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Some Log-Type Classes of Estimators using Multiple Auxiliary Information

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Abstract: ***In this paper, some classes of log-type estimators using information on two auxiliary variables have been proposed for estimating the population mean of the study variable It has been shown that these classes of log-type estimators have lesser mean square error under the optimum values of the characterizing scalars as compared to some of the commonly used estimators available in the literature. Further, an extension of the proposed classes using multiple auxiliary information have also initiated in this article. A numerical study is included as an illustration using two auxiliary variables.***

Keywords: Multiple auxiliary information, logarithmic function, bias, mean square error

1. Introduction

In sampling theory, it is a popular trend to use auxiliary information to obtain more efficient estimators for the population parameters to increase the precision of the estimator. Estimators obtained using auxiliary information are supposed to be more efficient than the estimators obtained without using auxiliary information. The ratio, regression, product and difference methods take advantage of the auxiliary information at the estimation stage. Many authors like, Pandey and Dubey (1988), Upadhyaya and Singh (1999) , Kalidar and Cingi (2003), Singh and Tailor (2003), Singh (2003), Sisodia and Dwivedi (1981), Koyuncu and Kalidar along with many others have proposed various estimators using auxiliary information on various population parameters like coefficient of skewness, kurtosis, variation, standard deviation, correlation coefficient, etc. Sometimes, it is more economical to obtain information on more than one auxiliary information; this would probably help in improvising the efficiency of the estimator, used to estimate the parameter under consideration. The literature deals with a wide range of ratio, product, difference and exponential estimators proposed by various renowned authors using multiple auxiliary information (Olkin (1958), Raj (1965), Singh (1967), Shukla(1966), etc.). Recently, Bhushan, Gupta and Pandey (2015) had made the use of logarithmic relationship between the study variable and auxiliary variable, extending the work, in this paper we have made the use of multiple auxiliary variables *x's* for estimating the population mean $\overline{Y}$. The proposed estimators would work in case when the study variable is logarithmically related to the auxiliary variable.

Consider a finite population $U=(U_1,U_2\ldots,U_N)$ of size N from which a sample of size n is drawn according to simple random sampling without replacement (SRSWOR). Let y_i and x_i denotes the values of the study and auxiliary variables for the ith unit, $(i=1,2,\ldots,N)$, of the population. Further let $\overline{y}$ and $\overline{x}$ be the sample means of the study and auxiliary variables, respectively.

We suggest the following new classes of log-type estimators for the population mean $\overline{Y}$ as:

$$T_{r1}=\overline{y}\left[1+\log\left(\frac{\overline{x}_1}{\overline{X}_1}\right)\right]^{\alpha_1}\left[1+\log\left(\frac{\overline{x}_2}{\overline{X}_2}\right)\right]^{\alpha_2} \tag{1.1}$$

$$T_{r2}=\overline{y}\left[1+\beta_1\log\left(\frac{\overline{x}_1}{\overline{X}_1}\right)\right]\left[1+\beta_2\log\left(\frac{\overline{x}_2}{\overline{X}_2}\right)\right] \tag{1.2}$$

$$T_{r3}=\overline{y}\left[1+\log\left(\frac{\overline{x}_1^*}{\overline{X}_1^*}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\overline{x}_2^*}{\overline{X}_2^*}\right)\right]^{\delta_2} \tag{1.3}$$

$$T_{r4}=\overline{y}\left[1+\theta_1\log\left(\frac{\overline{x}_1^*}{\overline{X}_1^*}\right)\right]\left[1+\theta_2\log\left(\frac{\overline{x}_2^*}{\overline{X}_2^*}\right)\right] \tag{1.4}$$

where,

$$\left.\begin{aligned}\overline{X}_i^* &= a_i\overline{X}_i+b_i\\ \overline{x}_i^* &= a_i\overline{x}_i+b_i\end{aligned}\right\}\quad i=1,2$$

such that $\alpha_i,\beta_i,\delta_i$ and θ_i are the optimizing scalars, $a_i(\neq 0),b_i$ are either real numbers or functions of the known parameters of the auxiliary variable $x_i's$ such as the standard deviations S_{x_i}, coefficient of variation C_{x_i}, coefficient of kurtosis $\beta_2(x_i)$ and correlation coefficient $\rho_{x_ix_j}$ of the population. $(i\neq j=1,2)$

2. Properties of the Suggested Classes of Log-Type Estimators

In order to obtain the bias and mean square error (MSE), let us consider

$\overline{y}=\overline{Y}(1+e_0)$; $\overline{x}_1=\overline{X}_1(1+e_1)$; $\overline{x}_2=\overline{X}_2(1+e_2)$

with $E(e_0)=E(e_1)=E(e_2)=0$

Using the results given in Sukhatme and Sukhatme (1991), we have the following theorems:

Theorem 2.1

The biases of the suggested classes of estimators are given by

$$Bias(T_{r1}) = \lambda_n \bar{Y}\left(\alpha_1\rho_{yx_1}C_yC_{x_1} + \alpha_2\rho_{yx_2}C_yC_{x_2} - \alpha_1C_{x_1}^2 - \alpha_2C_{x_2}^2 + \alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} + \frac{\alpha_1^{\,2}}{2}C_{x_1}^2 + \frac{\alpha_2^{\,2}}{2}C_{x_2}^2 \right) \quad (2.1)$$

$$Bias(T_{r2}) = \lambda_n \bar{Y}\left(\beta_1\rho_{yx_1}C_yC_{x_1} + \beta_2\rho_{yx_2}C_yC_{x_2} + \beta_1\beta_2\rho_{x_1x_2}C_{x_1}C_{x_2} - \frac{\beta_1}{2}C_{x_1}^2 - \frac{\beta_2}{2}C_{x_2}^2 \right) \quad (2.2)$$

$$Bias(T_{r3}) = \lambda_n \bar{Y}\begin{pmatrix} \delta_1\upsilon_1\rho_{yx_1}C_yC_{x_1} + \delta_2\upsilon_2\rho_{yx_2}C_yC_{x_2} - \delta_1\upsilon_1^{\,2}C_{x_1}^2 - \delta_2\upsilon_2^{\,2}C_{x_2}^2 \\ +\delta_1\delta_2\upsilon_1\upsilon_2\rho_{x_1x_2}C_{x_1}C_{x_2} + \frac{\delta_1^{\,2}}{2}\upsilon_1^{\,2}C_{x_1}^2 + \frac{\delta_2^{\,2}}{2}\upsilon_2^{\,2}C_{x_2}^2 \end{pmatrix} \quad (2.3)$$

$$Bias(T_{r4}) = \lambda_n \bar{Y}\begin{pmatrix} \theta_1\upsilon_1\rho_{yx_1}C_yC_{x_1} + \theta_2\upsilon_2\rho_{yx_2}C_yC_{x_2} + \theta_1\theta_2\upsilon_1\upsilon_2\rho_{x_1x_2}C_{x_1}C_{x_2} \\ -\frac{\theta_1}{2}\upsilon_1^{\,2}C_{x_1}^2 - \frac{\theta_2}{2}\upsilon_2^{\,2}C_{x_2}^2 \end{pmatrix} \quad (2.4)$$

Theorem 2.2

The mean square errors of the suggested classes of estimators up to the terms of order n^{-1} are given by

$$MSE(T_{r1}) = \lambda_n \bar{Y}^2\left(C_y^2 + \alpha_1^{\,2}C_{x_1}^2 + \alpha_2^{\,2}C_{x_2}^2 + 2\alpha_1\rho_{yx_1}C_yC_{x_1} + 2\alpha_2\rho_{yx_2}C_yC_{x_2} + 2\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} \right) \quad (2.5)$$

$$MSE(T_{r2}) = \lambda_n \bar{Y}^2\left(C_y^2 + \beta_1^{\,2}C_{x_1}^2 + \beta_2^{\,2}C_{x_2}^2 + 2\beta_1\rho_{yx_1}C_yC_{x_1} + 2\beta_2\rho_{yx_2}C_yC_{x_2} + 2\beta_1\beta_2\rho_{x_1x_2}C_{x_1}C_{x_2} \right) \quad (2.6)$$

$$MSE(T_{r3}) = \lambda_n \bar{Y}^2\begin{pmatrix} C_y^2 + \delta_1^{\,2}\upsilon_1^{\,2}C_{x_1}^2 + \delta_2^{\,2}\upsilon_2^{\,2}C_{x_2}^2 + 2\delta_1\upsilon_1\rho_{yx_1}C_yC_{x_1} \\ +2\delta_2\upsilon_2\rho_{yx_2}C_yC_{x_2} + 2\delta_1\delta_2\upsilon_1\upsilon_2\rho_{x_1x_2}C_{x_1}C_{x_2} \end{pmatrix} \quad (2.7)$$

$$MSE(T_{r4}) = \lambda_n \bar{Y}^2\begin{pmatrix} C_y^2 + \theta_1^{\,2}\upsilon_1^{\,2}C_{x_1}^2 + \theta_2^{\,2}\upsilon_2^{\,2}C_{x_2}^2 + 2\theta_1\upsilon_1\rho_{yx_1}C_yC_{x_1} \\ +2\theta_2\upsilon_2\rho_{yx_2}C_yC_{x_2} + 2\theta_1\theta_2\upsilon_1\upsilon_2\rho_{x_1x_2}C_{x_1}C_{x_2} \end{pmatrix} \quad (2.8)$$

Corollary 2.3

The minimum value of MSE is obtained for the optimum value of α_i given by

$$\alpha_{1(opt)} = \frac{\left(\rho_{yx_2}\rho_{x_1x_2} - \rho_{yx_1}\right)}{\left(1-\rho_{x_1x_2}^2\right)}\frac{C_y}{C_{x_1}} = \beta_{1(opt)} \quad (2.9)$$

$$\alpha_{2(opt)} = \frac{\left(\rho_{yx_1}\rho_{x_1x_2} - \rho_{yx_2}\right)}{\left(1-\rho_{x_1x_2}^2\right)}\frac{C_y}{C_{x_2}} = \beta_{2(opt)} \quad (2.10)$$

$$\delta_{1(opt)} = \frac{\left(\rho_{yx_2}\rho_{x_1x_2} - \rho_{yx_1}\right)}{\left(1-\rho_{x_1x_2}^2\right)}\frac{C_y}{\upsilon_1C_{x_1}} = \theta_{1(opt)} \quad (2.11)$$

$$\delta_{2(opt)} = \frac{\left(\rho_{yx_1}\rho_{x_1x_2} - \rho_{yx_2}\right)}{\left(1-\rho_{x_1x_2}^2\right)}\frac{C_y}{\upsilon_2C_{x_2}} = \theta_{2(opt)} \quad (2.12)$$

where $\upsilon_i = a_i\bar{X}_i / \left(a_i\bar{X}_i + b_i\right)$

and the minimum value of MSE is

$$MSE_{\min}(T_{ri}) = \lambda_n.\bar{Y}^2.C_y^2\left(1-R_{y.12}^2\right) = M \quad (2.13)$$

where $R_{y.12}^2$ is the multiple correlation coefficient between y and x_1, x_2

Note: It is important to note that if the parameters ρ, S_x^2, S_y^2 are not known then they can be estimated by their unbiased and consistent estimators and following Sampath (2005) it can be easily shown that the resultant estimators attain the same MSE given in (2.13).

3. Multivariate extension of the suggested classes of estimators using multiple auxiliary information

Let there are k auxiliary variables then we can use the variables by taking a linear combination of these k estimators of the form given in section 2, calculated for every auxiliary variable separately, for estimating the population mean. Then the estimators for population mean will be defined as,

$$T_{r1}^* = \bar{y}\prod_{i=1}^{k}\left[1+\log\left(\frac{\bar{x}_i}{\bar{X}_i}\right)\right]^{\alpha_i} \quad (3.1)$$

$$T_{r2}^* = \bar{y}\prod_{i=1}^{k}\left[1+\beta_i\log\left(\frac{\bar{x}_i}{\bar{X}_i}\right)\right] \quad (3.2)$$

$$T_{r3}^* = \bar{y}\prod_{i=1}^{k}\left[1+\log\left(\frac{\bar{x}_i^*}{\bar{X}_i^*}\right)\right]^{\delta_i} \quad (3.3)$$

$$T_{r4}^* = \bar{y}\prod_{i=1}^{k}\left[1+\theta_i\log\left(\frac{\bar{x}_i^*}{\bar{X}_i^*}\right)\right] \quad (3.4)$$

where $\alpha_i, \beta_i, \delta_i$ and θ_i are the optimizing scalars $(i = 1, 2, ..., k)$

4. Properties of the adapted classes of estimators using multiple auxiliary information

For obtaining Biases and Mean square errors(s) of the adapted classes, let us denote

$\bar{y}=\bar{Y}(1+e_0)$; $\bar{x}_i=\bar{X}_i(1+e_i)$

with $E(e_0)=E(e_i)=0$; $(i=1,2,...,k)$

and using the results given in Sukhatme and Sukhatme (1991), we have the following theorems:

Theorem 4.1

$$Bias\left(T_{r1}^*\right)=\lambda_n\bar{Y}\left(\sum_{i=1}^{k}\alpha_i\rho_{yx_i}C_yC_{xi}-\sum_{i=1}^{k}\alpha_iC_{x_i}^2+\sum_{i\neq j=1}^{k}\alpha_i\alpha_j\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\frac{\alpha_i^2}{2}C_{x_i}^2\right) \quad (4.1)$$

$$Bias\left(T_{r2}^*\right)=\lambda_n\bar{Y}\left(\sum_{i=1}^{k}\beta_i\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}\beta_i\beta_j\rho_{x_ix_j}C_{x_i}C_{x_j}-\sum_{i=1}^{k}\frac{\beta_i}{2}C_{x_i}^2\right) \quad (4.2)$$

$$Bias\left(T_{r3}^*\right)=\lambda_n\bar{Y}\left(\sum_{i=1}^{k}\delta_i\upsilon_i\rho_{yx_i}C_yC_{xi}-\sum_{i=1}^{k}\delta_i\upsilon_i^2C_{x_i}^2+\sum_{i\neq j=1}^{k}\delta_i\delta_j\upsilon_i\upsilon_j\rho_{x_ix_j}C_{x_i}C_{x_j}+\sum_{i=1}^{k}\frac{\delta_i^2}{2}\upsilon_i^2C_{x_i}^2\right) \quad (4.3)$$

$$Bias\left(T_{r4}^*\right)=\lambda_n\bar{Y}\left(\sum_{i=1}^{k}\theta_i\upsilon_i\rho_{yx_i}C_yC_{x_i}+\sum_{i\neq j=1}^{k}\theta_i\theta_j\upsilon_i\upsilon_j\rho_{x_ix_j}C_{x_i}C_{x_j}-\sum_{i=1}^{k}\frac{\theta_i}{2}\upsilon_i^2C_{x_i}^2\right) \quad (4.4)$$

Theorem 4.2

The mean square errors of the suggested classes of estimators up to the terms of order n^{-1} are given by

$$MSE\left(T_{r1}^*\right)=\lambda_n\bar{Y}^2\left(C_y^2+\sum_{i=1}^{k}\alpha_i^2C_{x_i}^2+2\sum_{i=1}^{k}\alpha_i\rho_{yx_i}C_yC_{x_i}+2\sum_{i\neq j=1}^{k}\alpha_i\alpha_j\rho_{x_ix_j}C_{x_i}C_{x_j}\right) \quad (4.5)$$

$$MSE\left(T_{r3}^*\right)=\lambda_n\bar{Y}^2\left(C_y^2+\sum_{i=1}^{k}\beta_i^2C_{x_i}^2+2\sum_{i=1}^{k}\beta_i\rho_{yx_i}C_yC_{x_i}+2\sum_{i\neq j=1}^{k}\beta_i\beta_j\rho_{x_ix_j}C_{x_i}C_{x_j}\right) \quad (4.6)$$

$$MSE\left(T_{r3}^*\right)=\lambda_n\bar{Y}^2\left(C_y^2+\sum_{i=1}^{k}\delta_i^2\upsilon_i^2C_{x_i}^2+2\sum_{i=1}^{k}\delta_i\upsilon_i\rho_{yx_i}C_yC_{x_i}+2\sum_{i\neq j=1}^{k}\delta_i\delta_j\upsilon_i\upsilon_j\rho_{x_ix_j}C_{x_i}C_{x_j}\right) \quad (4.7)$$

$$MSE\left(T_{r4}^*\right)=\lambda_n\bar{Y}^2\left(C_y^2+\sum_{i=1}^{k}\theta_i^2\upsilon_i^2C_{x_i}^2+2\sum_{i=1}^{k}\theta_i\upsilon_i\rho_{yx_i}C_yC_{x_i}+2\sum_{i\neq j=1}^{k}\theta_i\theta_j\upsilon_i\upsilon_j\rho_{x_ix_j}C_{x_i}C_{x_j}\right) \quad (4.8)$$

Corollary 4.3

Minimum value of the mean square error is obtained as

$$MSE_{\min}\left(T_{ri}^*\right)=\lambda_n.\bar{Y}^2.C_y^2\left(1-R_{y.12.....k}^2\right)$$

where $R_{y.12....k}^2$ is the multiple correlation coefficient between y and $x_1,x_2,...,x_k$

5. Some members of the class of estimators T_{r3} & T_{r4}

It can be easily observed that the classes T_{r3} & T_{r4} are more generalized form of class of estimators in the sense of the constants such that $a_i(\neq 0),b_i$ are either real numbers or functions of the known parameters of the auxiliary variable x_i's such as the standard deviations S_x, coefficient of variation C_{x_i}, coefficient of kurtosis $\beta_2(x_i)$ and correlation coefficient $\rho_{x_ix_j}$ of the population. $(i\neq j=1,2,...,k)$ .Thus a wide range of estimators can be formed using these known population parameters. In this sub-section, some members of the suggested classes of estimators are given for the case of two auxiliary variables only. As an illustration the members belonging to the classes (3.3), (3.4), we describe the members of the class (3.3) only. The members of the remaining (3.4) can be written on similar lines. Some of them are listed below:

Table 1

Log type estimators T_{r3}	a_1	b_1	a_2	b_2
$T_{r3_1}^*=\bar{y}\left[1+\log\left(\frac{\bar{x}_1}{\bar{X}_1}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\bar{x}_2}{\bar{X}_2}\right)\right]^{\delta_2}$	1	0	1	0
$T_{r3_2}^*=\bar{y}\left[1+\log\left(\frac{\bar{x}_1+C_{x_1}}{\bar{X}_1+C_{x_1}}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\bar{x}_2+C_{x_2}}{\bar{X}_2+C_{x_2}}\right)\right]^{\delta_2}$	1	C_{x_1}	1	C_{x_2}

$T^*_{r3_3} = \bar{y}\left[1+\log\left(\frac{\beta_2(x_1)\bar{x}_1 + C_{x_1}}{\beta_2(x_1)\bar{X}_1 + C_{x_1}}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\beta_2(x_2)\bar{x}_2 + C_{x_2}}{\beta_2(x_2)\bar{X}_2 + C_{x_2}}\right)\right]^{\delta_2}$	$\beta_2(x_1)$	C_{x_1}	$\beta_2(x_2)$	C_{x_2}
$T^*_{r3_4} = \bar{y}\left[1+\log\left(\frac{C_{x_1}\bar{x}_1 + \beta_2(x_1)}{C_{x_1}\bar{X}_1 + \beta_2(x_1)}\right)\right]^{\delta_1}\left[1+\log\left(\frac{C_{x_2}\bar{x}_2 + \beta_2(x_2)}{C_{x_2}\bar{X}_2 + \beta_2(x_2)}\right)\right]^{\delta_2}$	C_x	$\beta_2(x_1)$	C_{x_2}	$\beta_2(x_2)$
$T^*_{r3_5} = \bar{y}\left[1+\log\left(\frac{\bar{x}_1 + S_{x_1}}{\bar{X}_1 + S_{x_1}}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\bar{x}_2 + S_{x_2}}{\bar{X}_2 + S_{x_2}}\right)\right]^{\delta_2}$	1	S_{x_1}	1	S_{x_2}
$T^*_{r3_6} = \bar{y}\left[1+\log\left(\frac{\beta_1(x_1)\bar{x}_1 + S_{x_1}}{\beta_1(x_1)\bar{X}_1 + S_{x_1}}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\beta_1(x_2)\bar{x}_2 + S_{x_2}}{\beta_1(x_2)\bar{X}_2 + S_{x_2}}\right)\right]^{\delta_2}$	$\beta_1(x)$	S_{x_1}	$\beta_1(x_2)$	S_{x_2}
$T^*_{r3_7} = \bar{y}\left[1+\log\left(\frac{\beta_2(x_1)\bar{x}_1 + S_{x_1}}{\beta_2(x_1)\bar{X}_1 + S_{x_1}}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\beta_2(x_2)\bar{x}_2 + S_{x_2}}{\beta_2(x_2)\bar{X}_2 + S_{x_2}}\right)\right]^{\delta_2}$	$\beta_2(x_1)$	S_{x_1}	$\beta_2(x_2)$	S_{x_2}
$T^*_{r3_8} = \bar{y}\left[1+\log\left(\frac{\bar{x}_1 + \rho}{\bar{X}_1 + \rho}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\bar{x}_2 + \rho}{\bar{X}_2 + \rho}\right)\right]^{\delta_2}$	1	ρ	1	ρ
$T^*_{r3_9} = \bar{y}\left[1+\log\left(\frac{\bar{x}_1 + \beta_2(x_1)}{\bar{X}_1 + \beta_2(x_1)}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\bar{x}_2 + \beta_2(x_2)}{\bar{X}_2 + \beta_2(x_2)}\right)\right]^{\delta_2}$	1	$\beta_2(x_1)$	1	$\beta_2(x_2)$
$T^*_{r3_{10}} = \bar{y}\left[1+\log\left(\frac{C_{x_1}\bar{x}_1 + \rho}{C_{x_1}\bar{X}_1 + \rho}\right)\right]^{\delta_1}\left[1+\log\left(\frac{C_{x_2}\bar{x}_2 + \rho}{C_{x_2}\bar{X}_2 + \rho}\right)\right]^{\delta_2}$	C_{x_1}	ρ	C_{x_2}	ρ
$T^*_{r3_{11}} = \bar{y}\left[1+\log\left(\frac{\rho\bar{x}_1 + C_{x_1}}{\rho\bar{X}_1 + C_{x_1}}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\rho\bar{x}_2 + C_{x_2}}{\rho\bar{X}_2 + C_{x_2}}\right)\right]^{\delta_2}$	ρ	C_{x_1}	ρ	C_{x_2}
$T^*_{r3_{12}} = \bar{y}\left[1+\log\left(\frac{\beta_2(x_1)\bar{x}_1 + \rho}{\beta_2(x_1)\bar{X}_1 + \rho}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\beta_2(x_2)\bar{x}_2 + \rho}{\beta_2(x_2)\bar{X}_2 + \rho}\right)\right]^{\delta_1}$	$\beta_2(x_1)$	ρ	$\beta_2(x_2)$	ρ
$T^*_{r3_{13}} = \bar{y}\left[1+\log\left(\frac{\rho\bar{x}_1 + \beta_2(x_1)}{\rho\bar{X}_1 + \beta_2(x_1)}\right)\right]^{\delta_1}\left[1+\log\left(\frac{\rho\bar{x}_2 + \beta_2(x_2)}{\rho\bar{X}_2 + \beta_2(x_2)}\right)\right]^{\delta_2}$	ρ	$\beta_2(x_1)$	ρ	$\beta_2(x_2)$

6. Comparison with the Available Estimators

In this section a comparison of the suggested classes of estimators with some of the known estimators in terms of biases and mean square error up to order of is given. We consider the following estimators:

(a) Mean per unit estimator
It is an unbiased estimator of population mean and its variance is given by,

$$Var(\bar{y}) = \lambda_n \bar{Y}^2 C_y^2$$

$$MSE(\bar{y}) - M = R^2_{y.12} C_y^2 \geq 0 \quad (6.1)$$

(b) Ratio estimator $\bar{y}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$

where x may be chosen as x_1 or x_2

$$MSE(\bar{y}_R) = \lambda_n \bar{Y}^2 \left(C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x\right)$$

$$MSE(\bar{y}_R) - M = \lambda_n \bar{Y}^2 \left[\left(C_x - \rho_{yx} C_y\right)^2 + \left(R^2_{y.12} - \rho^2_{yx}\right) C_y^2\right] \geq 0 \quad (6.2)$$

Since $R^2_{y.12} \geq \rho^2_{yx}$

(c) Product estimator $\bar{y}_P = \bar{y}\frac{\bar{x}}{\bar{X}}$

$$MSE(\bar{y}_P) = \lambda \bar{Y}^2 \left(C_y^2 + C_x^2 + 2\rho_{yx} C_y C_x\right)$$

$$MSE(\bar{y}_P) - M = \lambda_n \bar{Y}^2 \left[\left(C_x + \rho_{yx} C_y\right)^2 + \left(R^2_{y.12} - \rho^2_{yx}\right) C_y^2\right] \geq 0 \quad (6.3)$$

Since $R^2_{y.12} \geq \rho^2_{yx}$

(d) The usual regression estimator $\bar{y}_{lr} = \bar{y} + \hat{\beta}\left(\bar{X} - \bar{x}\right)$

$$MSE(\bar{y}_{lr}) = \lambda \bar{Y}^2 C_y^2 \left(1 - \rho^2_{yx}\right)$$

$$MSE(\bar{y}_{lr}) - M = \lambda_n \bar{Y}^2 \left[\left(R^2_{y.12} - \rho^2_{yx}\right) C_y^2\right] \geq 0 \quad (6.4)$$

Since $R^2_{y.12} \geq \rho^2_{yx}$

(e) Log-type estimators

$$T_{G1} = \bar{y}\left[1+\log\left(\frac{\bar{x}}{\bar{X}}\right)\right]^{\alpha_1}$$

$$T_{G2} = \bar{y}\left[1+\alpha_2 \log\left(\frac{\bar{x}}{\bar{X}}\right)\right]$$

$$T_{G3} = \bar{y}\left[1+\log\left(\frac{\bar{x}^*}{\bar{X}^*}\right)\right]^{\alpha_3}$$

$$T_{G4} = \bar{y}\left[1+\alpha_4 \log\left(\frac{\bar{x}^*}{\bar{X}^*}\right)\right]$$

$$MSE_{\min}\left(T_{Gi}\right) = \lambda_n.\bar{Y}^2.C_y^2\left(1-\rho_{yx}^2\right) \qquad i=1,2,3,4$$

$$MSE\left(T_{Gi}\right) - M = \lambda_n \bar{Y}^2\left[\left(R_{y.12}^2 - \rho_{yx}^2\right)C_y^2\right] \geq 0 \qquad (6.5)$$

Since $R_{y.12}^2 \geq \rho_{yx}^2$

7. Numerical Illustration

The comparison among these estimators is given in this section using a real data set. The data for this study is taken from [1], District Handbook of Aligarh, India. The population contains 332 villages. A simple random sample 80 villages is taken for the study. We consider the variables Y , X_1 and X_2 as the number of cultivators, area of the village and number of household in the village respectively. We compute the bias and the MSE for all estimators. The following values were obtained using the whole data sets:

$\bar{Y} = 1093.1$, $\bar{X}_1 = 181.57$, $\bar{X}_2 = 143.31$

$C_y = 0.7626$, $C_{x_1} = 0.7684$, $C_{x_2} = 0.7616$

$\rho_{yx_1} = 0.973$, $\rho_{yx_2} = 0.862$, $\rho_{x_1x_2} = 0.842$

Using the above results we have calculated the MSE and PRE for all the estimators in section 2. The PRE for each estimator with respect to the sample mean of a SRS is defined as follows:

$$e\left(y'\right) = \left[\frac{MSE\left(\bar{y}\right)}{MSE\left(y'\right)}\right]*100$$

Table 2

Estimators	Auxiliary Variables	MSE	Percent Relative Efficiency (PRE)
$\bar{y}$	None	6593.042	100
$\bar{y}_R$	x_1	359.1134	1835.922
$\bar{y}_R$	x_2	1817.305	362.7923
$\bar{y}_P$	x_1	26214.39	25.15047
$\bar{y}_P$	x_2	24520.31	26.88809
$\bar{y}_{lr}$	x_1	351.218	1877.194
$\bar{y}_{lr}$	x_2	1694.122	389.1717
T_{Gi}	x_1	351.218	1877.194
T_{Gi}	x_2	1694.122	389.1717
T_{ri}	x_1, x_2	309.8479	2127.832

8. Conclusion

The present study extends the idea regarding the effective use of single auxiliary information on variable to the use of multiple auxiliary information. The biases and mean square error of the suggested classes of estimators are obtained up to the terms of order n^{-1}. It has been shown that the suggested classes have minimum MSE(s) as compared to some renowned estimators available in the literature of sampling. The results are quite convincing both theoretically and empirically.

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