Revisions for Distance Measures of Xu

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Abstract: ***Xu (2017) developed a new distance that was published in the Journal of Intelligent and Fuzzy Systems. Xu (2018) provided a revision for his proof proposed in Xu (2017). However, we find that still contained questionable results such that we present a further amendment for Xu (2018). Our results will help researchers study distance and similarity measures in the future.***

Keywords: intuitionistic fuzzy sets, distance measure, similarity measure

1. Introduction

To construct new distance measures and similarity measures is an important task to enhance the theoretical development under the intuitionistic fuzzy sets environment that was proposed by Atanassov [1], which is an extension for the fuzzy sets originally proposed by Zadeh [15] to deal uncertainty dealing with decision-making problems. Recently, Xu [12] first pointed out that similarity measures constructed by Li and Cheng [6], Wang and Xin [11], Papakostas et al. [9], Li et al. [7], Hatzimichailidis et al. [3], Mitchell [8], Hung and Yang [4, 5], Szmidt and Kacprzyk [10], Yang and Chiclana [14], all encountered unsolvable pattern recognition problems. Hence, Xu [12] tried to develop a new distance and then a new similarity measure can be induced. Xu [13] claimed that the proof of Theorem 1 in Xu [12] contained dubious results, and then Xu [13] provided his revisions. However, we find that the revisions in Xu [13] still contained questionable findings such that in this paper, we present a further improvement.

2. An outline of Xu [12]

For an intuitionistic fuzzy set (IFS), denoted as $A=\{\langle x_i,\mu_A(x_i),v_A(x_i)\rangle : x_i\in X\}$ where X is the universe of discourse, with $X=\{x_1,x_2,...,x_n\}$. μ_A is the membership function and v_A is the non-membership function that satisfies $0\le\mu_A(x_i)\le 1$, $0\le v_A(x_i)\le 1$, with $0\le\mu_A(x_i)+v_A(x_i)\le 1$, for $i=1,2,...,n$. The hesitancy is denoted as π_A that satisfies $\pi_A(x_i)=1-\mu_A(x_i)-v_A(x_i)$, for $i=1,2,...,n$.

Xu [12] evenly partitioned hesitancy to membership and non-membership functions to develop a point in $\Re^4$ as

$$\left(\mu_A(x_i), v_A(x_i), \mu_A(x_i)+(\pi_A(x_i)/2), v_A(x_i)+(\pi_A(x_i)/2)\right). \quad (1)$$

Xu [12] defined two distances as follows

$$D^p_{IFSs}(A,B)=\left\{\frac{1}{4}\sum_{i=1}^{4}\left[\left|\Delta_\mu(i)\right|^p+\left|\Delta_v(i)\right|^p+\left|\Delta_{\pi\mu}(i)\right|^p+\left|\Delta_{\pi v}(i)\right|^p\right]\right\}^{1/p}, \quad (2)$$

and

$$D^{np}_{IFSs}(A,B)=\left\{\frac{1}{4n}\sum_{i=1}^{4}\left[\left|\Delta_\mu(i)\right|^p+\left|\Delta_v(i)\right|^p+\left|\Delta_{\pi\mu}(i)\right|^p+\left|\Delta_{\pi v}(i)\right|^p\right]\right\}^{1/p}, \quad (3)$$

where those auxiliary expressions are denoted as

$$\Delta_\mu(i)=\mu_A(x_i)-\mu_B(x_i), \quad (4)$$

$$\Delta_v(i)=\mu_v(x_i)-\mu_v(x_i), \quad (5)$$

$$Assign_A^{\pi\mu}(x_i)=\mu_A(x_i)+(\pi_A(x_i)/2), \quad (6)$$

$$Assign_B^{\pi\mu}(x_i)=\mu_B(x_i)+(\pi_B(x_i)/2), \quad (7)$$

$$Assign_A^{\pi v}(x_i)=v_A(x_i)+(\pi_A(x_i)/2), \quad (8)$$

$$Assign_B^{\pi v}(x_i)=v_B(x_i)+(\pi_B(x_i)/2), \quad (9)$$

$$\Delta_{\pi\mu}(i)=Assign_A^{\pi\mu}(x_i)-Assign_B^{\pi\mu}(x_i), \quad (10)$$

and

$$\Delta_{\pi v}(i)=Assign_A^{\pi v}(x_i)-Assign_B^{\pi v}(x_i). \quad (11)$$

In the following, we cite the definition of the conventional distance by Definition 2 of Xu [12].

Definition 2 of Xu [12].

A metric distance D defined in a non-empty set X is a real function,

$$D: X\times X\to[0,\infty), \quad (12)$$

that satisfies the following three conditions,

(MD 1) $D(x,y)=0$ id and only if $x=y$.

(MD 2) $D(x,y)=D(y,x)$.

(MD 3) $D(x,y)+D(y,z)\ge D(x,z)$, for $x,y,z\in X$.

Conditions (MD 1) and (MD 2) are easy to verified, such that Xu [12] only focused on his discussions for (MD 3).

To verify his new distance satisfies the property (MD 3), Xu [12] mentioned the following relations

$$\left(\Delta_{\mu}^{AC}\right)^2=\left(\Delta_{\mu}^{AB}+\Delta_{\mu}^{BC}\right)^2\le\left(\Delta_{\mu}^{AB}\right)^2+\left(\Delta_{\mu}^{BC}\right)^2, \quad (13)$$

$$\left(\Delta_{v}^{AC}\right)^2=\left(\Delta_{v}^{AB}+\Delta_{v}^{BC}\right)^2\le\left(\Delta_{v}^{AB}\right)^2+\left(\Delta_{v}^{BC}\right)^2, \quad (14)$$

$$\left(\Delta_{\pi\mu}^{AC}\right)^2=\left(\Delta_{\pi\mu}^{AB}+\Delta_{\pi\mu}^{BC}\right)^2\le\left(\Delta_{\pi\mu}^{AB}\right)^2+\left(\Delta_{\pi\mu}^{BC}\right)^2, \quad (15)$$

$$\left(\Delta_{\pi v}^{AC}\right)^2=\left(\Delta_{\pi v}^{AB}+\Delta_{\pi v}^{BC}\right)^2\le\left(\Delta_{\pi v}^{AB}\right)^2+\left(\Delta_{\pi v}^{BC}\right)^2. \quad (16)$$

Thus, Xu [12] claimed that

$$D_{IFSs}(A,C)\le D_{IFSs}(A,B)+D_{IFSs}(B,C), \quad (17)$$

to show that D_{IFSs} of Equations (2) and (3) both satisfy (MD 3).

3. A Review for Xu [13]

We cite from Xu [13] where the difference of index in Xu [13] and our paper is explicitly illustrated, "However, the conclusion $D_{IFSs}(A,C)\le D_{IFSs}(A,B)+D_{IFSs}(B,C)$ is derived from the formula (3) (that is formulas (3-6) of this paper), which is a wrong logical reasoning. Where, it should be noted that the formula (3) is correct. In fact, from the formula (3) and the property of inequality, we can obtained

$$\left[\left(\Delta_{\mu}^{AC}\right)^2+\left(\Delta_{v}^{AC}\right)^2+\left(\Delta_{\pi\mu}^{AC}\right)^2+\left(\Delta_{\pi v}^{AC}\right)^2\right]\le \left[\left(\Delta_{\mu}^{AB}\right)^2+\left(\Delta_{v}^{AB}\right)^2+\left(\Delta_{\pi\mu}^{AB}\right)^2+\left(\Delta_{\pi v}^{AB}\right)^2\right]+ \left[\left(\Delta_{\mu}^{BC}\right)^2+\left(\Delta_{v}^{BC}\right)^2+\left(\Delta_{\pi\mu}^{BC}\right)^2+\left(\Delta_{\pi v}^{BC}\right)^2\right]. \quad (18)$$

While, from the Definition 4 (that is Definition 2 of Xu [12].), we have

$$4\left[D_{IFSs}(A,C)\right]^2 =\left(\Delta_{\mu}^{AC}\right)^2+\left(\Delta_{v}^{AC}\right)^2+\left(\Delta_{\pi\mu}^{AC}\right)^2+\left(\Delta_{\pi v}^{AC}\right)^2, \quad (19)$$

$$4\left[D_{IFSs}(A,B)\right]^2 =\left(\Delta_{\mu}^{AB}\right)^2+\left(\Delta_{v}^{AB}\right)^2+\left(\Delta_{\pi\mu}^{AB}\right)^2+\left(\Delta_{\pi v}^{AB}\right)^2, \quad (20)$$

$$4\left[D_{IFSs}(B,C)\right]^2 =\left(\Delta_{\mu}^{BC}\right)^2+\left(\Delta_{v}^{BC}\right)^2+\left(\Delta_{\pi\mu}^{BC}\right)^2+\left(\Delta_{\pi v}^{BC}\right)^2. \quad (21)$$

Therefore,

$$\left[D_{IFSs}(A,C)\right]^2 \le\left[D_{IFSs}(A,B)\right]^2+\left[D_{IFSs}(B,C)\right]^2. \quad (22)$$

However, based on the formula (5) (that is the formula (12) of this paper), it is not obtained

$$D_{IFSs}(A,C)\le D_{IFSs}(A,B)+D_{IFSs}(B,C). \quad (23)$$

This shows that the proof of the paper [5] ([5] was indexed in Xu [13], that is Xu [12] in this paper) is incorrect."

On the other hand, Xu [13] provided his new proof to show that his new distance satisfying (MD 3).

4. Our comments for Xu [13]

We agree that Xu [13] and Chu et al. [2] already provided a right proof to show that D_{IFSs} of Equations (2) and (3) both satisfy (MD 3). Moreover, Chu et al. [2] showed that there are other questionable findings with respect to Xu [12], but they are out off the scope of this paper. We suggest interested readers directly refer to Chu et al. [2] for their further improvement for Xu [12].

In the following, we focus on Xu [13]. Even through Xu [13] offered a valid verification to show the new distance proposed by Xu [12] satisfying (MD 3), however, there are three questionable issues that should be revised for Xu [13].

First, we will show that based on the inequality of Equation (22), researchers can obtain the inequality of Equation (23) to indicate the reason proposed by Xu [13] to criticize the proof of Xu [12] is wrong.

We recall that

$$0\le D_{IFSs}(A,B), \quad (24)$$

and

$$0\le D_{IFSs}(B,C). \quad (25)$$

Hence, we derive that

$$\left[D_{IFSs}(A,B)\right]^2+\left[D_{IFSs}(B,C)\right]^2 \le\left[D_{IFSs}(A,B)+D_{IFSs}(B,C)\right]^2. \quad (26)$$

For the present moment, if we accept that the inequality of Equation (22) is valid for the moment, then we consider Equations (22) and (26) to obtain that

$$\left[D_{IFSs}(A,C)\right]^2 \le\left[D_{IFSs}(A,B)+D_{IFSs}(B,C)\right]^2. \quad (27)$$

Now, we take square root on both sides of Equation (27), then Equation (23) appears. Therefore, we demonstrate that if Equation (22) is valid, then we can verify Equation (23). We summarize our discussion in the next lemma 1.

Lemma 1.

If Equation (22) is valid, then we can prove Equation (23).

Second, we claim that if inequalities of Equations (13-16) are valid then we can show that inequality of Equation (22) is valid. We recall the expressions of Equations (13-16) and then we add them together to yield the inequality of Equation (28). We recall the expressions provided by Xu [13] of Equations (19-21), then we can accept the result of the Equation (22) proposed by Xu [13]. Hence, we summarize our above discussion in the next lemma 2.

Lemma 2.

If inequalities of Equations (13-16) are valid, then the inequality of Equation (22) is also valid.

Third, we will show that inequalities of Equations (13-16) contained questionable results.

We provide a counterexample with $X=\{x_1\}$, $\mu_A(x_1)=0.1$, $v_A(x_1)=0.6$, $\mu_B(x_1)=0.2$, $v_B(x_1)=0.5$, $\mu_C(x_1)=0.3$, and $v_C(x_1)=0.4$ to construct three intuitionistic fuzzy sets to develop our counterexample.

We know that

$$Assign_A^{\pi\mu}(x_1)=0.25, \quad (28)$$

$$Assign_A^{\pi v}(x_1)=0.75, \quad (29)$$

$$Assign_B^{\pi\mu}(x_1)=0.35, \quad (30)$$

$$Assign_B^{\pi v}(x_1)=0.65, \quad (31)$$

$$Assign_C^{\pi\mu}(x_1)=0.45, \quad (32)$$

and

$$Assign_C^{\pi v}(x_1)=0.55. \quad (33)$$

Inequalities of Equations (13-16) proposed by Xu [13] claimed that

$$\left(\Delta_{\mu}^{AC}\right)^2=0.04\le\left(\Delta_{\mu}^{AB}\right)^2+\left(\Delta_{\mu}^{BC}\right)^2=0.02, \quad (34)$$

$$\left(\Delta_{v}^{AC}\right)^2=0.04\le\left(\Delta_{\mu}^{AB}\right)^2+\left(\Delta_{\mu}^{BC}\right)^2=0.02, \quad (35)$$

$$\left(\Delta_{\pi\mu}^{AC}\right)^2=0.04\le\left(\Delta_{\pi\mu}^{AB}\right)^2+\left(\Delta_{\pi\mu}^{BC}\right)^2=0.02, \quad (36)$$

and

$$\left(\Delta_{\pi v}^{AC}\right)^2=0.04\le\left(\Delta_{\pi v}^{AB}\right)^2+\left(\Delta_{\pi v}^{BC}\right)^2=0.02. \quad (37)$$

We find that inequalities of Equations (13-16) proposed by Xu [13] are wrong to indicate inequalities of Equations (13-16) proposed by Xu [13] contained severe questionable findings. We summarize our findings in the next lemma 3.

Lemma 3.

Inequalities of Equations (13-16) proposed by Xu [13] that is the formula (3) in Xu [13] which also appeared in Xu [12], are both false.

From the above discussions, we point out that the motivations of Xu [13] to revise his findings in Xu [12] as cited in this paper as Equations (13-16) are invalid.

Xu [13] did not realize his questionable results as Equations (13-16) in this paper. The same mistake committed as Xu [12] and Xu [13].

5. Conclusion

We pay attention to a revised paper, Xu [13] to improve Xu [12]. However, Xu [13] still contained questionable results such that it is worthy to present further revisions to help researchers realize the genuine problem for the proof in Xu [12] and Xu [13].

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