

The study of this project concludes that the values we calculated generated and the values gathered from ARAI were almost equal. Even though there was a 5% of error which is negligible because we didn't consider the stator friction that is unavoidable for the dynamometer that is used in ARAI.

The successful completion of the project begins in a scope of improving the working efficiency of CI engine. The advanced methodology followed during the whole process of the project right from the scratch depicts a considerable innovative thought of calculating the rotational moment of inertia by comparison strategy with the conventional method of determining the same.

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