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Logistic Black-Scholes-Merton Partial Differential Equation: A Case of Stochastic Volatility

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Abstract: ***Real world systems have been created using differential equations, this has made it possible to predict future trends and behavior. Specifically stochastic differential equations have been fundamental in describing and understanding random phenomena. So far the Black-Scholes-Merton partial differential equation used in deriving the famous Black-Scholes-Merton model has been one of the greatest breakthroughs in finance as far as prediction of asset prices in the stock market is concerned. In this model we use the Logistic Brownian motion as opposed to the usual Brownian motion and we also consider volatility to be stochastic. In this study we have incorporated the stochastic nature of volatility and derived a Logistic Black-Scholes-Merton partial differential equation with stochastic volatility. This has been done by analyzing the Logistic Brownian motion and the Brownian motion, using the Ito process, Ito's lemma, stochastic volatility model and reviewing the derivation of the Black-Scholes-Merton partial differential equation. The formulated Differential equation may enhance reliable decision making based on more rational prediction of asset prices.***

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1. Introduction

Logistic Geometric Brownian motion model

In relaxing one of the assumptions of the Black-Scholes-Merton partial differential equation and using the Walrasian law and the excess demand function $ED(S(t)) = Q_D(S(t))$-$Q_S(S(t))$, where $ED(S(t))$ represents the excess demand, $Q_D(S(t))$ and $Q_S(S(t))$ are the quantities demanded and supplied respectively, the price of an asset follows a logistic geometric Brownian motion given by equation ;

$$dS - \mu S(S^* - S)dt + \sigma S(S^* - S)dZ$$

$$\frac{1}{S}\frac{dS}{\left(S^* - S\right)} = \mu dt + \sigma dZ \qquad (1)$$

where S^* is the Walrasian market equilibrium price, S is the stock price at any given time t, μ is the drift rate and σ is the volatility of the stock price at any given time t. Here, volatility σ is constant, [37].

We use the Logistic Geometric Brownian Motion in equation (1) and a choice of portfolio in equation $\Pi = -C + \frac{\partial C}{\partial S}S$ and the change in portfolio equation $\delta\Pi = -\delta C + \frac{\partial C}{\partial S}\delta S$ to derive to derive the Logistic Black-Scholes-Merton Partial differential equation give as,[37]

$$\frac{\partial C}{\partial t} + rS(S^* - S)\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \left(S^* - S\right)^2 \frac{\partial^2 C}{\partial S^2} = rC \qquad (2)$$

Volatility

Volatility is the measure of how uncertain we are about future stock price movement. The volatility of a stock price σ is defined so that $\sigma\sqrt{\delta t}$ is the standard deviation of the return on stock in a short period of time δ .As volatility increases therefore, the chance that a stock will do very well or very poorly increases, which results in both the call and put options rising or falling respectively.

Stochastic volatility

One assumption In the Black-Scholes-Merton model is that volatility is always constant. However Hull and White [16],[17], among others considered stochastic volatility models. They considered the fact that in a real markets situation volatility may follow a stochastic process of the following forms among others,

$$d\sigma = \mu_\sigma \sigma dt + \nu_\sigma \sigma dZ \qquad (3)$$

or

$$d\sigma = \mu_\sigma \sigma\left(b - \sigma\right)dt + \nu_\sigma \sigma dZ \qquad (4)$$

where μ, b and ν are constants and dZ refers to a Wiener process, σ is the asset volatility while μ_σ and ν_σ are the mean and variance of asset volatility respectively. In equation (4) the variance rate has a drift that pulls it back to a level b at a rate μ_σ ..

Multidimensional $It\hat{o}'s$ lemma

When functions have more than one random variable from which we can get a family of differential equations using the price of an underlying assets as

$$dX_i = \mu_i X_i dt + \sigma_i X_i dZ_i \qquad (5)$$

Where x_i is the stock price of the i^{th} asset, $i = 1,\ldots\ldots,N$, and μ_i and σ_i the drift and volatility of the i^{th} asset respectively, while dZ_i is the respective increase in the Wiener process. We have dZ_i is equal to $\varepsilon_i\sqrt{dt}$ where ε_i is a random drawing from the normal distribution table. Thus dZ_i has a mean of zero and a standard deviation of $\sqrt{dt}$ hence

$E\left(dZ_i\right) = 0$ and $E\left(dZ_i^2\right) = dt$

If Z_i and Z_j are correlated , the Wiener processes are dZ_i and dZ_j ,where $\operatorname{var}(dZ_i, dZ_j) = E(dZ_i dZ_j) = \rho_{ij}$, in this case ρ_{ij} is the correlation coefficient between th i^{th} and j^{th} Wiener processes. To manipulate the functions $G(X_1, X_2, ...X_N, t)$ of many stochastic variables $X_1, X_2, ...X_N$ and t then by the $It\hat{o}'s$ lemma we have

$$dG = \left(\frac{\partial G}{\partial t} + \frac{1}{2}\sum_{i=1}^{N}\sum_{i=2}^{N}\sigma_i\sigma_j\rho_{ij}X_iX_j\frac{\partial^2 G}{\partial X_i \partial X_j}\right)dt + \sum_{i=1}^{N}\frac{\partial G}{\partial X_i}dX_i \quad (6)$$

where $dZ_i^2 = dt, dZ_j^2 = dt$ and $dZ_i dZ_j = \rho_{ij}dt$ [16],[18],[37],[57],

By $It\hat{o}'s$ Multiplication table we have

*	dZ_i	dt
dZ_j	$\rho_{ij}dt$	0
dt	0	0

In case of two random variables X_1 and X_2 and a deterministic variable t , that is

$dX_1 = m_1(X_1, X_2, t) + n_1(X_1, X_2, t)dZ_1$ and

$dX_2 = m_2(X_1, X_2, t) + n_2(X_1, X_2, t)dZ_2$

In which dZ_1 and dZ_2 are Brownian increments, both normally distributed with variance dt and correlation ρ , $-1 \le \rho \le 1$, therefore from equation (6), we have

$$dG = \left(\frac{\partial G}{\partial t} + \frac{1}{2}n_1^2\frac{\partial^2 G}{\partial X_1^2} + \frac{1}{2}n_2^2\frac{\partial^2 G}{\partial X_2^2} + \rho n_1 n_2\frac{\partial^2 G}{\partial X_1 \partial X_2}\right)dt + \frac{\partial G}{\partial X_1}dX_1 + \frac{\partial G}{\partial X_2}dX_2 \quad (7)$$

The Logistic Black-Scholes-Merton Partial differential equation: A case of stochastic volatility

$It\hat{o}'s$ lemma can be used to transform two stochastic differential equations to obtain a pricing model in a case where volatility is stochastic. We assume that the asset price S follows a logistic geometric Brownian Motion of the form

$$dS - \mu S(S^* - S)dt + \sigma S(S^* - S)dZ \quad (8)$$

and the stochastic volatility also follows a Geometric brownian motion of the form,

$$d\sigma = \mu_\sigma S dt + \nu_\sigma \sigma dZ_2 \quad (9)$$

where μ_σ and ν_σ are the mean and variance of asset volatility respectively, and dZ_1 and dZ_2 are correlated Wiener processes (with the correlation coefficient $\rho \ne 1$) associated with the two differential equations (8) and (9) respectively. We let the Wiener processes have a correlation ρ .Considering equations (8) and equation (9), the value of an option is therefore a function of three variables, $C(S, \sigma, t)$where C is the price of the call option and S is the asset price. Since volatility is not a traded asset, its randomness cannot be easily traded away. Having two other sources of randomness therefore, we need to hedge our options against two other contracts, one being the Underlying asset as usual but the other to hedge the volatility risk. Consider a portfolio containing one option with values $C(S, \sigma, t)$, another quantity $-\delta$ (or $-\frac{\partial C}{\partial S}$) of the asset and finally $-\delta_1$ (or $-\frac{\partial C_1}{\partial S}$) of another option with a value $C_1(S, \sigma, t)$. Here δ and δ_1 of the option in this case represent the sensitivity of the option or portfolio to the underlying. The value of the portfolio will therefore be

$$\Pi = C - \delta S - \delta_1 C_1 \quad (10)$$

The change in the portfolio $d\Pi$ will be given by

$$d\Pi = dC - \delta dS - \delta_1 dC_1 \quad (11)$$

Using $It\hat{o}'s$ lemma on S , σ and t and the application in equation (7) from equation (8) and (9), we obtain

$$dC = \left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C}{\partial S \partial \sigma}\right)dt + \frac{\partial C}{\partial S}dS + \frac{\partial C}{\partial \sigma}d\sigma \quad (12)$$

The change in portfolio at time dt s therefore given as,

$$\begin{aligned} d\Pi = &\left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C}{\partial S \partial \sigma}\right)dt + \frac{\partial C}{\partial S}dS + \frac{\partial C}{\partial \sigma}d\sigma - \delta dS \\ &-\delta_1\left(\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C_1}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C_1}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C_1}{\partial S \partial \sigma}\right)dt - \delta_1\frac{\partial C_1}{\partial S}dS - \delta_1\frac{\partial C_1}{\partial \sigma}d\sigma \end{aligned} \quad (13)$$

Collecting the terms in dS and $d\sigma$ in equation (13) we obtain,

$$\begin{aligned} d\Pi = &\left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C}{\partial S \partial \sigma}\right)dt \\ &-\delta_1\left(\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C_1}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C_1}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C_1}{\partial S \partial \sigma}\right)dt - \left(\frac{\partial C}{\partial S} - \delta_1\frac{\partial C_1}{\partial S} - \delta\right)dS \\ &+\left(\frac{\partial C}{\partial \sigma} - \delta_1\frac{\partial C_1}{\partial \sigma}\right)d\sigma \end{aligned} \quad (14)$$

In order to eliminate all randomness we choose $\frac{\partial C}{\partial S} = \delta_1\frac{\partial C_1}{\partial S} + \delta$ and $\frac{\partial C}{\partial \sigma} = \delta_1\frac{\partial C_1}{\partial \sigma}$ making dS and $d\sigma$ terms to be equal to zero. After eliminating dS and $d\sigma$ which contain the Wiener Process dZ_1 and dZ_2 respectively, equation (14) becomes a non stochastic differential equation

$$\begin{aligned} d\Pi = &\left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C}{\partial S \partial \sigma}\right)dt \\ &-\delta_1\left(\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C_1}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C_1}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C_1}{\partial S \partial \sigma}\right)dt \end{aligned} \quad (15)$$

We use the no arbitrage arguments to set the return of the portfolio to be equal to the risk free interest rate r as follows,

$$d\Pi = r\Pi dt \quad (16)$$

Substituting equations (10) and (15) into equation (16) we obtain,

$$\begin{aligned} d\Pi = &\left(\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C}{\partial S \partial \sigma}\right)dt \\ &-\delta_1\left(\frac{\partial C_1}{\partial t} + \frac{1}{2}\sigma^2 S^2(S^* - S)^2\frac{\partial^2 C_1}{\partial S^2} + \frac{1}{2}\nu_\sigma^2\sigma^2\frac{\partial^2 C_1}{\partial \sigma^2} + \rho\sigma^2 S(S^* - S)\nu_\sigma\frac{\partial^2 C_1}{\partial S \partial \sigma}\right)dt \\ &= r(C - \delta S - \delta_1 C_1)dt \end{aligned} \quad (17)$$

We now have a situation where we have one equation with two unknowns C and C_1. Given that $\delta = \frac{\partial C}{\partial S}$ and $\delta_1 = \frac{\partial C_1}{\partial S}$ and that both are affected by a hedge ratio $\frac{\partial C}{\partial \sigma}$ and $\frac{\partial C_1}{\partial \sigma}$ (which are also the
Sensitivities of option price to volatility) respectively, we Collect the terms in C on one side and those in C_1 to be on the otherto obtain,

$$\frac{\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2(S^*-S)^2\frac{\partial^2 C}{\partial S^2}+\frac{1}{2}v_\sigma^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2}+\rho\sigma^2 S(S^*-S)v_\sigma\frac{\partial^2 C}{\partial S\partial\sigma}+rS\frac{\partial C}{\partial S}-rC}{\frac{\partial C}{\partial \sigma}}$$

$$\frac{\delta_1\left(\frac{\partial C_1}{\partial t}+\frac{1}{2}\sigma^2 S^2(S^*-S)^2\frac{\partial^2 C_1}{\partial S^2}+\frac{1}{2}v_\sigma^2\sigma^2\frac{\partial^2 C_1}{\partial \sigma^2}+\rho\sigma^2 S(S^*-S)v_\sigma\frac{\partial^2 C_1}{\partial S\partial\sigma}+-rC_1\right)}{\frac{\partial C_1}{\partial \sigma}}$$

Since the two different options will have different payoffs, this possibility can only be obtained if the left hand side and the right hand side are independent of the contract type. Both sides therefore can only be functions of the independent variables, S, σ and t and thus we have

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2(S^*-S)^2\frac{\partial^2 C}{\partial S^2}+\frac{1}{2}v_{S\sigma}^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2}+\rho\sigma^2 S(S^*-S)v_\sigma\frac{\partial^2 C}{\partial S\partial\sigma}+rS\frac{\partial C}{\partial S}-rC$$
$$=-(\mu_\sigma-\lambda v_\sigma)\frac{\partial C}{\partial \sigma} \quad (18)$$

for some function $\lambda(S,\sigma,t)$ which is the market price of volatility risk and $\mu_\sigma - \lambda v_\sigma$ is the risk neutral drift rate of volatility. Rewriting this equation we obtain

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2(S^*-S)^2\frac{\partial^2 C}{\partial S^2}+\frac{1}{2}v_{S\sigma}^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2}+\rho\sigma^2 S(S^*-S)v_\sigma\frac{\partial^2 C}{\partial S\partial\sigma}+rS\frac{\partial C}{\partial S}+$$
$$(\mu_\sigma-\lambda v_\sigma)\frac{\partial C}{\partial \sigma}-rC=0 \quad (20)$$

This equation gives us the equivalent of the Black-Scholes-Merton partial differential equation but with stochastic volatility.

If we let Z_1 and Z_2 to be of the the same distribution, then $dZ_1=dZ_2$, hence $\rho=1$ since $dZ_1^2=dt$ thus equation (20) becomes,

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2(S^*-S)^2\frac{\partial^2 C}{\partial S^2}+\frac{1}{2}v_{S\sigma}^2\sigma^2\frac{\partial^2 C}{\partial \sigma^2}+\sigma^2 S(S^*-S)v_\sigma\frac{\partial^2 C}{\partial S\partial\sigma}+rS\frac{\partial C}{\partial S}+$$
$$(\mu_\sigma-\lambda v_\sigma)\frac{\partial C}{\partial \sigma}-rC=0 \quad (21)$$

Equation 20 is therefore the Logistic Black-Scholes-Merton Partial Differential equation with stochastic volatility.

A solution to this equation based on various boundary conditions may enhance reliable decision making based on a rational prediction of future asset prices.

2. Conclusion and Recommendations

In this papers, we have managed to derive a Logistic Black-Scholes-Merton Partial differential equation with stochastic volatility (equation 20) . This is a major breakthrough in the study of the Black-Scholes-Merton Partial differential equation and its application in the prediction of future asset prices where volatility is Stochastic rather than constant as has been the assumption in all other previous studies

We recommend that this differential equation be solved by interested scholars in order to enhance prediction of future asset prices

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