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An Analysis of Unsteady MHD Fluid Flow in a Vertical Semi-Infinite Plate with Constant Transverve Magnetic Field

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Abstract: ***An MHD stokes free convection of an incompressible electrically conducting fluid flowing on a vertical porous semi-infinite plate has been considered. A uniform magnetic field is applied perpendicularly in the positive y-direction to the flow. An analysis of the effect of the uniform magnetic field, Hartmann and Prandtl numbers on both velocity profiles and temperature distribution are presented. The momentum and energy equations are both coupled and have been simultaneously solved numerically by the central finite difference approximations. The results obtained are discussed and presented both in tabular and graphical form. An increase in Hartmann is found to cause a decrease in velocity profile while an increase in Prandtl leads to a fall in temperature distribution. These results are found to merge with the physical situation of the flow***

Keywords: porous, incompressible, free convection

1. Introduction

Hydro magnetic is the science of the motion or flow of electrically conducting fluid in the presence of a magnetic field. It involves a mutual interaction between the fluid velocity field and the electromagnetic field. The study of an incompressible heat generating fluid past an infinite vertical porous plate has applications in many areas of science and engineering. Recently, considerable attention has been focused on new applications of magneto hydrodynamics (MHD) and heat transfer such as metallurgical processing. Melt refining involves magnetic field applications to control excessive heat transfer rate. Other applications of MHD heat transfer include MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems.This includes MHD generators, plasma studies, nuclear reactors, geothermal extractors and boundary layer control in the field of aeronautics and aerodynamics. In addition, this flow in a porous media is used to study the migration of underground water, movement of oil, gas and water through the reservoir, water purification, ceramic engineering and powder metallurgy.

The hydro magnetic flows and heat transfer have become more important in recent years because of its varied applications in agricultural engineering and petroleum industries. A lot of investigations have been done on MHD flows in a porous medium. Considerable progress on modifications and improvements on designs and manufacture of scientific equipments and machines has been made. However, more researches need to be done to produce better scientific equipments to cope with the ever changing technologies. Among the recent studies made on this topic includes a research done by Adel *et al* [2] on heat and mass transfer along a semi-infinite vertical flat plate under the combined buoyancy force effects of thermal and species diffusion in the presence of a strong non-uniform magnetic field. The similarity equations were solved numerically by using a fourth-order Runge-Kutta scheme with the shooting method. Kinyanjui, *et al*[8] presented work on MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite porous plate with Hall currents and radiation absorption while Emad *et al* [5] studied Hall current effect on magneto hydrodynamics free-convection flow past a semi-infinite vertical plate with mass transfer. They discussed the effects of magnetic parameter, Hall parameter and the relative buoyancy force effect between species and thermal diffusion on the velocity, temperature and concentration.

Jagadeeswara *et al* [7] studied viscous fluid flow past a hot vertical porous plate. The flow parameters in this study were analyzed under the assumptions that the suction velocity was constant and the wall temperature was span wise cosinusoidal. The solutions for the velocity, the temperature, skin friction and rate of heat transfer were obtained using perturbation method. The study observed that both the velocity and the skin friction decrease as the Magnetic parameter increases. The values of all flow quantities in the magnetic case were less than the values in the non-magnetic case. The study also found that the velocity and the skin friction increased with increasing suction. Further, Kwanza *etal*[9] presented their work on MHD stokes free convection past an infinite vertical porous plate subjected to a constant heat flux with ion-slip and radiation absorption. They discussed their tabulated results on concentration, velocity profiles and temperature distributions both theoretically and graphically whilst Palani and Abbas[11] carried out an investigation on Free Convection MHD Flow with Thermal Radiation from an Impulsively Started Vertical Plate. They established that velocity increases with a decrease in magnetic field parameter. In addition they realized that dimensionless temperature decreases with an increase in thermal radiation. Further, Loganathan and Sivapoornapriya [10] investigated on unsteady natural convective flow over an impulsively started semi-infinite vertical plate in the presence of porous medium with chemical reaction.

Abuga *et al* [1], Conducted an investigation on unsteady transient free convection MHD flow between two long vertical parallel plates with constant temperature and variable mass diffusion. They established that velocity and skin friction of the fluid increase with increase with the value of time but decrease with increasing the value of the Prandtl number, Schmidt number and magnetic parameter. Similarly they found that an increase in Rotational parameter led to a decrease in velocity profile when the Eckert number was 0.01 and an increase in velocity profile when Eckert number was 0.02. Furthermore they realized that an increase in time led to an increase in both primary and secondary velocity profiles in case of cooling of the plates by convection currents but led to a decrease in velocity profiles in case of heating the plates by convection currents. In addition [4] carried an investigation on unsteady free convection MHD flow and heat transfer between two heated vertical plates with heat source. It was found that an increase in Hartmann number caused high velocity profiles near the walls and low velocity profiles at the centre between the walls. Youn J [13] investigated the unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid in the vicinity of semi-infinite vertical porous moving plate in the presence of a transverse magnetic field. The plate moves with constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small permutation law, the effect of increasing values of the suction velocity parameter results into a slight increase in surface skin friction for lower values of plate moving velocity. It was also observed that for several values of Prandtl number, the surface heat transfer decreases by increasing the magnitude of suction velocity. Haque and Alam [6] studied the transient heat and mass transfer by mixed convection flow from a vertical porous plate with induced magnetic field, constant heat and mass fluxes. Pauline *et al* [12] carried out a study on unsteady hydro magnetic free convection flow past an infinite vertical porous plate in a porous medium. The governing equations neglected joule heating parameter in the energy equation and Hall Effect. The resulting partial differential equations were solved by Crank-Nicolson technique in which it was found that an increase in injection parameter accelerates the velocity of the flow field. Further, an increase in heat source parameter was found to increase the temperature of the flow field. Alireza *et al*[3] conducted a study on Heat transfer and MHD flow of non-Newtonian Maxwell fluid through a parallel plate channel. They used fourth order Runge Kutta numerical method to solve their governing equations and found that an increase in Hartmann number cause a decrease in velocity although its increase caused an increase in temperature.

In view of the above studies and due to the significance of flows over a vertical porous plate more investigations need to be done on this topic. This study has been carried out to analyze the effect of Hartmann and Prandtl numbers on velocity profiles and temperature distribution on an unsteady flow in semi-infinite porous plates.

2. Formulation of the Problem

An unsteady free convection flow of an electrically conducting, viscous, incompressible fluid past a vertical infinite porous plate is considered. A uniform magnetic field **B**, assumed unaltered, is applied perpendicular to the plates in the positive y-direction. It is assumed that the magnetic Reynolds number is so small such that the induced magnetic field is neglected in comparison to the applied one.

The -axis is taken along the infinite vertical porous wall in the upward direction and y -axis normal to the wall as shown in figure (1)

The governing equations of the problem are as follow

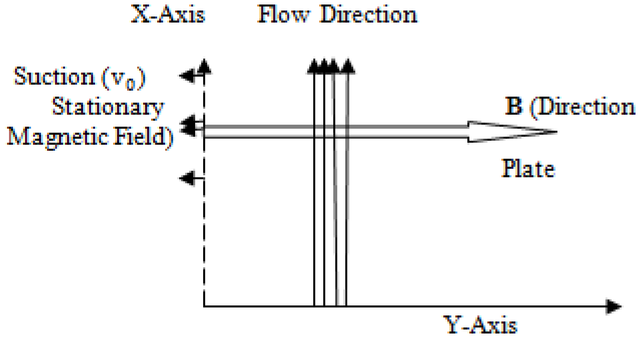


Figure 1

In order to derive the governing equations for this problem, the considerations are made;

- The plate is electrically non-conducting and very long in the x-z direction.
- The fluid is assumed to be incompressible with constant density and small Reynolds number.
- Effects of Hall current are ignored since a weak magnetic field is applied hence generalized Ohm's law negligible.
- Thermal conductivity, electrical conductivity and co-efficient of viscosity are constant.
- The fluid is Newtonian, does not undergo any chemical reaction and observes no slip conditions at the plates.

Since the plates are infinite in the x and z-directions, all the physical quantities except pressure do not change in these directions and hence simplifying the problem to a one-dimensional problem. The governing equations of the problem are expressed as follow;

$$\frac{\partial v}{\partial y} = 0 \quad \text{Continuity equation} \quad (1)$$

Integrating the equation (1) gives

$$V = -v_0 \quad (2)$$

where $v_0 > 0$ is the suction velocity.

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{1}{\rho}\nabla p + \vartheta\nabla^2 q + F \quad \text{Momentum} \quad (3)$$

The equation of continuity coupled with the simplified momentum equation result into Navier-Stokes equation of the form

$$\rho\left(-v_0\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \rho g + \vec{J} X \vec{B} \quad (4)$$

Energy equation is also expressed as

$$\rho C_p \frac{D_T}{D_t} = \nabla.(K\nabla.T) + \frac{1}{\sigma}j^2 + \mu\varphi \quad (5)$$

In regard to the restrictions considered in this problem, the equation simplifies to

$$\rho C_p \left(\frac{\partial T}{\partial t}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 \qquad (6)$$

The initial conditions are

$$t \le 0,\ u = 0\ ,\ T = 0 \ \text{ at } y = 0$$
$$t > 0,\ u = 0 \text{ and } T = T_1 \quad \text{for all } y \qquad (7)$$

In order to reduce the complexity of the problem, let us introduce the following non-dimensional variables

$$t = t^{*}\frac{U^2}{\vartheta},\quad y = y^{*}\frac{U}{\vartheta},\quad \theta = \frac{T^{*} - T_\infty^{*}}{T_0^{*} - T_\infty^{*}} \quad P_r = \frac{C_p\mu}{k}$$
$$u = \frac{u^{*}}{U},\quad v_0 = \frac{v_0^{*}}{U},$$
$$E_c = \frac{U^2}{C_p(T_w^{*} - T_\infty^{*})}\ x^{*} = \frac{x}{l}$$

In dimensional form the equation (4) is written as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + v_0\frac{\partial u}{\partial y} + Gr\theta - M^2 u \qquad (8)$$

Similarly equation (6) can be expressed in dimensional form as

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad (9)$$

The corresponding boundary conditions are;

$$t \le 0,\ u = 0\ ,\theta = 0 \ \text{ at } y = 0$$
$$t > 0,\ \ u = 1\ ,\theta = 1 \ \text{ for all } y \qquad (10)$$

3. Method of Solution

The partial differential equation (8) and (9) are nonlinear and hence cannot be solved analytically. These equations together with initial and boundary conditions (7) and (10) are solved numerically using the finite difference scheme. This scheme is chosen because it is relatively simple, accurate, and efficient and has a better stability characteristics. In addition, the technique is based on an iterative procedure and a tridiagonal matrix manipulation which provides satisfactory results but may be bound fail when applied to problems in which the differential equations are very sensitive to the choice of initial conditions. In all numerical solutions the continuous partial differential equation is replaced with a discrete approximation that is the numerical solution is known only at a finite number of points in the physical domain. The number of those points can be selected by the user of the numerical method. An increase in the number of points not only increases the resolution but also the accuracy of the numerical solution. The discrete approximations yield into a set of algebraic equations that are solved for the values of the discrete unknowns. The mesh is the set of locations where the discrete solution is computed. These points are called nodes and if one were to draw lines between adjacent nodes in the domain the resulting image would resemble a net or mesh. In order to use this scheme, the first and second order partial derivatives are replaced by their respective Taylors series finite difference approximations. Therefore, equations (8) and (9) when expressed in finite difference form become.

$$\frac{U_{ij+1} - 2U_{ij} + U_{ij-1}}{k^2} + v_0\frac{U_{ij+1} - U_{ij-1}}{2k} + Gr\theta_{ij} - M^2 u_{ij} = 0 \quad (11)$$

$$\frac{\theta_{ij+1} - \theta_{ij-1}}{k} = \frac{1}{P_r}\frac{\theta_{ij+1} - 2\theta_{ij} + \theta_{ij-1}}{k^2} \qquad (12)$$

The finite difference equations (11) and (12) are then solved using MATLAB PDE software.

4. Results and Discussions

The equations (11) and (12) are solved by the MATLAB software and graphs plotted for different values of dimensionless Hartmann number, M^2,Prandtl number, p_r and Grashof number,Gr.With these mentioned parameters, velocity profiles as well as the temperature profiles has been expedited in Fig.1 to Fig.6. In Fig.1, it has been depicted that as the Hartmann number increases the velocity profile decreases. This decrease occurs due to the presence of magnetic field in the conducting fluid. The influence of the magnetic field causes a resistive force called Lorentz force. This force has a tendency to slow down the lateral velocity. It is also noted that an increase in Hartmann numbers cause a decrease in temperature distribution of the flow as shown in Fig. 6. On the other hand, an increase in Grashof number was found to cause an increase in temperature of the flow field as indicated in Fig. 4. However, a rise in Grashof number was found to cause a decrease in velocity profiles.

The effect of Prandlt number on velocity profile has been shown in Fig. 5. It was found that the velocity profile decreases drastically for the increase of Prandtl number. Similarly, an increase in Prandtl number was found to cause a fall of temperature of the flow field.

Figure1: Effect of Varying Hartmann numbers on Velocity

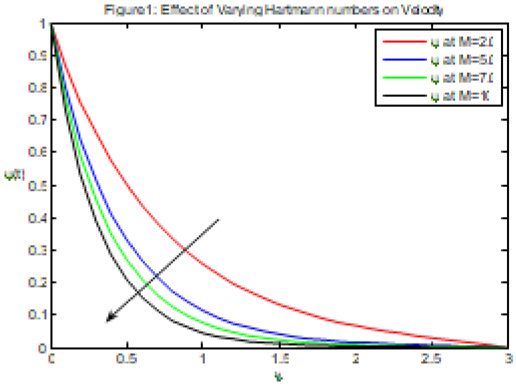
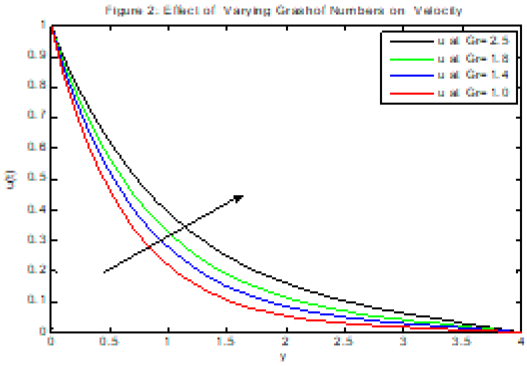


Figure 2: Effect of Varying Grashof Numbers on Velocity



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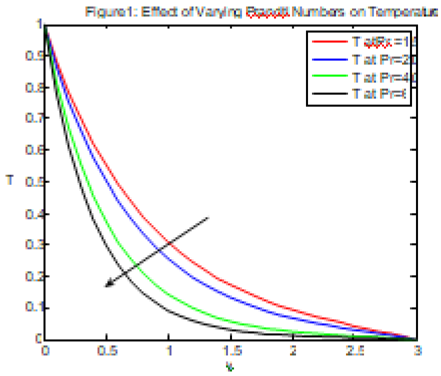

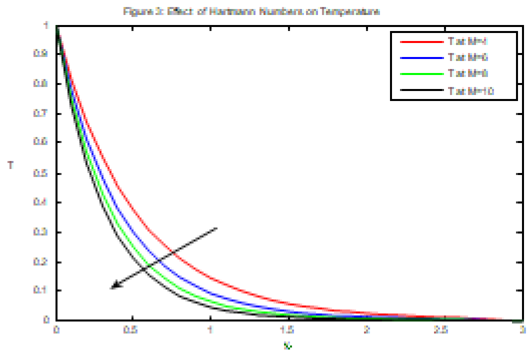

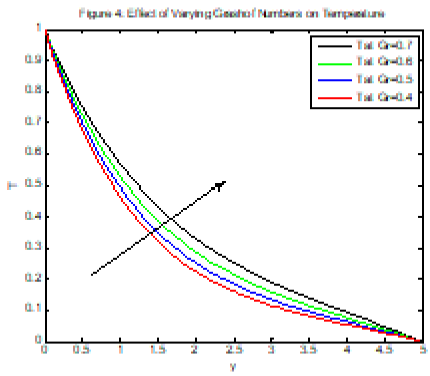

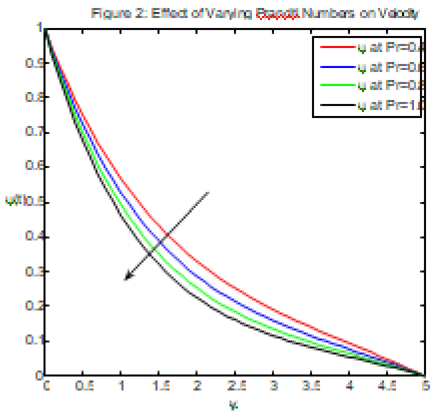

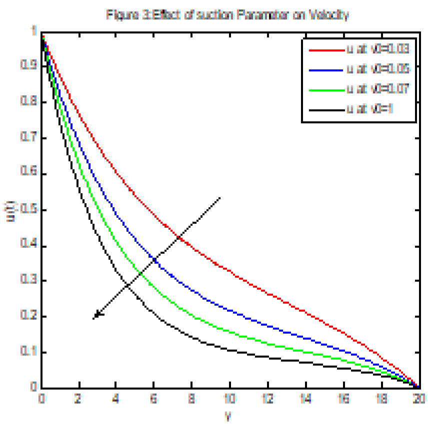

5. Conclusion

Velocity profiles and temperature distribution on a unsteady flow of an incompressible, viscous and electrically conducting fluid over a vertical plate in the presence of a uniform magnetic field have been investigated. Hartmann and Prandtl Numbers are found to have a great effect on velocity profiles and temperature distribution respectively. Results obtained for various values of these flow parameters have been found to suitably agree with the physical situation of the flow.

It has been noted that an increase in Hartmann number causes a decrease in velocity profiles while an increase in Prandtl Number leads to a fall in temperature distribution.

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