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Variable Viscosity and Prandtl Number Effects on Natural Convection Water Boundary Layers about a Vertical Plate

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Abstract: ***The aim of this article is to study the effect of variable viscosity and Prandtl number on the steady, laminar flow (of water) past a vertical porous flat plate. The coupled parabolic partial difference equations governing the self-similar flow have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization technique. Results indicate that variable viscosity (and Prandtl number) has a major role on skin friction and heat transfer parameters as well as velocity and temperature fields. Further, it is observed that the effect of variable fluid properties plays a significant role in the control of laminar boundary layer. investigation reveals the fact that when the working fluid is sensitive to the temperature, the effect of variable viscosity and Prandtl number has to be taken into the consideration in order to predict the skin friction and heat transfer rate, accurately.***

Keywords: Skin friction, Heat transfer, Temperature-dependent Viscosity, Velocity, Temperature

MSC 2010 Codes – 76M20, 76N20, 76R10

1. Introduction

In the analysis of boundary layer phenomenon, applications of heat transfer are generally based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially the fluid viscosity and hence, the Prandtl number. Several researchers have studied the effect of variable viscosity on different geometries under various situations [1-7].

Free or natural convection boundary layer flows frequently encountered in environmental and engineering devices. Extensive literature is available on the topic of the laminar boundary layer flow over a porous vertical plate with suction and injection, having wide range of engineering applications. In fact, the case of uniform suction and blowing (injection) through an isothermal vertical wall was treated first by Sparrow and Cess [8]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [9], who obtained asymptotic solutions, valid at large distances from the leading edge, for both suction and blowing (injection).The main objective of the present study is to investigate the effect of variable viscosity and Prandtl number on the free convection boundary layer flow(of water) over a vertical porous plate with suction.

2. Governing Equations

Consider a semi-infinite porous plate, which is played vertical in a quiescent fluid (water) of infinite extent maintained at an uniform temperature. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge (origin) in the stream wise direction and y is measured normal to the surface of the plate. Indeed, the flow is assumed to be in the x-direction i.e., along the vertical plate in the upward direction and the y-axis is taken to be normal to the plate.

The fluid properties are assumed to be isotropic and constant except for the fluid viscosity. The temperature difference between the surface of the plate(T_w) and the ambient fluid(T_∞) is taken to be small ($< 50^0$C). In the range of temperature (T) considered (i.e. 0-50^{0}C), the variation of both density (ρ) and specific heat (c_p) of water with temperature, is less than 1% (See Table I) and hence they are taken as constants. However, since the variation of thermal conductivity (k) and viscosity (μ) [and hence the Prandtl number (Pr)] with temperature is quite significant, the viscosity (μ) and Prandtl number are assumed to vary as an inverse linear function of temperature:

$$\mu = 1/\left(b_1 + b_2 T\right) \tag{1}$$

$$\mathrm{Pr} = 1/\left(c_1 + c_2 T\right) \tag{2}$$

where

$$b_1 = 53.4100, b_2 = 2.4300, c_1 = 0.068 \text{ and } c_2 = 0.0040 \tag{3}$$

Table I: Values of thermo-physical properties of water at different temperature [10].

Temperature (T) (^{0}C)	Density (ρ) (gr./cm^3)	Specific heat(c_p) (J × 10^7/kg^{0}K)	Thermal conductivity (k)(erg× 10^5/cm.s-^{0}K)	Viscosity (μ) (gr. × 10^{-2}/ cm-s)	Prandtl number (Pr)
0	1.00228	4.2176	0.5610	1.7930	13.48
10	0.99970	4.1921	0.5800	1.3070	9.45
20	0.99821	4.1818	0.5984	1.0060	7.03
30	0.99565	4.1784	0.6154	0.7977	5.12
40	0.99222	4.1785	0.6305	0.6532	4.32
50	0.98803	4.1806	0.6435	0.5470	3.55

The relation (1) and (2) are reasonably holds good approximations for liquids such as water, particularly for small wall and ambient temperature differences. Further, the fluid added (injection) or removed (suction) is the same as that involved in flow. Under the above-mentioned assumptions with Boussinesq's approximation, the boundary layer equations governing the steady, two-dimensional flow are [11]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (4)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \qquad (5)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left[\left(\Pr^{-1}\mu\frac{\partial T}{\partial y}\right)\right] \qquad (6)$$

The initial and boundary conditions are

$$x=0,\ y>0;\ u=v=0,\ T=T_w$$
$$x>0;\ y=0;\ u=0,\ v=-v_0(\ for\ suction),$$
$$x>0;\ y=0;\ u=0,\ v=+v_0(\ for\ injection),$$
$$y\to\infty;\ x>0;\ u=0,\ T=T_\infty \qquad (7)$$

Introducing the following transformations

$$u=\frac{\partial\psi}{\partial y};\ v=-\frac{\partial\psi}{\partial x};\ \psi=\frac{v^2 g\beta(T_w-T_\infty)\xi^3}{V_0^3}\left[f(\eta,\xi)\pm\frac{\xi}{4}\right]$$

$$T=T_\infty+\left(T_w-T_\infty\right)G(\eta,\xi);\ \eta=\frac{V_o y}{v\xi};\ \xi=V_0\left[\frac{4x}{v^2 g\beta\left(T_{wo}-T_\infty\right)}\right]^{1/4} \qquad (8)$$

to Eqns.(4) – (6), we see that the continuity Eq.(4) is identically satisfied and Eqns.(5) – (6) reduces, respectively, to

$$\left(NF'\right)'+3fF'-2F^2\pm\xi F'+G=\xi\left(F F_\xi-F' f_\xi\right) \qquad (9)$$

$$\left(N\Pr^{-1}G'\right)'+3fG'\pm\xi G'=\xi\left(FG_\xi-G' f_\xi\right) \qquad (10)$$

where

$$u=\frac{V_0^2 4x}{v\xi^2}F;\quad v=-\frac{V_0}{\xi}\left(3f+\xi f_\xi-\eta F\pm\xi\right)$$

$$f=\int_0^\eta F\,d\eta\ ;\ N=\left(\frac{\mu}{\mu_\infty}\right)=\frac{b_1+b_2T_\infty}{b_1+b_2T}=\frac{1}{1+a_1G},$$

$$\Pr=\frac{1}{c_1+c_2T}=\frac{1}{a_2+a_3G},\quad a_1=\left(\frac{b_2}{b_1+b_2T_\infty}\right)\Delta T_w,$$

$$a_2=c_1+c_2T_\infty,\quad a_3=c_2\Delta T_w,\quad \Delta T_w=\left(T_w-T_\infty\right) \qquad (11)$$

It is noted here that the upper and lower signs in Eqns. (9) and (10) is taken thought for suction and injection, respectively. The present study, however, restricted to the case of suction only.

The transformed boundary conditions are:

$$\begin{aligned} &F=0; \quad G=1 \quad at \quad \eta=0\\ &F=0; \quad G=0 \quad as \quad \eta\to\infty \end{aligned} \qquad (12)$$

The local skin friction parameter and heat transfer parameter can be expressed as

$$\tau_w=\frac{V_0}{g\beta\left(T_{w0}-T_\infty\right)}\left(\frac{\partial u}{\partial y}\right)_{y=0}=\xi\left(F'\right)_{\eta=0} \qquad (13)$$

$$Q=\frac{v}{V_0\left(T_{w0}-T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\frac{1}{\xi}\left(G'\right)_{\eta=0} \qquad (14)$$

Here, u and v are velocity components in x and y-directions respectively; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ and η are transformed co-ordinates; ψ and f are the dimensional and dimension- less stream functions respectively; Pr is the Prandtl number; $a_1,a_2,a_3,b_1,b_2,c_1,\&\ c_2$ are constants; g is the gravitational acceleration; β is the coefficient of thermal expansion; w and ∞ denote conditions at the edge of the boundary layer on the wall and in the free stream respectively, and prime $(')$ denotes derivatives with respect to η.

3. Results and Discussion

The set of partial differential Eqns.(9) and (10) along with the boundary conditions (12) has been solved numerically employing an implicit finite difference scheme with a quasilinearization technique [11,12]. The detailed description is omitted here to conserve the space. In order to assess the accuracy of the numerical method which we have used, the skin friction and heat transfer parameters (τ_w, Q) for suction have been obtained by solving Eqs.(9) and (10) for constant viscosity [N=1] case, taking Pr=1.0, and compared with those of Merkin [9]. Our results are found to be in good agreement with those of [9], validating the accuracy of the numerical method used in the present study. The results for variable fluid properties have been presented in the graphical form and analyzed.

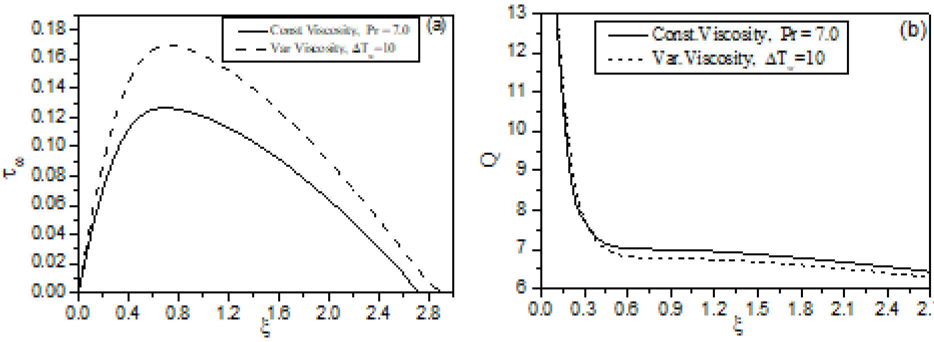

Figure 1: Variation of (a) skin friction and (b) heat transfer parameters along the stream wise direction

Figure 1 describes the variation of skin friction (τ_w) and heat transfer parameters (Q) with the stream wise coordinate ξ, in the presence of both variable fluid properties [T_∞ =18.7°C, ΔT_w= 10.0] and constant fluid properties [N =1and Pr = 7.0], in the presence of suction. It is observed from Fig.1(a) that skin friction (τ_w) increases from zero to a maximum value in a certain range of ξ say ξ =1.2, and then decreases as ξ further increases. It is also observed that the effect of variable fluid properties is to increase the skin friction and to decrease the heat transfer. In fact, τ_w for variable fluid properties differs from that of constant fluid fluid properties by about 23.78% at ξ =0.5 and about 42.50% at ξ =2.5. On the other hand, the percentage of difference between constant and variable fluid properties, in the case of Q [Fig.1(b)] is 2.79% at ξ =0.5 and about 2.11% at ξ =2.5.Further, it is important to observe that in the case variable fluid properties, the point of zero skin friction moved downstream as compared constant fluid properties. Indeed, the point of zero skin friction occurs at ξ =2.8 for constant fluid properties whereas it is visible at ξ =2.9 [Fig.(a)] for variable fluid properties. This vindicates the controlling of laminar boundary layer avoiding boundary layer separation.

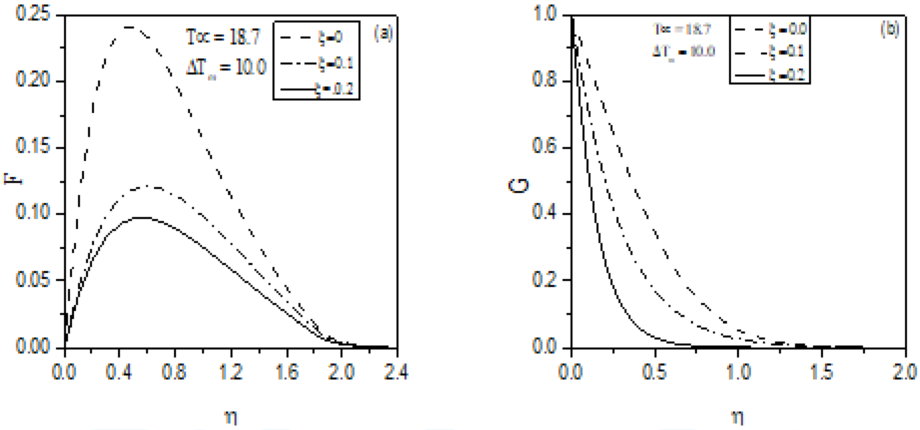

Figure 2: Behavior of (a) Velocity and (b) Temperature profiles at different stream wise locations

The relevant velocity (*F)* and temperature (*G*) profiles are shown in Fig.2,for the case of variable fluid properties. It is observed that the thickness of momentum boundary layer increases with the increase of streamwise coordinate (ξ), which results in the velocity of the fluid inside the boundary layer. On the other hand, the thermal boundary layer thickness decreases as ξ increases, enhancing the temperature inside the boundary layer.

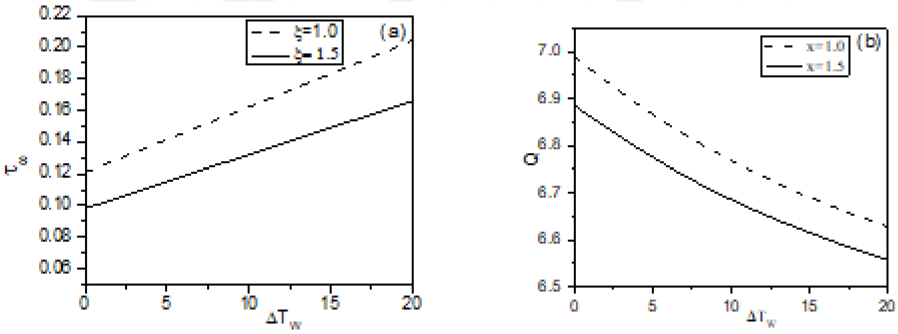

Figure 3: Effect of ΔT_W on (a) Skin friction and (b) Heat transfer parameters at stream wise locations

The variation of skin friction (τ_w) and heat transfer parameters (Q) with ΔT_w difference temperature between the surface of the plate and the ambient fluid,) is depicted in Fig.3, to see the effect of variation of viscosity and Prandtl number with temperature across the boundary layer. Since T_∞ =18.7°C, the maximum value of ΔT_w taken is 20°C so as to keep the temperature within the allowed value (< 40°C), during numerical computations. In these figures, we observe that τ_w increases while, Q decreases with the increase of ΔT_w. Also, Q is found to decrease with the increase of ξ. Further, it is observed that the heat transfer decreases by about 1.30% when ΔT_w = 5°C and, about 1.07% when ΔT_w= 20°C, at ξ =1 and ξ =1.5, respectively. On the other hand, the skin friction decreases by about 22.93% when ΔT_w= 5°C and, about 23.16% when ΔT_w= 20°C, at same stream wise locations of the vertical plate. It is clear from these figures that temperature-dependent viscosity as well as Prandtl number plays a key role in providing strong coupling of energy and momentum transfer, exposing their supremacy within the laminar boundary layer region.

4. Conclusions

The steady, laminar water boundary layer flow past a vertical porous flat plate is numerically investigated assuming both viscosity and Prandtl number as linear inverse functions of temperature. The computed results show that the flow/temperature fields, skin friction and heat transfer characteristics are significantly affected by the temperature-dependent viscosity and Prandtl number. From the present study, it is concluded that the effect of variable fluid properties plays a significant role in the control of laminar boundary layer.

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