Energy Parameters of Symmetrical Optical System Apodized with Gaussian Filter

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Abstract: *In this paper, we studied energy parameters of the Point Spread Function (PSF) namely, encircled energy, relative encircled energy, excluded energy and displaced energy of an optical system apodized with Gaussian filter against the specific radius in the focal plane of imaging system. These parameters are important in the performance analysis of optical system in presence of apodisation.*

Keywords: Symmetrical optical system, Apodization, Gaussian filter, Point spread function, Encircled energy, Excluded energy, Displaced energy, Relative encircled energy

1. Introduction

The PSF of an optical device is the image of a single point object. The Point Spread Function (PSF) plays an important role in the image formation theory in the optical system such as Fluorescence Microscope. It is a main brick that builds up the whole acquired image. The degree of spreading (blurring) in the image of this point object is a measure for the quality of an optical system. The imaging in the optical system is completely described by its PSF. The term encircled energy refers to a measure of concentration of energy in an optical image. The encircled energy of the resulting image gives the distribution of energy in that PSF. Encircled energy is used to quantify the spreading of a laser beam at a given distance. The encircled energy is the quantity that also serves as an index of the performance of an optical system. The total illumination is a measure of the fraction of the energy of a point source diffraction image contained in a specified radius. Lord Rayleigh [1], first realized the importance of the encircled energy factor to find the illuminations in the various rings of the diffraction pattern and presented a formula for calculating the same. Lansraux and Boivin[2] proposed computing techniques for the numerical evaluation of the encircled energy in the diffraction pattern of an optical system having aberrations. Vishwanatham .et.al [3] have studied the fraction of the encircled energy with regard to desperation factor in the diffraction pattern of circular apertures. Murthy [4] investigated PSF based corollaries of encircled energy and its related parameters such as excluded energy, displaced energy, and zonal energy increment in the presence of defocusing by employing co-sinusoidal amplitude filters. Biswas and Boivin [5] studied the performance of optimum apodizer for rotationally symmetric optical systems in presence of spherical aberrations. Aberrations have significant influence on the performance of optimum apodizer, particularly on the values of encircled energy. Optimum apodisation is used in an optical system to maximize the concentration of energy within specified area in the diffraction pattern formed by the systems. Some corollaries of the PSF such as the minimization with the second order moment are studied by Asakura [6]. Mondal

et.al [7] have studied the encircled energy and its complementary quantity excluded energy using Straubel apodisation filters. Keshavulu [8] has investigated the encircled energy in the presence of individual and combined effects of defocusing and primary spherical aberration in the case of optical system apodised with shaded aperture systems. Role of Excluded energy and relative encircled energy in the point image quality assessment is studied by Srisailam [9].Thirupathi [10] has studied encircled energy factor in impulse response functions of optical systems with first order parabolic filters. Karunasagar [11] has studied the encircled energy, relative encircled energy, excluded energy, Zonal energy increment for rotationally symmetric system. In this work, we studied energy parameters of symmetrical optical system apodized with Gaussian filter.

The rest of the paper is organized as follows. In the section 2, mathematical expressions for encircled energy and related parameters of PSF of symmetrical optical system with Gaussian filter is derived. In the section 3, numerical results are presented. Finally conclusion is given in section 4.

2. Encircled Energy and Related Parameters

Encircled energy is the fraction of the total energy of the diffraction pattern contained within a specified circle of radius. Encircled energy pertaining to the circle of radius δ ,EE (δ) is given by [2]

$$EE(\delta) = \frac{\int_0^{2\pi} \int_0^{\delta} |G_F(0,Z)|^2 Z dZ d\varphi}{\int_0^{2\pi} \int_0^{\infty} |G_F(0,Z)|^2 Z dZ d\varphi} \ , \qquad \dots (1)$$

where ϕ is the azimuthal angle, G_F (0, Z) is the light amplitude in the image plane at a point Z units away from the diffraction head. Since the integral with respect to ϕ is definite, the above equation reduced to

$$EE(\delta) = \frac{\int_0^{\delta} |G_F(0,Z)|^2 Z dZ}{\int_0^{\infty} |G_F(0,Z)|^2 \ Z dZ} \ . \qquad \dots (2)$$

The subsequent G_F indicates that the optical system is apodised with given filter. The amplitude of the light diffracted in the far field region associated with rotationally

symmetric pupil function f(r) can be expressed by the well known equation. [12].

$$G_F(0, Z) = 2 \int_0^1 f(r) J_0(Zr) r dr , \qquad \dots (3)$$

In the Eq.(3) J_0 (Z r) is the Bessel function of the first kind and zero order .For Gaussian apodisation filter , the pupil function f(r) is

$$f(r) = e^{-\frac{r^2}{\sigma^2}} , \qquad \dots (4)$$

where σ is the apodisation parameter which controls the degree of the uniformity of the transmission r is the normalized distance of a point on the pupil from its center. From the Eqs. (2) _ (4), we get

$$EE(\delta) = \frac{\int_0^\delta \left| \int_0^1 e^{-\frac{r^2}{\sigma^2}} J_0(Zr) r dr \right|^2 Z dZ}{\int_0^1 \left| e^{-\frac{r^2}{\sigma^2}} \right|^2 r dr} . \qquad \dots (5)$$

For annular apertures with central obscuration, the limits are from 0 to 1.It is monotonically decreasing positive function. It vanishes at the origin δ =0 and increases monotonically approaching unity asymptotically as δ tends to ∞ Relative encircled energy $EE_r(\delta)$ factor is defined as the ratio of the light energy within a specified circle of radius δ centered on the diffraction head due to the non-Airy pupil to the total light energy in the diffraction pattern due to the Airy pupil. Mathematically, [13] it can be written as

$$EE_r(\delta) = \frac{\int_0^\delta |G_F(0,Z)|^2 Z dZ}{\int_0^\infty |G_A(0,Z)|^2 Z dZ} , \qquad \dots (6)$$

where the subscripts A and F stand for Airy and non-Airy pupils, respectively. Excluded energy is the energy difference between total energy and energy concentrated within a specified circle of radius δ. The distribution of energy in the outer ring structure is also called the excluded energy $EE_X(\delta)$. According to [13], it can be written as

$$EE_X(\delta) = 1 - EE(\delta) , \qquad \dots (7)$$

It is an important quality factor to examine the performance of the pupil function and also to study the structure of a specified ring in detail. Minimum value of this factor indicates well functioning of the pupil function. The best pupil is that which can minimize the excluded energy and maximize the encircled energy. This is a useful and convenient factor for various techniques of apodisation to change the ring structure in diffraction pattern and also useful in some photometric situations. The Eq.(7) can be written as

$$EE_X(\delta) = \frac{\int_\delta^\infty |G_F(0,Z)|^2 Z dZ}{\int_0^\infty |G_F(0,Z)|^2 z dZ} . \qquad \dots (8)$$

Displaced energy $EE_d(\delta)$ is the difference between the encircled energy of the perfect system and that of the apodised case [13], and is written as

$$EE_d(\delta) = EE_A(\delta) - EE_F(\delta). \qquad \dots (9)$$

This factor is useful to compare the energy distribution in the case of actual optical imaging systems to that of perfect systems. This is a more sensitive quality factor in the case of aperture obscuration and less sensitive in the case of image motion. The positive sign of this factor indicates that the energy displacement is outward while the negative sign indicates the energy displacement is inward.

3. Numerical Results and Discussions

In this section, parameters of physical interest namely, encircled energy, relative encircled energy, excluded energy, and displaced energy are computed using the equations (5), (6), (8) and (9). Since the equations (5), (6), (8) and (9) involve Bessel functions under integral sign, the closed form solutions are not possible. Hence, for numerical process, MATHEMATICA Version: 8.0.1 software has been employed. Encircled energy against δ for apodisation parameter σ =0.25, 0.5, 0.75 and 1.0 are computed and results are presented in Fig: 1. from this figure, it is clear that the encircled energy increases rapidly for first values of δ and later on increases slowly before finally approaching unity asymptotically. As apodisation parameter σ increases, encircled energy value increases and approaches unity. The curves pertaining to σ =0.5, 0.75 and 1.0 are closer and little far from that of the case σ =0.25. Fig:2 shows the relative encircled energy values against radii δ ranging from 0 to 12 with increment of 0.2 for σ =0.25, 0.5, 0.75 and 1.0.This energy values rapidly increases for lower values of δ and there after plateau region is obtained. The curve pertaining to σ =0.25 almost coincide with x-axis and it closer to the curve σ =0.5 and far from that of the curves σ = 0.75 and 1.0. Fig:3 shows that excluded energy values against δ ranging from 0 to12 in step of 0.2 for σ =0.2,0.4,0.6,0.8 and 1.0 .The excluded energy curves show a rapid decrease for lower values of δ , except for the case σ =0.2 curve remaining all are closer and they coincide with x-axis after δ =4. The curves Fig: 1 and Fig: 3 are mirror images of each other. The minimum excluded energy gives a better image. Displaced energy against δ for apodisation parameter σ = 0.2, 0.4, 0.6, 0.8 and1.0 are computed and results are presented in Fig: 4. from this figure it is clear that the positive values of displaced energy means here is that encircled energy of Airy pupil is greater than apodised Gaussian pupil, similarly, in negative values, means here the encircled energy of Airy pupil is less than the apodised Gaussian pupil.
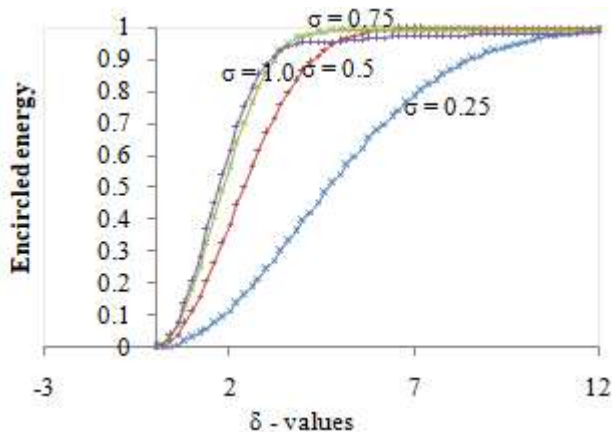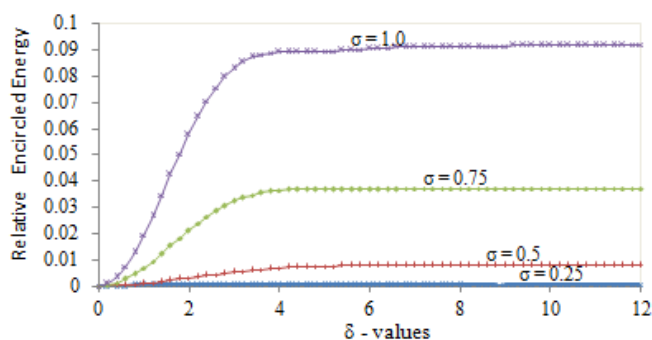
Figure 1: Encircled Energy curves

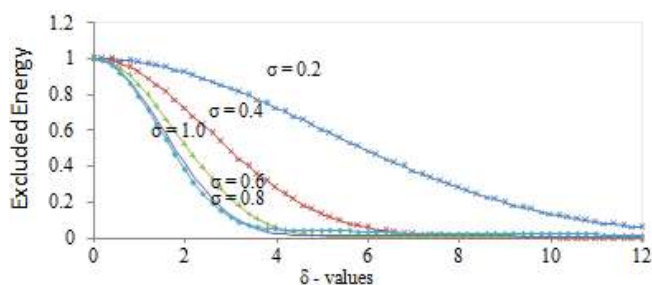**Figure 2:** Relative Encirlced Energy
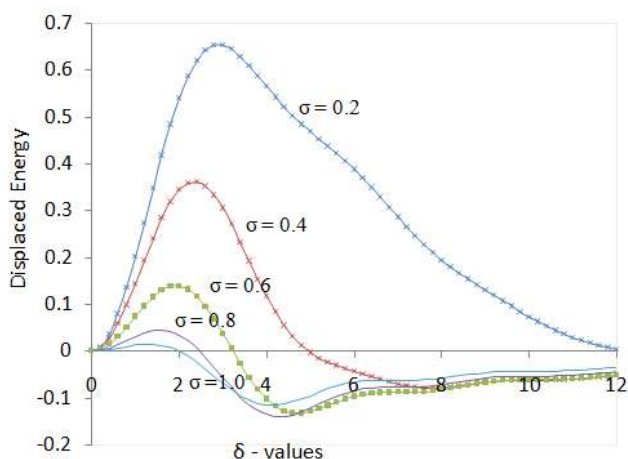

Figure 3: Excluded Encircled Energy

Figure 4: Displaced Energy

4. Conclusion

Various energy parameters encircled energy, relative encircled energy; excluded energy and displaced energy of symmetrical optical system apodized with Gaussian filter are studied. The said parameters are computed against the radius

of PSF for various values of apodisation parameter. This kind of analysis is useful in imaging process of health care sector.

References

[1] Lord Rayleigh, Study of encircled energy in the diffraction pattern as an image quality parameter, Encyl.Brit, Vol.24, p.430, 1888.

[2] 2)G. Lansraux, G.Boivin, Computing techniques for the numerical evaluation of the encircled energy in the diffraction pattern with aberrated system, Canada Journal of Physics, Vol. 36, p. 1696,1958.

[3] S. Vishvanatham, K.P.Rao, P.K.Mondal, Study of fraction of the encircled energy in the diffraction pattern of circular aperture, Indian Journal of Pure and Applied Physics, Vol.17, p .678, 1979.

[4] Murthy, PSF based corollaries in the presence of defocusing by employing co-sinusoidal amplitude filter, Ph.D .Thesis, Osmania University, Hyderabad, 1993.

[5] S.C.Biswas, A. Boivin, Study of influence of wave aberrations on the performance of optimum apodisers, Journal of Optics (Paris), Vol.4, p. 1, 1975.

[6] T.Asakura, Energy corollaries of the PSF with optimum apodisation used in an optical system, Oyo Buturi, Vol.31, p.730, 1962.

[7] K.P.Rao, P.K.Mondal, and T.Seshagiri Rao, Encircled energy and its complementary quantity excluded energy using Straubel apodisation filter, Pramana, vol. 7, no. 6, pp. 389–396,1976.

[8] Keshavulu, Encircled energy with the presence of defocusing effects apodised with shaded aperture systems, Ph.D. Thesis, Osmania University, India, 2001.

[9] A. Srisailam, Role of Excluded energy and relative encircled energy in the point image quality assessment, Ultra Scientist, Vol.25 (2) A, pp.333-338, 2013.

[10] P.Thirupathi, Encircled energy factor in impulse response functions of optical systems with first order parabolic filters, Advances in Applied Science Research, Vol. 3(6), pp.3935-3943, 2012.

[11] D.Karunasagar, - Ph.D -Thesis, Studies on the performance of Optical System Apodised with generalized Hanning amplitude filters, p.62, 2003.

[12] A.Boivion, The amplitude of the light diffracted in the far field region associated with rotationally symmetric pupil function, Gautheir-Villars, Paris, p.97, 1964.

[13] Wetherill, The calculation of image quality parameters, Applied optics and Applied Engineering, Vol.8, Ch.6, pp.173-283, 1980.