

Supply Chain with Coordination and a Dominant Retailer

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Abstract: *We studied a supply chain replenishment policy by an algebraic approach such that some practitioners without the knowledge of calculus still can realize this kind of research topic. Under two extra conditions and one weaker condition proposed by us, we show that our algebraic approach obtained the same formulated optimal solution as that of an analytic method proposed by previous papers. Hence, we point out previous findings are incomplete and then we provide our revisions.*

Keywords: Supply chain, a dominant retailer, formulated optimal solution

1. Introduction

Chen and Xiao [1] published a paper related to a supply chain with demand disruption and coordination along with a dominant retailer. This important paper has been cited for 88 times. We just list some of them for the last three years, Cao et al. [2] for coordinating a supply chain under demand and cost disruptions, Mahdiraji et al. [3] for game-theoretic approach for coordinating unlimited multi-echelon supply chains, Yuan et al. [4] for advertising and pricing decisions in a manufacturer retailer channel with demand and cost disruptions, Jiang et al. [5] for decision and coordination in a competing retail channel involving a third-party logistics provider, Lingling [6] for manufacturer's pricing strategies for a supply chain with fairness concern, Shu and Mao [7] for research on quantitative methods of supply chain based on disruptions management framework, Chen and Xiao [8] for reordering policy and coordination of a supply chain with a loss-averse retailer, Li et al. [9] for double marginalization and coordination in the supply chain with uncertain supply, Wang et al. [10] for pricing and effort investment for a newsvendor type product, to illustrate that this topic is a hot research spot. Consequently, following this research trend, we will study this supply chain problem with an algebraic method to find the optimal solution such that those practitioners without the knowledge of calculus still can learn this important research issue.

2. Notation and Assumptions

Chen and Xiao [1] considered a supply chain consisting of one manufacturer, one dominant retailer and N fringe retailers, $N \geq 2$. They assumed that the dominant retailer, acting like a monopolist, has market power in the market, and only the dominant retailer can provide the demand stimulating service, which goes beyond what fringe retailers can do. For example, the dominant retailer can carry on some propagating advertisement to promote the product. At the same time, the dominant retailer is a price leader. Once the retail price is settled down, all fringe retailers regard it as the market retail price, which is generally consistent with the marketing operation described earlier where some small retailers use the pricing book of a large retailer. They took the following notation and assumptions:

- s The service investment of the dominant retailer;
- q_T The total market demand in the final market;
- q_d The market demand for the dominant retailer;
- q_r The market demand for each fringe retailer;
- p The retail price decided by the dominant retailer. We add a stronger condition: $p > c$;
- a The market scale, $a > 0$. We add a stronger condition: $a > p > 0$;
- c The unit production cost, $a > c$;
- c_u The unit penalty cost for a unit increased quantity, $c_u \geq 0$;
- c_s The unit penalty cost for a unit decreased quantity, $c_s \geq 0$;
- γ The share of the market demand for the dominant retailer;
- θ The demand sensitivity to the service level of the dominant retailer, similar to Desiraju and Moorthy [11], and Tsay and Agrawal [12], $2\sqrt{\sqrt{2}-1} > \theta > 0$. In this paper, we only need a weak condition: $2 > \theta > 0$;
- λ The dominant retailer's share of the supply chain's profit, $0 < \lambda < 1$.

3. Review of Chen and Xiao [1]

Chen and Xiao [1] tried to maximize the channel profit without demand disruption

$$\pi_T(p, s) = (a - p + \theta\sqrt{s})(p - c) - s. \quad (1)$$

Here, we need two extra conditions $a > p > 0$ and $p > c$ to guarantee the positivity of the profit.

They used the analytic approach to find the maximum value. For completeness, we recall their analytic approach. We compute the system of the first partial derivatives to find that

$$\frac{\partial}{\partial p} \pi_T(p, s) = a + c - 2p + \theta\sqrt{s}, \quad (2)$$

and

$$\frac{\partial}{\partial s} \pi_T(p, s) = \frac{\theta(p-c)}{2\sqrt{s}} - 1. \tag{3}$$

$$= \frac{(a-c)^2}{4-\theta^2}. \tag{10}$$

If we solve the system of $\frac{\partial}{\partial p} \pi_T(p, s) = 0$ and

$$\frac{\partial}{\partial s} \pi_T(p, s) = 0, \text{ then}$$

$$2p = a + c + \theta\sqrt{s}, \tag{4}$$

and

$$\theta p = \theta c + 2\sqrt{s}. \tag{5}$$

From Equations (4) and (5), we derive that

$$\frac{a + c + \theta\sqrt{s}}{2} = \frac{\theta c + 2\sqrt{s}}{\theta}, \tag{6}$$

and then it yields

$$(4 - \theta^2)\sqrt{s} = \theta(a - c). \tag{7}$$

Here, we need our weaker condition of $2 > \theta > 0$ to guarantee the positivity of the left-hand side of Equation (7).

We derive the optimal service investment of the dominant retailer,

$$s^* = \frac{\theta^2(a-c)^2}{(4-\theta^2)^2}. \tag{8}$$

We plug the finding of Equation (8) into Equation (5) to find that

$$p^* = c + \frac{2(a-c)}{4-\theta^2}$$

$$= \frac{2(a+c) - c\theta^2}{4-\theta^2}. \tag{9}$$

Owing to $2 > \theta > 0$, we obtain that $2(a+c) > 4c > c\theta^2$ to guarantee the positivity of p^* .

Next, we plug the results of Equations (8) and (9) into Equation (1) to obtain that

$$\pi_T(p^*, s^*) = (a - p^* + \theta\sqrt{s^*})(p^* - c) - s^*$$

$$= \left[a - \frac{2(a+c) - c\theta^2}{4-\theta^2} + \theta \frac{\theta(a-c)}{4-\theta^2} \right]$$

$$\left[\frac{2(a+c) - c\theta^2}{4-\theta^2} - c \right] - \frac{\theta^2(a-c)^2}{(4-\theta^2)^2}$$

$$= \left[\frac{4a - a\theta^2 - 2(a+c) + c\theta^2 + a\theta^2 - c\theta^2}{4-\theta^2} \right]$$

$$\left[\frac{2(a+c) - c\theta^2 - 4c + c\theta^2}{4-\theta^2} \right] - \frac{\theta^2(a-c)^2}{(4-\theta^2)^2}$$

$$= \left[\frac{2(a-c)}{4-\theta^2} \right] \left[\frac{2(a-c)}{4-\theta^2} \right] - \frac{\theta^2(a-c)^2}{(4-\theta^2)^2}$$

$$= \frac{4(a-c)^2}{(4-\theta^2)^2} - \frac{\theta^2(a-c)^2}{(4-\theta^2)^2}$$

In this paper, to introduce inventory models of the supply chain to practitioners without a background of calculus, we will apply an algebraic method to find the optimal solution.

4. Our Algebraic Approach

To simplify the expression, we denote $y = \sqrt{s}$, and then we rewrite Equation (1) in the descending order of y to derive that

$$\pi_T(p, y) = -y^2 + \theta(p-c)y + (a-p)(p-c). \tag{11}$$

We complete the square of y in Equation (11) to obtain that

$$\pi_T(p, y) = -(y - (\theta(p-c)/2))^2 + (\theta^2(p-c)^2/4) + (a-p)(p-c). \tag{12}$$

Because the coefficient of $(y - (\theta(p-c)/2))^2$ is $-1 < 0$, to attain the maximum value, we should have

$$y = \theta(p-c)/2. \tag{13}$$

We plug the findings of Equation (13) into Equation (12) and then rewrite it in the descending order of p . We will express Equation (12) with

$$\pi_T(p) = \pi_T(p, y = \theta(p-c)/2), \tag{14}$$

and then we derive that

$$\pi_T(p) = \frac{-(4-\theta^2)}{4} p^2$$

$$+ \frac{2(a+c) - c\theta^2}{2} p + \frac{c^2}{4} \theta^2 - ac. \tag{15}$$

We complete the square of p in Equation (4.5) to yield that

$$\pi_T(p) = \frac{-(4-\theta^2)}{4} \left[p - \frac{2(a+c) - c\theta^2}{4-\theta^2} \right]^2$$

$$+ \frac{(2(a+c) - c\theta^2)^2}{4(4-\theta^2)} + \frac{c^2}{4} \theta^2 - ac. \tag{16}$$

Owing to the coefficient of $\left[p - \frac{2(a+c) - c\theta^2}{4-\theta^2} \right]^2$ is $\frac{-(4-\theta^2)}{4} < 0$, to achieve the maximum value, we should derive

$$p = \frac{2(a+c) - c\theta^2}{4-\theta^2}. \tag{17}$$

We plug the results of Equation (17) into Equation (16) to find

$$\pi_T(p^*, s^*) = \frac{4(a+c)^2 - 4c(a+c)\theta^2 + c^2\theta^4}{4(4-\theta^2)}$$

$$+ \frac{4c^2\theta^2 - c^2\theta^4}{4(4-\theta^2)} + \frac{-16ac + 4ac\theta^2}{4(4-\theta^2)}$$

$$= \frac{4(a+c)^2 - 16ac}{4(4-\theta^2)}$$

$$= \frac{(a-c)^2}{4-\theta^2}, \quad (18)$$

with $p^* = \frac{2(a+c)-c\theta^2}{4-\theta^2}$ of Equation (17) and

$$s^* = (y^*)^2 = \frac{\theta^2(a-c)^2}{(4-\theta^2)^2} \text{ of Equation (13).}$$

Our findings are identical with that proposed by Chen and Xiao [1] by an analytic approach as Equations (8-10). However, we point out that there are two extra conditions: $a > p > 0$ and $p > c$ to guarantee the well-define of the maximum profit model. Moreover, we show that a weaker condition: $2 > \theta > 0$ that is enough for this supply chain problem.

5. Conclusions

Our algebraic approach not only derived the same optimal solution but also obtained the optimal value at the same time. This special feature will be an important factor for an efficient computation algorithm. Two missing conditions are added by us and a too strong condition is reduced to justify our derivations.

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